

M.A. Gordon
R.L. Sorochenko

Radio Recombination Lines

Their Physics and Astronomical
Applications

AS
SL

 Springer

Radio Recombination Lines

Astrophysics and Space Science Library

EDITORIAL BOARD

Chairman

W. B. BURTON, *National Radio Astronomy Observatory, Charlottesville, Virginia, U.S.A.* (bburton@nrao.edu) and *University of Leiden, The Netherlands* (burton@strw.leidenuniv.nl)

F. BERTOLA, *University of Padua, Italy*

J. P. CASSINELLI, *University of Wisconsin, Madison, U.S.A.*

C. J. CESARSKY, *European Southern Observatory, Garching bei München, Germany*

P. EHRENFREUND, *Leiden University, The Netherlands*

O. ENGVOLD, *University of Oslo, Norway*

A. HECK, *Strasbourg Astronomical Observatory, France*

E. P. J. VAN DEN HEUVEL, *University of Amsterdam, The Netherlands*

V. M. KASPI, *McGill University, Montreal, Canada*

J. M. E. KUIJPERS, *University of Nijmegen, The Netherlands*

H. VAN DER LAAN, *University of Utrecht, The Netherlands*

P. G. MURDIN, *Institute of Astronomy, Cambridge, UK*

F. PACINI, *Istituto Astronomia Arcetri, Firenze, Italy*

V. RADHAKRISHNAN, *Raman Research Institute, Bangalore, India*

B. V. SOMOV, *Astronomical Institute, Moscow State University, Russia*

R. A. SUNYAEV, *Space Research Institute, Moscow, Russia*

Radio Recombination Lines

Their Physics and Astronomical Applications

by

M.A. Gordon

and

R.L. Sorochenko



Springer

M.A. Gordon
National Radio Astronomy Observatory
Tucson
NRAO
949 North Cherry Avenue
Tucson AZ 85721-0655
USA
mgordon@nrao.edu

R.L. Sorochenko
Russian Academy of Sciences
P.N. Lebedev Physical Institute
Laninsky Prospect 53
Moskva
Russia 119991

ISSN: 0067-0057

ISBN: 978-0-387-09604-9

e-ISBN: 978-0-387-09691-9

DOI: 10.1007/978-0-387-09691-9

Library of Congress Control Number: 2008931183

© Springer Science+Business Media, LLC 2009

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC, 233 Spring Street, New York, NY 10013, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use in this publication of trade names, trademarks, service marks, and similar terms, even if they are not identified as such, is not to be taken as an expression of opinion as to whether or not they are subject to proprietary rights.

Printed on acid-free paper

springer.com

*To our children and grandchildren, to
their children, and to all of the next
generations, who may benefit from a
deeper understanding of the universe in
which they live.*

Preface

Recombination lines at radio wavelengths have been – and still are – a powerful tool for modern astronomy. For more than 30 years, they have allowed astronomers to probe the gases from which stars form. They have even been detected in the Sun.

In addition, observations of these spectral lines facilitate basic research into the atom, in forms and environments that can only exist in the huge dimensions and extreme conditions of cosmic laboratories.

We intend this book to serve as a tourist's guide to the world of Radio Recombination Lines. It contains three divisions: a history of their discovery, the physics of how they form and how their voyage to us influences their spectral profiles, and a description of their many astronomical contributions to date. The appendix includes supplementary calculations that may be useful to some astronomers. This material also includes tables of line frequencies from 12 MHz to 30 THz ($\lambda = 10\ \mu\text{m}$) as well as FORTRAN computer code to calculate the fine-structure components of the lines, to evaluate radial matrix integrals, and to calculate the departure coefficients of hydrogen in a cosmic environment. It also describes how to convert observational to astrophysical units. The text includes extensive references to the literature to assist readers who want more details.

We appreciate the help of L.W. Avery, D.S. Balser, T.M. Bania, T. Bastian, J.H. Bieging, H.J.J. Blom, N.G. Bochkarev, R.L. Brown, L.A. Bureeva (Minaeva), W.B. Burton, T. Alan Clark, Z.F. Dravskikh, W.C. Erickson, P.A. Feldman, H.C. Goldwire, W.M. Goss, H.R. Griem, C. Heiles, D. Hoang-Binh, D. Hollenbach, S. Japenga, V.S. Lisitsa, F.J. Lockman, J.M. Moran, P. Palmer, Y.N. Parijskij, H.E. Payne, P.A. Pinto, V.O. Ponomarev, B. Robinson, A. Rosh, E.R. Seaquist, M. Simon, P.J. Smiley, G.T. Smirnov, I.I. Sobelman, V.S. Strel'nitski, Y. Terzian, R.I. Thompson, J.K.G. Watson, A. Wettstein, and T.L. Wilson.

We are grateful to N.S. Kardashev and P.A. Vanden Bout for providing the support that enabled us to write this book.

This book was written mainly by exchanging Email; we authors did not meet during this collaboration. We thank V.R. Sorochenko (RLS's physicist son) for his great help with these communications as well as for his draft translation of the Russian part of the text.

Contents

| | | |
|----------|---|-----------|
| 1 | Introduction | 1 |
| 1.1 | The Cosmos as a Laboratory | 1 |
| 1.2 | Spectral Lines in Astronomy | 1 |
| 1.3 | The Bohr Atom | 3 |
| 1.3.1 | Bohr Lines at Radio Wavelengths | 6 |
| 1.3.2 | Other Line Series | 6 |
| 1.4 | Spectral Lines in Radio Astronomy | 7 |
| 1.4.1 | Theoretical Studies | 7 |
| 1.4.2 | Detection of Radio Recombination Lines | 12 |
| 1.4.3 | Other Searches and Detections | 17 |
| 2 | RRLs and Atomic Physics | 23 |
| 2.1 | The First Surprising Results: The Absence of Stark Broadening | 23 |
| 2.2 | The Broadening of Radio Recombination Lines | 25 |
| 2.2.1 | Natural Broadening | 25 |
| 2.2.2 | Doppler Broadening | 28 |
| 2.2.3 | Stark Broadening of RRLs | 30 |
| 2.2.4 | Elastic and Inelastic Impact Broadening | 43 |
| 2.2.5 | Combining Profiles: The Voigt Profile | 46 |
| 2.2.6 | Observational Test of the Revised Theory | 49 |
| 2.3 | Intensity of Radio Recombination Lines | 56 |
| 2.3.1 | Radiation Transfer | 56 |
| 2.3.2 | Continuum Emission | 61 |
| 2.3.3 | Transfer Equation for Continuum Radiation | 64 |
| 2.3.4 | Comparison with Continuum Observations | 65 |
| 2.3.5 | Line Absorption and Emission Coefficients | 67 |
| 2.3.6 | Transfer Equation for RRLs | 71 |
| 2.3.7 | The First Measurements of RRL Intensity | 72 |
| 2.3.8 | Departures from LTE | 74 |
| 2.3.9 | Non-LTE Line Intensities | 76 |

| | | |
|----------|--|------------|
| 2.3.10 | Calculating Departure Coefficients | 78 |
| 2.3.11 | Line Intensities in Terms of Transfer Theory | 86 |
| 2.3.12 | Line Enhancement: A More General View | 88 |
| 2.3.13 | Classification of a Non-LTE Transition | 91 |
| 2.4 | The Range of RRL Studies | 94 |
| 2.4.1 | High-Frequency RRLs | 94 |
| 2.4.2 | Low-Frequency RRLs | 99 |
| 2.5 | How Many Atomic Levels Can Exist? | 107 |
| 2.5.1 | Radiation Broadening of RRLs | 108 |
| 2.5.2 | Existence as well as Detectability | 111 |
| 2.6 | Summary | 113 |
| 3 | RRLs: Tools for Astronomers | 117 |
| 3.1 | Physical Conditions in H II Regions | 121 |
| 3.1.1 | Electron Temperature of H II Regions | 121 |
| 3.1.2 | Electron Density of H II Regions | 131 |
| 3.1.3 | Velocities of Turbulent Motion | 134 |
| 3.2 | Ionized Hydrogen and Helium in the Galaxy | 135 |
| 3.2.1 | Distribution of H II Regions | 135 |
| 3.2.2 | Low-Density Ionized Hydrogen | 141 |
| 3.2.3 | Thickness of the Ionized Hydrogen Layer | 152 |
| 3.2.4 | Helium in the Galaxy | 155 |
| 3.3 | Exploration of the Cold ISM by RRLs | 165 |
| 3.3.1 | C II Regions at the Boundaries of ISM Clouds | 166 |
| 3.3.2 | C II Regions: Information from Carbon RRLs | 170 |
| 3.3.3 | The Relationship Between H II, H ⁰ , and Molecular Gas | 176 |
| 3.3.4 | C RRLs, IR Fine-Structure Lines of C ⁺ , and O I Lines | 182 |
| 3.3.5 | Carbon RRLs from Atomic and Molecular Clouds | 187 |
| 3.3.6 | Estimates of the Galactic Cosmic Ray Intensity | 209 |
| 3.4 | RRLs from Stars and Stellar Envelopes | 213 |
| 3.4.1 | Planetary Nebulae | 213 |
| 3.4.2 | The Sun | 218 |
| 3.4.3 | MWC349 | 222 |
| 3.5 | RRLs from Extragalactic Objects | 231 |
| | Appendixes | 239 |
| | Appendix A Constants | 241 |
| A.1 | Miscellaneous Constants | 241 |
| A.2 | Rydberg Constants | 241 |
| A.2.1 | Reduced Mass | 241 |
| A.2.2 | Table of Rydberg Constants | 243 |

| | |
|---|-----|
| Appendix B Tables of Line Frequencies | 245 |
| B.1 Frequencies Below 100 GHz | 245 |
| B.2 Frequencies Above 100 GHz | 258 |
| B.3 FORTRAN Code for Fine-Structure Frequencies | 274 |
| Appendix C Supplemental Calculations | 281 |
| C.1 Early Estimates of Stark Broadening | 281 |
| C.2 Refinements to the Bohr Model | 283 |
| Appendix D Hydrogen Oscillator Strengths | 285 |
| D.1 Population of Atomic Sublevels | 285 |
| D.2 Calculation of Oscillator Strengths | 286 |
| D.3 Radial Matrix Integrals Code | 288 |
| D.3.1 Radial Matrix Integrals $\Re_H(n_2, \ell_1 - 1; n_1, \ell_1)$ | 288 |
| D.3.2 Radial Matrix Integrals $\Re_H(n_2, \ell_1 + 1; n_1, \ell_1)$ | 292 |
| Appendix E Departure Coefficients | 297 |
| E.1 FORTRAN Code for Calculating b_n Values | 297 |
| Appendix F Observational Units | 329 |
| F.1 What Radio Telescopes Measure | 329 |
| F.2 How Radio Telescopes Measure | 330 |
| F.2.1 Sources Smaller Than the Beam Size | 330 |
| F.2.2 Sources Larger Than the Beam Size | 334 |
| F.2.3 Antenna Temperature Scale | 336 |
| References | 339 |
| Author Index | 353 |
| Subject Index | 359 |

List of Figures

| | | |
|------|---|----|
| 1.1 | Predictions of RRL strengths | 10 |
| 1.2 | Predicted line brightness temperatures vs. wavelength | 11 |
| 1.3 | Tentative spectra of radio recombination lines | 13 |
| 1.4 | The 22 m radio telescope at Pushchino | 15 |
| 1.5 | First detections of radio recombination lines | 16 |
| 1.6 | The NRAO 140-ft. radio telescope | 18 |
| 1.7 | Control room of NRAO 140-ft. telescope | 19 |
| 1.8 | Confirmation of RRLs, 1965 | 20 |
| | | |
| 2.1 | Predictions of RRL line widths | 24 |
| 2.2 | Lorentz line profile | 26 |
| 2.3 | Gaussian line profile | 29 |
| 2.4 | Turbulence widths of RRLs | 31 |
| 2.5 | Wave train distorted by phase shifts | 32 |
| 2.6 | Impact geometry | 33 |
| 2.7 | Compensation mechanism of RRL Stark broadening | 41 |
| 2.8 | Stark broadening of RRLs | 45 |
| 2.9 | Comparison of Gaussian, Lorentz, and Voigt profiles | 49 |
| 2.10 | Orion line widths | 50 |
| 2.11 | The 100-m radio telescope at Effelsberg, Germany | 53 |
| 2.12 | Variation of Voigt profiles with principal quantum number | 54 |
| 2.13 | Stark-induced widths of Lorentz profiles for RRLs | 54 |
| 2.14 | Basic radiation transfer | 57 |
| 2.15 | Geometry of specific intensity | 59 |
| 2.16 | Continuum emission from W3 | 62 |
| 2.17 | Continuum emission from the Orion nebula | 66 |
| 2.18 | Linear absorption coefficients | 70 |
| 2.19 | Ratio of line-to-continuum emission | 73 |
| 2.20 | Temperatures calculated from initial detections | 73 |
| 2.21 | Typical departure coefficients for H II regions | 83 |
| 2.22 | Evolution of departure coefficients | 84 |

| | | |
|------|--|-----|
| 2.23 | H departure coefficients at small values of n | 86 |
| 2.24 | Amplification of line intensities | 87 |
| 2.25 | Variation of opacities | 89 |
| 2.26 | Variation of line gain with n | 90 |
| 2.27 | Non-LTE line excitation as a function of population ratio | 92 |
| 2.28 | Excitation regions of RRL line with respect to n | 93 |
| 2.29 | First millimeter wave recombination line | 95 |
| 2.30 | Recombination lines from MWC349 | 96 |
| 2.31 | Net absorption coefficient for α -type RRLs | 98 |
| 2.32 | The $1,800 \times 900$ m UTR-2 Kharkov radio telescope | 100 |
| 2.33 | Low-frequency carbon RRLs toward Cassiopeia A | 101 |
| 2.34 | Density limits for low-frequency RRLs | 103 |
| 2.35 | Non-LTE coefficients for low-temperature C and H | 105 |
| 2.36 | Carbon RRLs detected toward Cassiopeia A | 106 |
| 2.37 | Interline spacing vs. line widths | 111 |
| 2.38 | Superposition of low-frequency RRLs | 112 |
| 2.39 | Carbon β RRLs detected toward Cassiopeia A | 113 |
| 2.40 | Range of detected RRLs | 114 |
| | | |
| 3.1 | The range of objects studied with RRLs | 119 |
| 3.2 | Generalized spectrum of RRLs toward an H II region | 120 |
| 3.3 | L/C ratios of hydrogen RRLs from the Orion nebula | 122 |
| 3.4 | T_e/T_e^* as a function of frequency and emission measure | 124 |
| 3.5 | Frequency vs. emission measure for $T_e/T_e^* = 1$ | 125 |
| 3.6 | T_e/T_e^* vs. observing frequency | 126 |
| 3.7 | Galactocentric gradient of T_e | 129 |
| 3.8 | Galactocentric variation of O and N | 130 |
| 3.9 | Temperature gradient of ultracompact H II regions | 131 |
| 3.10 | Velocity gradients across Orion A | 135 |
| 3.11 | Differential rotation of the Milky Way Galaxy | 137 |
| 3.12 | Distance ambiguity of Galactic radial velocities | 138 |
| 3.13 | Galactic spiral arms determined from H II regions | 140 |
| 3.14 | Galactocentric distribution of H II regions | 141 |
| 3.15 | Galactocentric distribution of hydrogen | 142 |
| 3.16 | Detection of H157 α RRLs from diffuse gas | 142 |
| 3.17 | H166 α RRLs from diffuse gas | 146 |
| 3.18 | H166 α RRLs from diffuse gas compared with H I emission | 147 |
| 3.19 | Locus of low-frequency RRLs from the Galaxy | 148 |
| 3.20 | Ionized gas in the Galaxy | 150 |
| 3.21 | Evolution of naked OB associations | 151 |
| 3.22 | A supernovae-driven “chimney” in the ISM | 154 |
| 3.23 | First detection of helium RRLs | 155 |
| 3.24 | Variation of He and H ionizing photons with Stellar type | 157 |
| 3.25 | Distribution of y^+ in the H II region W3A | 159 |
| 3.26 | Theoretical enhancement zone for y^+ | 161 |

| | | |
|------|---|-----|
| 3.27 | $N(\text{He}^+)/N(\text{H}^+)$ as a function of Galactocentric radius | 162 |
| 3.28 | Spectra of $^3\text{He II}$ and RRLs | 164 |
| 3.29 | Detection of the $\text{C109}\alpha$ line in NGC2024 | 166 |
| 3.30 | The 166α of NGC2024 and W3 | 168 |
| 3.31 | Structure of a photodissociation region | 169 |
| 3.32 | Sulfur and carbon 166α recombination lines | 173 |
| 3.33 | Champagne flow in a star-forming region | 178 |
| 3.34 | The bar structure in the Orion nebula | 180 |
| 3.35 | 56α lines of hydrogen, helium, and carbon | 185 |
| 3.36 | Graphical determination of N_H and T in PDRs | 186 |
| 3.37 | Two carbon RRLs toward Cas A | 188 |
| 3.38 | Carbon line widths in Cas A as a function of n | 189 |
| 3.39 | ISM carbon RRL intensities in Cas A as a function of n | 192 |
| 3.40 | Comparison of $\text{C221}\alpha$ and molecular spectra toward Cas A | 195 |
| 3.41 | Structure of the Perseus arm | 198 |
| 3.42 | Radial velocities toward the Perseus arm from a TASS wave | 199 |
| 3.43 | A model of the interstellar material toward Cas A | 200 |
| 3.44 | Carbon RRLs toward the Galactic center | 201 |
| 3.45 | Carbon RRLs toward six directions of the Galactic plane | 202 |
| 3.46 | The carbon $\lambda = 158\text{ }\mu\text{m}$ line in the Galactic plane | 203 |
| 3.47 | Carbon $\lambda = 158\text{ }\mu\text{m}$ emission from NGC6334 | 204 |
| 3.48 | $\text{C}^+ \lambda = 158\text{ }\mu\text{m}$ emission compared with other observations | 205 |
| 3.49 | The averaged RRL spectrum of Cas A at 42 MHz | 212 |
| 3.50 | Continuum spectra of planetary nebulae | 215 |
| 3.51 | Departure coefficients for Fe XV in the Sun | 219 |
| 3.52 | $\text{H19}\alpha$ and $\text{Mg19}\alpha$ <i>radio</i> recombination lines in the Sun | 221 |
| 3.53 | Continuum spectrum of MWC349 | 222 |
| 3.54 | Radio recombination lines from MWC349 | 224 |
| 3.55 | Intensity variations of the $\text{H30}\alpha$ line from MWC349 | 225 |
| 3.56 | Velocity variations of the $\text{H30}\alpha$ line from MWC349 | 226 |
| 3.57 | Approximate locations of the MWC346 line peaks | 227 |
| 3.58 | Velocities of $\text{H}n\alpha$ recombination lines from MWC349A | 228 |
| 3.59 | Evaporating disk models for ultracompact H II regions | 228 |
| 3.60 | Possible outflow from MWC349A | 229 |
| 3.61 | Downward view of the circumstellar disk of MWC349A | 230 |
| 3.62 | $\text{H110}\alpha$ line from the galaxy M82 | 232 |
| 3.63 | Continuum image of NGC3628 | 233 |
| 3.64 | Higher-resolution image of NGC3628 | 234 |
| 3.65 | Comparison of $\text{H41}\alpha$ and HCO^+ (1–0) emission in M82 | 237 |
| B.1 | Fine-structure components of the $\text{H19}\alpha$ line | 274 |
| C.1 | van de Hulst’s calculations of I_L/I_C | 282 |
| D.1 | Critical electron density for LTE sublevels | 286 |

List of Tables

| | | |
|-----|--|-----|
| 3.1 | Electron temperatures derived from H II regions | 127 |
| 3.2 | Electron densities of H II regions | 133 |
| 3.3 | Velocity components affecting observed frequencies | 136 |
| 3.4 | Physical conditions of C II boundary regions from C RRLs | 172 |
| 3.5 | Observed parameters of the C166 α and S166 α lines | 173 |
| 3.6 | H II region radial velocities from various spectral lines | 177 |
| 3.7 | Characteristics of C II regions | 187 |
| 3.8 | Some planetary nebulae with detected RRLs | 216 |
| 3.9 | RRLs detected from extragalactic objects | 235 |
| | | |
| A.1 | Recommended values of physical units | 242 |
| A.2 | Masses and Rydberg constants | 242 |
| | | |
| B.1 | H, ^4He , ^{12}C , and ^{32}S RRL frequencies below 100 GHz | 245 |
| B.2 | Rydberg H, ^4He , ^{12}C , and ^{32}S frequencies above 100 GHz | 259 |
| B.3 | Dirac hydrogen RRL frequencies above 100 GHz | 269 |
| | | |
| F.1 | Beam shape correction factor | 332 |
| F.2 | Illumination taper factor | 333 |
| F.3 | Main beam efficiencies for the IRAM 30-m telescope | 337 |

Chapter 1

Introduction

Abstract This chapter describes the early theory and initial detection of radio recombination lines from astronomical objects. The focus is historical.

1.1 The Cosmos as a Laboratory

The history of science shows many close connections between physics and astronomy. It is well known that a number of physical laws evolved from a base of astronomical observations. For example, Kepler observed and, later Newton derived, the laws of gravitation while studying the motion of planets and their satellites. The existence of thermonuclear energy was solidly established when it explained the energy balance of the Sun and stars. The anomalous shift of Mercury's perihelion showed us that Newton's gravitational theory was incomplete; this observation helped lead Einstein to the more comprehensive theory of General Relativity.

The cosmos is a wonderful laboratory. There, physicists find that matter can have very high and very low temperatures. It can have ultrahigh and ultralow densities. It can occupy huge volumes. It can exist in states impossible to duplicate in a terrestrial laboratory – states that are not always in dynamical or thermal equilibrium. This extreme diversity of matter in the cosmos is one of the reasons that astronomical observations and astrophysical studies are so valuable.

1.2 Spectral Lines in Astronomy

Low-density cosmic matter gives us a unique opportunity to study elementary processes in atoms and molecules by means of the phenomenon of spectral lines. This is important for physics. It is worthwhile to remember that the

first spectral lines – the Fraunhofer lines – were first detected in astronomic objects, in the spectra of the Sun and the stars. These observations stimulated the development of laboratory spectroscopy.

Emission of spectral lines from cosmic objects became an essential tool in astronomy. The frequency of each line is unique and identifies the atom, ion, or molecule emitting that radiation. Knowing the line frequency through laboratory measurements or through calculations, astronomers can determine the velocity shift of the line and, by local kinematics and by the Hubble law, estimate the distance of the emitting region. The line intensities are related to the number of atoms along the line of sight within the telescope’s field of view. The line widths are produced by a combination of the motion of the emitting atoms, of perturbations to the radiation induced by magnetic fields, and by the difficulty that the photons experienced passing through the medium. In this way, the line shapes are the record of what the photons experienced when they were created and in their voyage to us.

The opportunities to investigate spectral lines in astronomy broadened considerably with the extension of astronomical observations into the radio regime, now known as “radio astronomy.” One enormous advantage of the radio regime relative to the optical was that the spectral window could be shifted from high to low frequencies, thereby obtaining high spectral resolution at easily managed frequencies. Called “superheterodyne” conversion, this process was developed in the early 1900s to enhance radio receivers for communications. Implementing this technique in the optical regime involves solving difficult physics problems. At present, only limited applications exist.

Spectral lines from a great number of cosmic atoms and molecules are now available throughout the electromagnetic spectrum. In this book, we consider a special class of these spectral lines, namely, spectral lines resulting from transitions between highly excited atomic levels. Conceptually, these lines appear after the recombination of ions and electrons to form atoms, leaving the electrons in levels with high principal quantum numbers n . These newly bound electrons jump downward from level to level much like going down a flight of stairs, losing energy in each jump by radiating it away in the form of a spectral line. When these lines appear in radio regime, they are called “radio recombination lines” (RRLs).

The study of RRLs has revealed a number of surprising new concepts for physics and astronomy. For example, in ultralow-density regions of the interstellar medium (ISM), an atom can exist with electrons in very high quantum levels – up to $n \approx 1,000$ and, correspondingly, with huge diameters approaching 0.1 mm. We can observe the spectral lines from these giant atoms over a wide range of radio waves, from millimeter to decameter wavelengths. Because interstellar atoms are sensitive to variations in gas densities and temperatures in any region, their RRL emission sends us information about the structure of their cosmic environments. And, as we shall see, the basic physics underlying these atomic lines are easy to understand.

1.3 The Bohr Atom

To understand the early searches for RRLs, we first need to discuss a basic physics model known today as the “Bohr atom.” This model explains atomic emission lines in a simple way.

Line radiation caused by transitions between atomic levels was detected about 100 years ago. These lines were grouped into series such as the then well-known Lyman, Balmer, and Paschen line series emitted by hydrogen in the ultraviolet (UV), visible, and infrared (IR) wavelength ranges. Physicists soon found empirically that the frequencies ν of the lines in these series could be represented by a simple formula:

$$\nu_{n_2 \rightarrow n_1} = Rc \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad n_2 > n_1 > 0, \quad (1.1)$$

where n_1 and n_2 are positive integers, c is the speed of light, and the constant R was called the Rydberg constant. Each line series could be fitted by choosing a value for n_1 and then sequentially entering values for n_2 . For example, $n_1 = 1$ would give the Lyman series; $n_1 = 2$, the Balmer series; $n_1 = 3$, the Paschen series; and so on. Examination of (1.1) shows that the lines of each series become closer together as n_2 increases, forming a “series limit” when $n_2 \Rightarrow \infty$ of

$$\nu_{n_2=\infty \rightarrow n_1} = \frac{Rc}{n_1^2} \quad (1.2)$$

beyond which the lines become a continuum clearly visible on the spectral plates.

What are the physics behind these empirical formulas? From these observations, Bohr (1913) developed his quantum theory of the atom – a mathematically simple theory that explained most of the series of atomic lines known at that time.

In this theory, Bohr postulated that atoms have discrete stationary energy levels; in other words, these energy levels are “quantized” rather than continuous. One can imagine a set of orbits of electrons circulating around the nucleus at quantized radii. Introducing discrete quantum numbers for angular momentum, Bohr assumed that only those orbits can exist for which the angular momentum L is a multiple of $h/2\pi$, i.e., described by following expression:

$$L = n \frac{h}{2\pi}, \quad n > 0 \quad (1.3)$$

$$= 1.0545919 \times 10^{-27} n \quad \text{erg sec}, \quad (1.4)$$

where h is a Planck’s constant and n is any positive integer. Bohr’s formulation allowed orbits of discrete diameters $2a$ given by

$$2a = \frac{n^2 h^2}{2\pi^2 m Z e^2} \quad (1.5)$$

$$= 1.05835 \times 10^{-8} n^2 \text{ cm}, \quad (1.6)$$

where m is the mass of the electron, e is the electronic charge in ESU, and Ze is the charge of the nucleus. Equation (1.6) indicates that the sizes of orbits as well as atom's sizes increase as n^2 . Setting $n = 1$ and $Z = 1$ produces the radius of the first orbit of hydrogen, known as the “Bohr radius,”

$$a_0 = \frac{h^2}{4\pi^2 m e^2} \quad (1.7)$$

often used as a parameter in equations involving atomic physics.

Classical electrodynamics predicted orbital diameters by equating the electrical attraction between each electron and the nucleus to the centripetal acceleration:

$$\frac{Ze^2}{a^2} = \frac{mv^2}{a}, \quad \text{or} \quad (1.8)$$

$$2a = \frac{2Ze^2}{mv^2}, \quad (1.9)$$

so that every orbital diameter would be allowed depending upon the orbital speed v or the kinematic energy mv^2 of the electron.

The total energy E of an electron in a circular orbit is the sum of the electrical potential and the kinetic energy:

$$E = -\frac{Ze^2}{a} + \frac{1}{2}mv^2 \quad (1.10)$$

$$= -\frac{Ze^2}{a} + \frac{Ze^2}{2a} \quad (1.11)$$

$$= -\frac{Ze^2}{2a} \quad (1.12)$$

after substitution of (1.9). Using the quantization of orbits described by (1.6), Bohr calculated the energy E_n associated with each electronic orbit¹ n :

$$E_n = -\frac{2\pi^2 m e^4}{h^2} \frac{Z^2}{n^2} \quad (1.13)$$

$$= -2.17989724 \times 10^{-11} \frac{Z^2}{n^2} \text{ ergs.} \quad (1.14)$$

¹ A fundamental difference between the Bohr theory and classical electrodynamics is that in the Bohr theory, electrons do not radiate even though they are technically accelerating by changing direction.

Note that the energy of bound electrons must be negative. Because the energy of a photon is $h\nu$, the frequency of each atomic line would then be

$$\nu_{n_2 \rightarrow n_1} = \frac{E_{n_2} - E_{n_1}}{h}, \quad \text{or} \quad (1.15)$$

$$= \frac{2\pi^2 m Z^2 e^4}{h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (1.16)$$

$$= R c Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \quad (1.17)$$

which is identical with the empirical formula given by (1.1) if the effective nuclear charge $Z = 1$ and the Rydberg constant is

$$R = \frac{2\pi^2 m e^4}{h^3 c}. \quad (1.18)$$

Substituting into (1.18) the values of the physical constants listed in Table A.1, we derive $R = 109,737.35 \text{ cm}^{-1}$ which is close to the value of $109,675 \text{ cm}^{-1}$ obtained by Rydberg (1890) from measurements of hydrogen spectral lines. Such close agreement leaves no doubt regarding the validity of (1.14).

Later, Bohr (1914) did even better. The original theory assumed an infinitely small electronic mass orbiting the nucleus. He refined his earlier equations to use the center of mass as the centroid of the orbit and the reduced mass m_R in place of the orbiting electronic mass, so that

$$R = R_\infty \left(\frac{M}{M + m} \right), \quad (1.19)$$

where the coefficient

$$R_\infty = \frac{2\pi^2 m e^4}{ch^3} \quad (1.20)$$

and is now called the Rydberg constant for infinite mass. Section A.2.1 gives details. With this correction, the Rydberg constant for hydrogen $R_H = 109,677.57 \text{ cm}^{-1}$. The calculated and measured values of R_H now agreed within 0.002%.

There are additional refinements to the Bohr model that improve generality. These include consideration of elliptical orbits and the quantization of angular momentum. Section C.2 describes these calculations in detail.

Although our discussion has so far concentrated on hydrogen, these equations can also describe RRL spectra of multielectron atoms *and* ions. This “hydrogenic” model assumes that only one electron is in an excited level; the $Z - 1$ other electrons lie in or near ground levels. For neutral atoms, the net negative charge of the inner electrons would screen the positive charge of the nucleus, so that a lone outer electron would see only a single nuclear

charge and $Z = 1$. For ions, a similar situation would obtain but with $Z > 1$. Table A.2 gives Rydberg constants for a few atoms² common to the cosmos.

1.3.1 Bohr Lines at Radio Wavelengths

The theory did not restrict the number of atomic levels nor the number of the line series. Bohr (1914) showed that, for large quantum numbers and for transitions from $n_2 = n + 1 \rightarrow n_1 = n$, (A.6) gives a series of line frequencies for Bohr lines of neutral hydrogen:³

$$\begin{aligned}\nu_H &\approx \frac{2R_H c Z^2}{n^3}, \quad n \gg 1 \\ &= 6.58 \times 10^{15} \frac{1}{n^3} \text{ Hz}\end{aligned}\tag{1.22}$$

with an accuracy of about 2–3% depending upon the frequency. Although unrealized at the time, substituting values of, say, $100 < n < 200$ into (1.22) will yield approximate frequencies for lines throughout the radio range.

1.3.2 Other Line Series

Bohr's model was a brilliant success. It not only explained the hydrogen line series observed up to the year 1913 but predicted new lines as well. However, research into spectral lines toward longer wavelengths proceeded slowly. The fourth atomic series for hydrogen with $n_1 = 4$ and the first line $\lambda_{5 \rightarrow 4} = 4.05 \mu\text{m}$ was detected by Brackett (1922) 9 years after Bohr's theory had appeared; the fifth series with $n_1 = 5$ and the first line $\lambda_{6 \rightarrow 5} = 7.46 \mu\text{m}$, by Pfund (1924) 11 years after; and the sixth series with $n_1 = 6$ and the first

² A property of the Bohr line series expressed by (1.17) and (1.19) is that the entire line series can be shifted in frequency by changing the Rydberg constant R . Radial velocities will cause similar shifts. This means that identification of the atomic species emitting these lines in a cosmic environment, in principle, cannot be made simply on the basis of the observed frequencies – as can be done for molecular emission lines with their less regularly spaced frequencies. In practice, radial velocities for observed optical and molecular lines along the same sight lines help identification of the atomic species of Bohr lines from cosmic gas.

³ The approximation comes from the first term of the binomial expansion of (1.17) when $n_2 \equiv n_1 + \Delta n$:

$$\nu_H \approx 2R_H c Z^2 \frac{\Delta n}{n^3} \left[1 - \frac{3}{2} \left(\frac{\Delta n}{n} \right) + 2 \left(\frac{\Delta n}{n} \right)^2 - \frac{5}{2} \left(\frac{\Delta n}{n} \right)^3 + \cdots \right], \quad \Delta n \ll n.\tag{1.21}$$

line $\lambda_{7\rightarrow 6} = 12.3 \mu\text{m}$, by Humphreys (1953) 40 years later as a result of a very fine measurements of spectra in a gas discharge.

With these studies, classical laboratory spectroscopy ran out of ability. Only new techniques could find new series and answer the question of how far the theory could go. In this quest, the frequencies of the lines moved from the optical through the infrared into the radio regime. Here, astronomy and, more exactly, its rapidly developing branch of radio astronomy came to the aid of physics.

1.4 Spectral Lines in Radio Astronomy

1.4.1 Theoretical Studies

The Dutch astronomer, van de Hulst (1945), was the first to consider the possibility of radio line radiation from transitions between highly excited levels of atoms in the ISM. In the same classical paper that predicted the $\lambda = 21 \text{ cm}$ line, van de Hulst also considered radiation from ionized hydrogen for both free-free and bound-bound transitions.

While calculation of the total emission in these lines is straightforward, the detectability of such RRLs would depend upon the distribution of this emission above the underlying continuum emission or, in other words, upon the shape of the emission lines. Although thermal conditions determine the amount of emission in the RRLs relative to the continuum emission, other effects like Stark broadening can widen the lines, spreading out the line emission in frequency, thereby reducing their peak intensities and, in turn, their detectability. van de Hulst derived an expression for the ratio of the peak intensity of the line I_L to the continuum intensity I_C :

$$\frac{I_L}{I_C} = 0.1 \frac{\nu}{\Delta\nu} \frac{h\nu}{kT}, \quad (1.23)$$

where $\Delta\nu$ is the full frequency width of the line, T is the temperature of medium, and k is a Boltzmann's constant. Conceptually, (1.23) describes the total emission in the line to be the product $I_L \Delta\nu$.

To estimate the Stark broadening, van de Hulst drew from an analysis by Inglis and Teller (1939) of optical and infrared line series in stellar spectra. Within the hydrogenic line series of stellar spectra (see Sect. 1.3), there is a wavelength, short of the series limit, at which distinct lines can no longer be seen. In frequency units, (1.17) models these series (for any given n_1) as $n_2 \rightarrow \infty$. At a critical value of n_2 , the lines of that series merge into a continuum that continues until the series limit is reached.

The explanation for this line merging is simple. At this critical frequency (or wavelength), Stark broadening within the stellar atmosphere broadens

the line to match the gap between it and the adjacent line at $n_2 + 1$. By counting lines, an astronomer determines this critical value of n_2 and calculates the wavelength (or frequency) separation to the next line at $n_2 + 1$. This separation must equal the amount of the line broadening and, consequently, is a measure of the electron density necessary to produce it. In this way, Inglis and Teller provided a method of determining gas densities in stellar atmospheres.

Using results from Inglis and Teller and an estimate for the density of the ISM, van de Hulst estimated the magnitude of Stark broadening to be

$$\frac{\Delta\nu}{\nu} = \left(\frac{\lambda}{100\text{ m}} \right)^{3/5}. \quad (1.24)$$

Although the original paper gives few details regarding this formula, the quantity $\Delta\nu/\nu \approx 0.02$ if $\lambda = 20\text{ cm}$, a typical wavelength considered for radio astronomy in 1944, e.g., the $\lambda = 21\text{ cm}$ line. At this wavelength, (1.24) shows the Stark broadening to be dramatically bigger than the thermal broadening that he correctly estimated as $\Delta\nu/\nu = 10^{-4}$. In the meter wavelength range where radio astronomers (Reber, 1944) were actually observing at that time, the Stark broadening would be even larger. Consequently, van de Hulst concluded that hydrogen lines caused by transitions between highly excited levels would be too broad and, therefore, too weak to be observed.⁴

Other astronomers were also pessimistic. Reber and Greenstein (1947) had considered hydrogen radio lines in their examination of the astronomical possibilities of radio wavelengths but had excluded them, “these [lines] have small intensity.” Wild (1952) also considered RRLs but dismissed them because “these lines are so numerous that, without the presence of some selection mechanism they may be regarded merely as contributing toward a continuous spectrum.”

Kardashev (1959) reached just the opposite conclusion. Although he was aware that Wild (1952) had dismissed the possibility of detecting lines, he was unaware of the very pessimistic van de Hulst (1945) study.⁵ Kardashev made

⁴ Many years later, Sullivan (1982) analyzed the working notes of van de Hulst while studying the history of radio astronomy. He found a place in these notes where van de Hulst appeared to have inverted the exponent in (1.24), i.e., the Stark broadening should vary as $(\lambda/100\text{m})^{5/3}$. In fact, combining expressions from the two relevant papers (van de Hulst, 1945; Inglis and Teller, 1939) show that the exponent indeed should have been 5/3. The correct formula – not (1.24) – would predict smaller Stark broadening at radio wavelengths and, correspondingly, more intense line intensities. Sullivan did not comment on the choice of electron density used to derive (1.24). Probably, van de Hulst assumed N_e to be 1 cm^{-3} . See Appendix C.1 for more details.

⁵ The van de Hulst paper was published in a very rare edition of “Nederlands Tijdschrift voor Natuurkunde” that Soviet libraries did not have. The disruption of scientific contact during the second world war and, later, during the cold war years also played a role. According to Shklovsky (1956b; 1960), Soviet scientists had learned about the $\lambda = 21\text{ cm}$ line only from references and comments that appeared much later in journals that were

detailed calculations of the expected line widths and intensities of excited hydrogen RRLs in ionized nebulae (H II regions).

The earlier papers by highly respected astronomers created a difficult climate for optimism with respect to detections of RRLs. Parijskij (2002) recalls an ad hoc meeting at the IAU General Assembly in Moscow in August 1958, where well-known radio astronomers discussed with the then young Nicolay Kardashev the validity of his new, encouraging calculations (Kardashev, 1959). These probably included W.L. Erickson, G.B. Field, L. Goldberg, F.T. Haddock, J.P. Hagen, D.S. Heesch, T.K. Menon, C.A. Muller, H.F. Weaver, and G.L. Westerhout (Kardashev, 2002). The discussion took place in a small room in a new building of Moscow University. Parijskij acted as interpreter. He recalls that the discussion was interesting but quite intense – one “of the deepest I have heard in my life” – with the experienced astronomers examining every calculation made by Kardashev. At the end of the 2-h meeting, they took some kind of a vote and decided that Kardashev might well be correct. This must have been a challenging experience for the young astronomer.

The principal difference between Kardashev and van de Hulst in these calculations lies in their approach to Stark broadening. Kardashev also used the Inglis–Teller relationship, but only for a rough estimate. Independently, he calculated Stark broadening from collisions of excited atoms with electrons as well as from quasistatic broadening. From this analysis, he concluded that, in H II regions with typical values of electron temperature $T_e = 10^4$ K and density $N_e = 10^2$ cm $^{-3}$, pressure broadening would have no significant influence on the line broadening at frequencies greater than 7,000 MHz. In other words, he concluded that line widths would be determined solely from thermal effects, i.e., from the frequency redistribution of emission from a Maxwellian gas according to Doppler effects giving rise to a Gaussian line shape.

After calculating an oscillator strength to determine the line intensities, Kardashev predicted that excited hydrogen radio lines would be observable by radio astronomical techniques in the range from the FIR to decimeter waves. He also showed that the $n \rightarrow n - 1$ transition would have highest intensity and, in addition to the hydrogen lines, the radio lines of helium would be detectable. The frequencies of the helium lines would be shifted relative to hydrogen because of the difference in the Rydberg constant (see Table A.2) due to its greater mass.

Subsequent calculations made it possible to define the intensities of expected radio lines more accurately and, thereby, to plan a search optimized in both frequency and in target sources (Sorochenko, 1965). To re-estimate the line intensities, the attention was again focused on Stark broadening. This time, the calculations used the theory of line broadening in a plasma as developed in early 1960s (Griem, 1960).

available in the USSR. In fact, the van de Hulst paper only appeared in the USSR after its translation (Sullivan, 1982) into English.

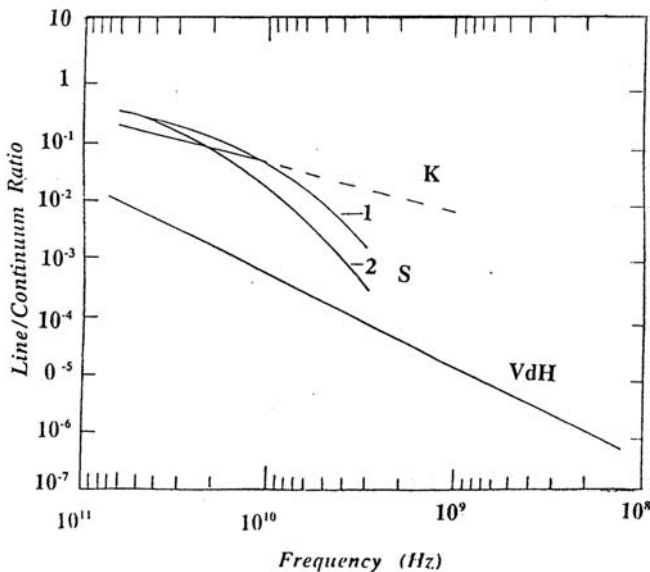


Fig. 1.1 Predicted line-to-continuum ratios for radio recombination lines as a function of frequency. *VdH* van de Hulst (1945) from (1.23) and (1.24), *K* Kardashev (1959) who calculated that Stark broadening may be neglected for $\nu > 7,000$ MHz, *S* Sorochenko (1965) who considered both thermal and Stark broadening for the two values of electron density $N_e = 100 \text{ cm}^{-3}$ (1) and $N_e = 1,000 \text{ cm}^{-3}$ (2). All calculations assume an electron temperature of $T_e = 10^4 \text{ K}$

Figure 1.1 summarizes the line-to-continuum ratios (I_L/I_C) from the papers mentioned. All calculations refer to the $n \rightarrow n-1$ transitions. One can see that van de Hulst (1945) strongly underestimated I_L/I_C , especially taking into account the probable adopted density $N_e = 1 \text{ cm}^{-3}$. For Doppler broadening alone as calculated by Kardashev (1959) for the centimeter wavelength range, the L/C ratio is a few percent, at values of $N_e = 10^2 \text{ cm}^{-3}$ appropriate for H II regions. If Stark broadening is taken into account, at $N_e \geq 10^2 \text{ cm}^{-3}$ the L/C ratio decreases noticeably at frequencies $\nu < 10 \text{ GHz}$ ($\lambda = 3 \text{ cm}$).

To estimate realistic circumstances for the detection of the radio lines, Sorochenko (1965) calculated their intensities in the units of brightness temperature T_b customary in radio astronomy. Figure 1.2 shows these expected values at line center $T_{b,l.c}$ as a function of wavelength λ and of N_e . These data are normalized to the value of the emission measure (*EM*) of the H II region. *EM* is a physical parameter calculated from observations of the continuum emission of an H II region and defined as $N_e^2 L \text{ cm}^{-6} \text{ pc}$, where L is the depth of an H II region in parsecs⁶ (pc) along the line of sight through the H II region.

⁶ Abbreviation for “parallactic arcsecond,” the distance at which the average Earth–Sun distance subtends an angle of $1''$. One pc = $3.0856 \times 10^{18} \text{ cm}$.

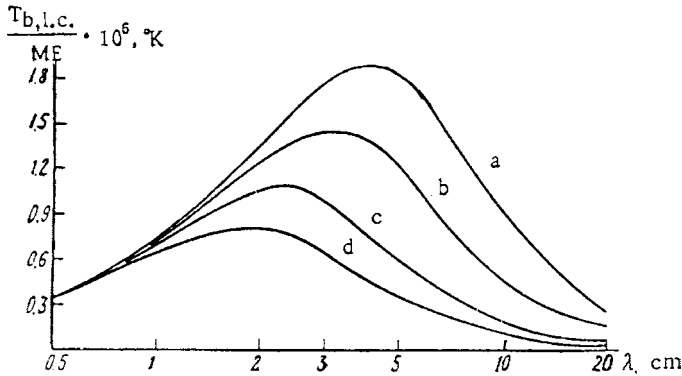


Fig. 1.2 Brightness temperature at the line center as a function of wavelength: (a) $N_e = 100 \text{ cm}^{-3}$, (b) $N_e = 200 \text{ cm}^{-3}$, (c) $N_e = 500 \text{ cm}^{-3}$, and (d) $N_e = 1,000 \text{ cm}^{-3}$

In the millimeter range, the effective size of the atoms is small, there is less collisional interaction with the ambient H II gas, and Stark (pressure) broadening is insignificant as a result. Here, only Doppler broadening determines the line widths and Fig. 1.2 shows the brightness temperature of the lines to increase with wavelength. For an electron density of $N_e > 100 \text{ cm}^{-3}$, Stark broadening of the lines begins to manifest itself at centimeter wavelengths, spreading the line emission over a broader wavelength (frequency) range and reducing the peak intensity of the lines. As the wavelength increases further, the line intensities decline sharply. There is a peak or “turnover” in each of the curves, with the maximum of the brightness temperature shifting toward shorter wavelengths with larger densities.

Simple Bohr atom physics easily explains this effect. Because the longer wavelength lines are generated by atoms whose electrons are in larger orbits, the effective size of these atoms is larger, and their larger sizes render them more likely to interact or collide with the charged particles of the ambient H II gas. These collisions strip the atoms of the outer electrons, thereby removing their ability to radiate and, correspondingly, reducing the aggregate line intensity emitted by the H II region. The wavelength of the turnover is directly related to the probability of these collisions and, therefore, decreases as the gas density increases.

From an experimental viewpoint, this analysis indicated that the search for RRLs would be more effective at low centimeter wavelengths where Stark broadening would be weakest and the line intensities would be the strongest. Specifically, it suggested that the search should take place at $\lambda = (2 - 5) \text{ cm}$ in the brightest, extended H II regions, the Omega and Orion nebulae. Furthermore, at these wavelengths, the angular sizes of these bright H II regions would be well matched to the beam of typical radio telescopes available at that time, thereby ensuring maximum sensitivity for the search.

The stage had now been prepared for the main act: the actual detection of RRLs. In actuality, of course, the stories of theoretical refinements and the searches were complex and intertwined.

1.4.2 Detection of Radio Recombination Lines

The first attempt to detect radio lines emitted by highly excited atoms was undertaken at the end of 1958 in Pulkovo by Egorova and Ryzkov (1960) just after they learned about Kardashev's calculations. Utilizing the receiver developed to search for the deuterium lines ($\lambda = 91.6$ cm), and the unmovable parabolic antenna 20×15 m, they searched for hydrogen radio line corresponding to the $n_{272} \rightarrow n_{271}$, or H271 α , transition in the Galactic plane over the longitude range $l = 60^\circ$ – 115° , but without success.

Five years later, Pulkovo radio astronomers repeated their attempt. At this time, the search was done in 1963 by Z.V. Dravskikh and A.F. Dravskikh (1964) during the testing of the new 32-m paraboloid antenna of the Space Research Center in the Crimea. A simple $\lambda = 5$ cm mixer receiver with filter width of 2 MHz and a tuning accuracy of about 1–3 MHz scanned over a 20-MHz band to search for the $n_{105} \rightarrow n_{104}$ hydrogen line at 5.76 GHz in the Omega and Orion nebulae.

According to Dravskikh (1994; 1996), a strong wind arose during their scheduled time, making it difficult to point the telescope and resulting in only eight spectrograms for the Omega nebula and five for Orion. The quality of these spectra were accordingly poor, and the Dravskikhs were reluctant to consider them further. However, a young colleague, Yuri Parijskij, insisted that spectra should be processed further, believing that the wind effects could be removed. After this processing, the lines appeared to be present in the spectra of each nebula – although too weak to convince everyone of the reality of their detection, shown in Fig. 1.3. The authors themselves estimated the detection probability to be 0.9, corresponding to a signal-to-noise (S/N) ratio of 2.

At that time, the situation was very competitive. Two Soviet groups had been preparing to search for RRLs. Besides A.F. Dravskikh and Z.V. Dravskikh in Pulkovo, the other group for detecting lines was located at the Lebedev Physical Institute in Moscow, where they had been preparing since 1963. The competition involved the quality as well as the timing of the searches. The Pulkovo group had been able to begin their observations earlier but the detections were marginal, having been achieved by necessarily salvaging the unfortunate wind-damaged spectra. On the other hand, the Lebedev group wanted to make detections that would be convincing to everyone and were willing to delay their observing until their specially designed equipment was ready.

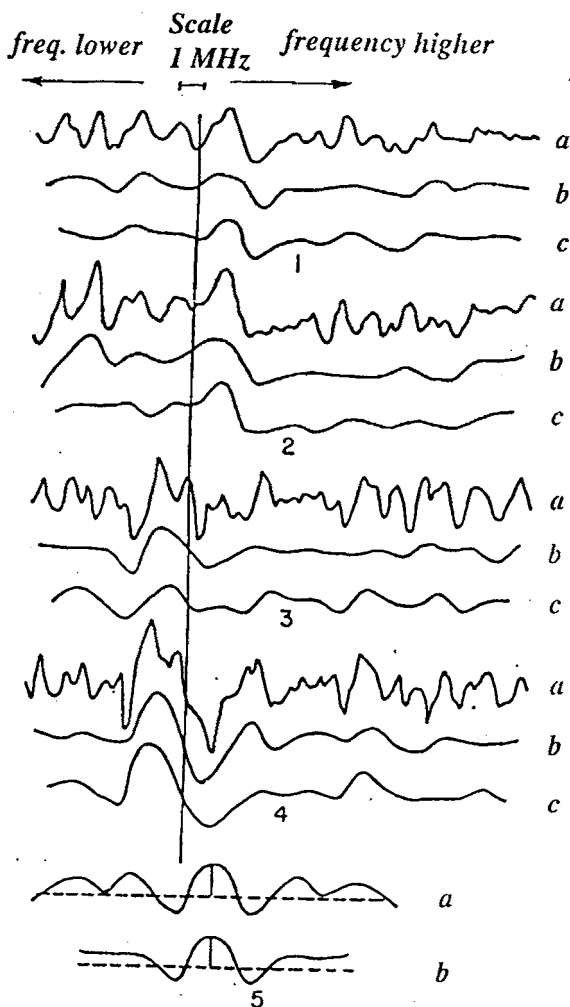


Fig. 1.3 Figures 1–5 of Z.V. Dravskikh and A.F. Dravskikh (1964): (1*a*) average of the eight spectra of the Omega nebula at 5.7 GHz, (1*b*) previous spectrum convolved with a 1-MHz filter, (1*c*) a different smoothing scheme; (2) same as (1), but only four spectra of the Omega nebula; (3) average of all five spectra of Orion; (4) average of only three spectrograms from Orion; and (5) the average of the left and right sides of the spectra (centered about the line proposed position) in the Omega (*a*) and Orion (*b*) nebulae. The vertical line marks the radial velocity position of 0 km s^{-1} with respect to the Local Standard of Rest (LSR) for the $n_{105} \rightarrow n_{104}$ line

Based upon a closer analysis of the lines' expected properties and intensities with regard to their 22-m radio telescope (Sorochenko, 1965), the Lebedev group came to the conclusion that a new receiver would be needed – one with a sensitivity at least an order of magnitude greater than the existing

spectrometers being used for observations of the $\lambda = 21$ cm line. The recombination lines were not only expected to be very weak in themselves but also expected to be weak with respect to the stronger background continuum emission emitted by the H II regions (see Fig. 1.1). In other words, the very objects in which the weak lines should appear would also be emitting strong background emission that would make detection more difficult.

To overcome these difficulties, the Lebedev staff developed a nulling-type spectral radiometer at a wavelength of $\lambda = 3.4$ cm using low noise parametric amplifiers. With great accuracy, this radiometer ensured that the noise in the 20-MHz band was the same for the source (antenna) and the reference load. In this way, it was insensitive to fluctuations of the background continuum such as pointing errors, changes in atmospheric emission, etc. At the same time, it was capable of detecting weak, narrow spectral lines superimposed upon the strong, background continuum emission.

On 27 April 1964, using this radiometer and the 22-m radio telescope of the Physical Institute in Pushchino shown in Fig. 1.4, Sorochenko and Borodzich (1965) detected the hydrogen radio line $n_{91} \rightarrow n_{90}$ (H90 α) at 8,872.5 MHz in the spectrum of the Omega nebula on their first attempt. Figure 1.5 shows these spectra. Unlike the earlier observations of the Pulkovo group 4 months earlier in December 1963, this line was clearly present even in the individual spectrograms. The specially designed receiver and the better observing conditions had made a definitive difference. Observations carried out over the next 3 months showed shifts in the line frequency corresponding to the Doppler shifts expected from the Earth's orbital rotation. These frequency shifts dispelled any doubts about the cosmic origin of the line.

Nearly simultaneously with the Lebedev group, the group (Dravskikh, Dravskikh, Kolbasov, Misezhnikov, Nikulin and Shteinshteyger, 1965) at Pulkovo observatory also convincingly detected an excited hydrogen line. Only a month separated these two detections. After improving their radiometer by installing a maser amplifier for the receiver, they were able to detect the hydrogen radio line $n_{105} \rightarrow n_{104}$ at 5,762.9 MHz with the 32-m radio telescope. This time, in May and July of 1964, there was no doubt. The H104 α line had definitely been detected in the Omega nebula. Figure 1.5 shows their spectra as well. The Doppler shift of line frequency due to orbital motion of Earth was also found, confirming this detection as well.

On 31 August 1964, the results of both groups were communicated to astronomers attending the XII General Assembly of the International Astronomical Union in Hamburg, Germany. In a joint session of Commissions 33, 34, and 40 organized by Westerhout, Yuri Parijskij presented a paper on behalf of Dravskikh et al. (1966), and Vitkevitch did the same for Sorochenko and Borodzich (1966). Figure 1.5 shows the first spectrograms of the excited hydrogen lines $n_{91} \rightarrow n_{90}$ and $n_{105} \rightarrow n_{104}$ with good S/R ratios that were presented to the IAU General Assembly.

At that presentation, there were a number of questions from the audience (Dravskikh, 1996). Accustomed to the much higher S/N ratios of



Fig. 1.4 The 22-m radio telescope of the P.N. Lebedev Physical Institute in Pushchino, 100 km south of Moscow. With this instrument, the excited hydrogen line $n_{91} \rightarrow n_{90}$ was clearly detected on 27 April 1964

the $\lambda = 21$ cm radio spectra of atomic hydrogen and having experienced the technical difficulties of observing weak spectral lines at that time, many astronomers were skeptical of these clearly noisy results (Price, 2002). In addition, the visual material at that time was less than ideal, and the presentations came near the end of the day, being the 23rd and 24th papers of the 26 presented. Alan H. Barrett, codiscoverer of the second known radio astronomical line (OH) during the previous October (Weinreb, Barrett, Meeks and Henry, 1963), asked Parijskij, “Are you saying that you detected the excited hydrogen line $n = 105 \rightarrow 104$?” Parijskij replied, “Yes.” Barrett repeated his question, “Are you saying that these lines can exist?” Parijskij again replied, “Yes.” Despite their reservations, this exchange (Parijskij, 2002) also shows

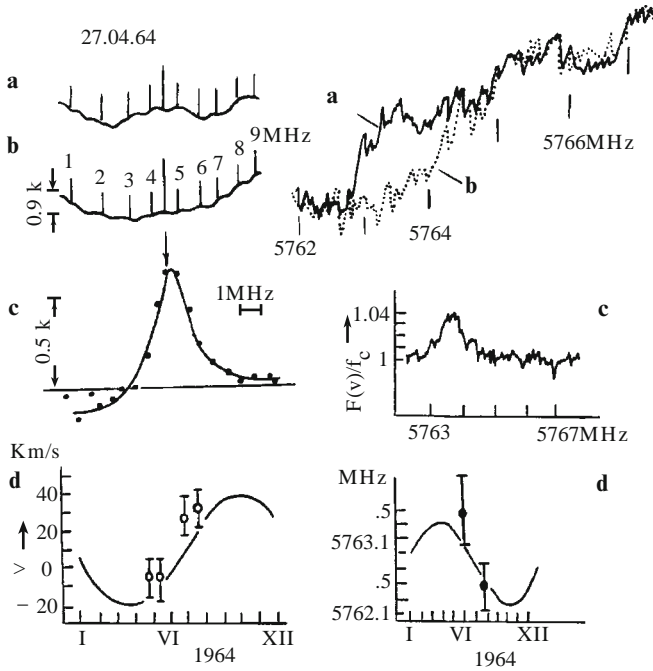


Fig. 1.5 The first spectrograms of excited hydrogen lines with good signal-to-noise ratios that were presented to the IAU General Assembly in Hamburg, Germany, in 1964. *On the left*, the $n_{91} \rightarrow n_{90}$ (H90 α) hydrogen line observations from Pushchino. **a** The spectrogram toward the Omega nebula. **b** The test spectrogram with the antenna off the source. **c** The average of seven spectra toward Omega and the five test spectra made in April 1964. The abscissa is frequency; the ordinate, the antenna temperature. The *large mark* indicates the calculated line frequency; the *vertical dashes*, 1-MHz intervals. **d** The measured Doppler shift during the year 1964. This *curve* indicates the nebula's calculated radial velocity relative to Earth (Sorochenko and Borodzich, 1965). *On the right*, the Pulkovo observations of the $n_{105} \rightarrow n_{104}$ (H104 α) hydrogen line. **a** The spectrogram toward Omega. **b** The test spectrogram. **c** The average of the 12 spectrograms obtained in May 1964. The abscissa is frequency; the ordinate, the ratio of nebula's line-to-continuum flux densities. **d** The measured Doppler shift during the year 1964, showing the calculated frequency shift (Dravskikh et al., 1965)

that the radio astronomers were very interested in these new results. The participants brought this news back to colleagues in their own radio astronomy groups. Yet, few of them followed up the detections immediately, possibly because of what they wrongly believed to be uncertain results.

The detections obtained by the P.N. Lebedev Physical Institute and the Pulkovo observatory groups confirmed each other. The initial detections differed by only one month or so, and the confirming Doppler observations overlapped each other. For this reason, both Soviet groups agreed to consider 31 August 1964 – the date the detections were presented to the IAU General Assembly – to be the official discovery date of radio lines emitted by excited

atoms. As described earlier in Sect. 1.3, these lines result from the process of recombination of ions and electrons and, when they occur in the radio wavelength regime, they are called “radio recombination lines.”

In the end, the competition between the two observational groups had produced a great success: the unambiguous detection of not just one recombination line but two. RRLs had arrived as a powerful new tool for astronomers everywhere.

1.4.3 Other Searches and Detections

Radio astronomers outside the Soviet Union had also read the Kardashev paper with interest. In particular, the German radio astronomer P.G. Mezger tried unsuccessfully to detect an excited hydrogen line at 2.8 GHz in 1960 with the 25-m Stockert radio telescope of the University of Bonn (Mezger, 1960), after having received a translation of the 1959 Kardashev paper. Mezger (1992) gives an account⁷ of this search in his autobiographical book, “Blick in das kalte Weltall” (“A Look into the Cold Universe”).

Some of the initial searchers for cosmic RRLs simply had bad luck. The Australian group of John Bolton, Frank Gardner, and Brian Robinson used the CSIRO 64-m telescope at Parkes to make a quick search for the lines near 5 GHz, probably in 1963 prior to the Soviet detections although the exact date has been lost (Robinson, 2001). Unfortunately, they used the n^{-3} approximation given by (1.21) to calculate the rest line frequency. The approximation error placed the actual line frequency outside of their narrowband spectrometer and, consequently, the line was not detected.

Mezger’s interest persisted and, in 1963 after moving to the US National Radio Astronomy observatory (NRAO) in Green Bank, WV, he tried again to detect these lines together with B. Höglund (Höglund and Mezger, 1965), a visiting Swedish astronomer. The first series of observations were made with the NRAO 85-ft. (26-m) telescope beginning in the Fall of 1964. He used a new receiver especially designed for the detection of RRLs. The spectrometer part of the instrument had been designed and built by B. Höglund principally for observations of the then well-known $\lambda = 21$ cm hydrogen line but had been borrowed for a search for RRLs. According to Mezger (1994), these attempts gave “ambiguous results, probably because of local oscillator instabilities.”

The second attempt brought success. Using the newly completed 140-ft. (43-m) telescope shown in Fig. 1.6, Höglund and Mezger (1965) were able to detect the $\text{H}109\alpha^8$ lines (5 GHz) with unexpectedly high signal-to-noise

⁷ See also an after-dinner talk given by Moran (1994) in celebration of Mezger’s 65th birthday.

⁸ From this point on, we shall exclusively use the standard convention (Lilley, Menzel, Penfield and Zuckerman, 1966) for naming RRLs: the elemental symbol from the Mendeleeev



Fig. 1.6 The 140-ft. (42.6-m) diameter, equatorial radio telescope at the Green Bank, WV, facility of the US National Radio Astronomy observatory. The telescope saw first light in 1964 and was taken out of service in 1999

ratios on 9 July 1965, the first day of the observations. Figure 1.7 shows the astronomers and Fig. 1.8 shows their results.

Two days after their successful observations, they presented their observations to the Scientific Visiting Committee, appointed annually by NRAO's management organization, Associated Universities, Inc., to assess NRAO operations. On this committee was Ed Purcell, a Nobel laureate in physics from

Periodic Table, followed by the principal quantum number of the lower level, and followed again by a Greek letter identifying the order of the transition. For example, H109 α corresponds to the hydrogen line from the transition $n_{110} \rightarrow n_{109}$. Similarly, H92 β corresponds to the hydrogen transition $n_{94} \rightarrow n_{92}$, etc.

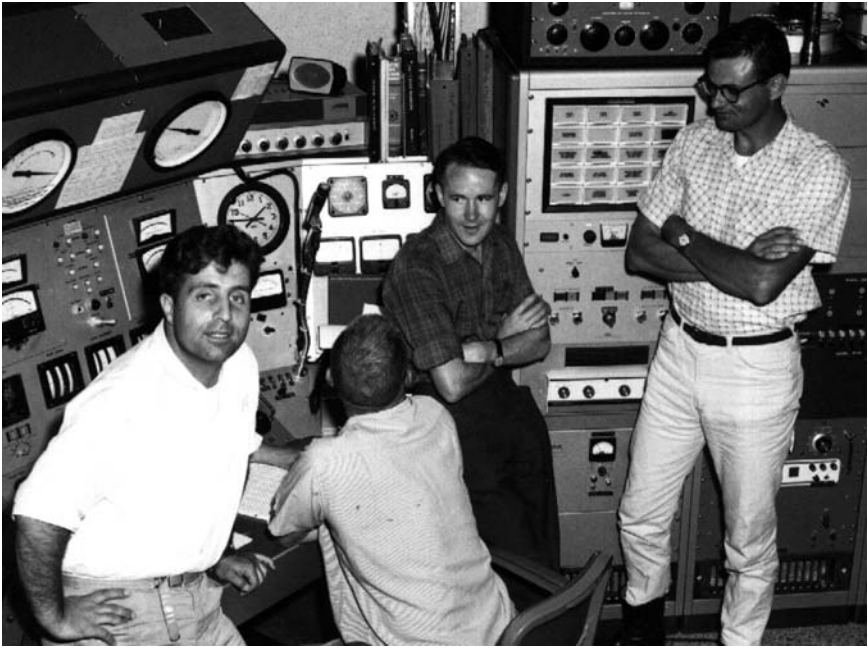


Fig. 1.7 Control room of the NRAO 140-ft. telescope in July 1965, near the time of the detection of the H109 α RRL. *Left to right:* P.G. Mezger, H. Brown (telescope operator), B. Höglund, and N. Albaugh (electronic technician). NRAO photograph

Harvard who was codiscoverer of the $\lambda = 21$ cm hydrogen line in 1951 – the first spectral line available to radio astronomers. According to Mezger (1992), Purcell was fascinated that atoms with size scales of μm could exist in interstellar space.

When, also according to Mezger (1992), Purcell returned to Harvard from the meeting, he shared the news with physics colleagues about the existence of such large atoms – to be called “Rydberg atoms” – in cosmic gases. He learned that Harvard’s radio astronomy group, which had the technical capacity for the RRL detections, had not searched for the lines. Later, Mezger heard a rumor that Donald Menzel, a brilliant theoretician within the astronomy department, earlier had convinced their radio astronomy group that pressure broadening would make RRLs impossible to detect, possibly the result of reading van de Hulst’s 1945 paper. Within a few days of hearing the news, the Harvard radio astronomy group returned their maser receiver to the nearest RRL frequency and detected the H156 α and H158 α lines near $\lambda = 18$ cm in the Omega and W51 nebulae also without difficulty (Lilley, Menzel, Penfield and Zuckerman, 1966).

However, the comparatively late participation of the Harvard astronomy group may have been due to other factors (Palmer, 2001). In 1964, OH maser lines had been discovered, and the 60-ft. (18-m) radio telescope at Harvard’s

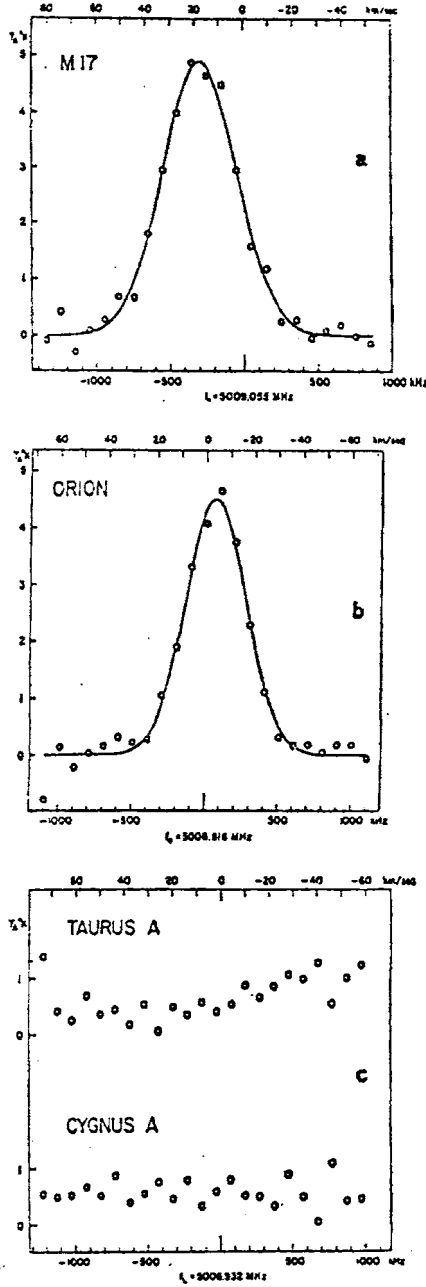


Fig. 1.8 Figure 1 of Höglund and Mezger (1965) showing the detections of the H109 α RRLs with the newly completed 140 ft. (43 m) telescope of the National Radio Astronomy observatory in Green Bank, WV. It is significant that, whereas the RRLs appeared in the gaseous nebulae Omega (M17) and Orion, they did not appear in the nonthermal sources Taurus A and Cygnus A

Agassiz observing station was fully engaged exploring their characteristics. In addition, Harvard's professor of radio astronomy, Ed Lilley, was devoting most of his energy promoting the design and construction of a huge radome-enclosed radio/radar astronomy telescope (Northeast Radio Observatory Corporation – NEROC) to be used by US northeastern universities, which, unfortunately, was never funded. Finally, according to Palmer, at that time all the Harvard astronomy graduate students were involved with other research projects; none were available to pursue the new discovery. Nonetheless, Purcell did persuade the group to redirect some resources toward searching for RRLs after returning from the Green Bank meeting, which led directly to the detection of the $\lambda = 18$ cm RRLs.

With these detections, the dam had broken, and soon other radio astronomical groups were detecting RRLs with excellent signal-to-noise ratios. Lilley et al. (1966) reported the detection of RRLs from helium at $\lambda = 18$ cm. Gardner and McGee (1967) and McGee and Gardner (1967) reported the detection of a number of hydrogen α and β RRLs at frequencies of 1.4 and 3.3 GHz with the 210-ft. (64-m) telescope in Parkes, Australia. These observations gave line ratios consistent with emission in thermodynamic equilibrium. Gordon and Meeks (1967) observed 94α lines at 7.8 GHz from both hydrogen and helium in Orion, deducing a kinetic temperature of 6,600 K from the line widths and, also, detecting the H148 δ line. Palmer et al. (1967) detected a recombination line with a somewhat higher frequency than the H109 α line in the NGC2025 and IC1795 nebulae; it was later identified as the C109 α line of atomic carbon (Goldberg and Dupree, 1967; Zuckerman and Palmer, 1968).

Observations of RRLs were quickly extended to frequencies as low as 404 MHz with the detection of the H253 α line (Penfield, Palmer and Zuckerman, 1967), while Dieter (1967) detected the H158 α line in 39 H II regions – thereby establishing the new lines as suitable tools for astronomical surveys.

It became evident that RRLs would be a source of vast information about the microworld of astrophysics such as the features of highly excited atoms, as well as about macroworld of the astronomical bodies such as the structure of surrounding cosmic space. Accordingly, astronomers at many observatories and institutes around the world began to observe RRLs.

Chapter 2

RRLs and Atomic Physics

Abstract This chapter derives the physics ab initio that underlie the formation of radio recombination lines in astronomical objects. It includes natural, thermal, and pressure broadening of the spectral lines; the radiation transfer of the spectra and their underlying free–free emission (Bremsstrahlung) through the ionized media; the excitation of the atomic levels; the frequency range over which lines may be detected from astronomical objects; and the sizes of excited atoms that can exist within interstellar environments.

2.1 The First Surprising Results: The Absence of Stark Broadening

The first observations of radio recombination lines (RRLs) gave surprising results. The newly detected line profiles (Sorochenko and Borodzich, 1965; Höglund and Mezger, 1965; Lilley, Menzel, Penfield and Zuckerman, 1966) showed neither the Stark broadening individually nor the variation in Stark broadening with increasing quantum number n as predicted by theory. All the RRLs observed in the Omega nebula (M17) with principal quantum numbers up to $n = 166$, within the accuracy of the measurements, had constant ratios of line width to frequency. This indicated pure Doppler broadening.

In a sense, the astronomers were pleased. RRLs would be observable. Evidently, van de Hulst's (1945) pessimistic prediction that pressure broadening of the excited hydrogen lines would render them unobservable in the radio range was incorrect. The more recent calculations of the line width based on the theory of spectral line broadening by plasmas (Sorochenko, 1965) seemed to fit the observations better, in the sense that the new theory allows their detectability.

Let us look at the data available in 1967. Figure 2.1 shows a plot of actual line widths observed for hydrogen RRLs over the years 1964–1967 together with the results of the calculations of Stark broadening. The revised theory

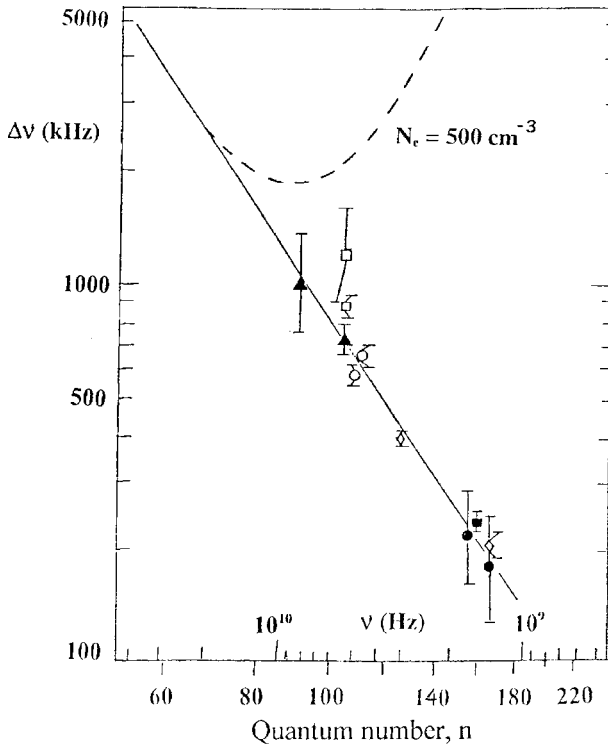


Fig. 2.1 The first observations of the RRL width $\Delta\nu$ as a function of quantum number n in Omega nebula. The *straight line* is the Doppler broadening with $\Delta\nu/\nu = 1.2 \times 10^{-4}$. The *dotted curve* shows what was expected based on the revised estimates of Stark broadening by Sorochenko (1965). The RRL observations shown are: *filled triangles* – Lebedev Physical Institute – H90 α (Sorochenko and Borodzich, 1965) and H104 α (Gudnov and Sorochenko, 1967); *open squares* – Pulkovo – H104 α (Dravskikh et al., 1965; Dravskikh and Dravskikh, 1967); *open circles* – Green Bank – H109 α (Höglund and Mezger, 1965; Mezger and Höglund, 1967); *open diamonds* – Parks – H126 α and H166 α (McGee and Gardner, 1967); *filled circles* – Harvard – average value of H156 α and H158 α (Lilley, Menzel, Penfield and Zuckerman, 1966) and H166 α (Palmer and Zuckerman, 1966); and *filled squares* – University of California – H158 α (Dieter, 1967)

predicts that, for $N_e = 500 \text{ cm}^{-3}$, the minimum possible value of electron density in gaseous nebulae (H II regions), Stark broadening should begin to manifest itself for $n\alpha$ -type RRLs with $n > 100$. However, the measured line widths showed no Stark broadening at all. At $n = 166$, the line width should be ten times the value actually observed.

The new results were exciting. RRLs would be observable at substantially higher transitions than previously assumed. This suggested that RRLs could be a new tool for astronomers.

However, the physics of the line broadening evidently had not yet been understood. With the increasing level (n) of excitation, the outer electron

becomes less connected to the atomic nucleus as shown by (1.6), (C.7), and (C.8). Correspondingly, the sensitivity of that electron to external electric fields of the ambient plasma must inevitably increase, manifesting this increasing influence by Stark broadening of the line widths. However, this was not observed. Consequently, one could suppose that the highly excited atoms have some mechanism of resisting the influence of the external fields, one that does not occur at the lower excitation levels that fit Doppler broadening very well. The solution of this problem required revision of the existing Stark broadening theory for RRLs and, probably, some additional careful experiments in atomic physics. Let us discuss this very interesting question further after a general discussion of broadening mechanisms.

2.2 The Broadening of Radio Recombination Lines

2.2.1 Natural Broadening

Several mechanisms determine the width of spectral emission lines. One is called “natural” broadening. It results from the finite length of the emitted wave train and the variation of its amplitude over the emission time. It is an intrinsic property of the atom.

2.2.1.1 Lorentz Profile

To predict the shape of a line due to natural broadening, we need to consider the broadening mechanism in more detail. Prior to quantum mechanics, physicists considered atoms as oscillating electric dipoles. From the moment that $t = 0$, the energy of an oscillator decreases as $E(t) = E_0 \exp(-\Gamma t/2)$, where the damping constant

$$\Gamma = \frac{8\pi^2 e^2}{3mc^3} \nu_0^2, \quad (2.1)$$

and ν_0 is the central frequency of oscillation and m is the mass of the oscillator.

This damping characteristic determines the frequency spectrum of the oscillator (Lorentz, 1906). If the intensity of the oscillator $f(t) = \exp(i2\pi\nu_0 t) \exp(-\Gamma t/2)$, then its complex spectrum results from the Fourier transform (Bracewell, 1965)

$$F(\nu) = \int_{-\infty}^{\infty} f(t) e^{-i2\pi\nu t} dt \quad (2.2)$$

and the real spectrum

$$\phi(\nu) \propto \int_{-\infty}^{\infty} F^*(\nu) F(\nu) dt \quad (2.3)$$

$$= \frac{\Gamma/\pi}{\pi [4(\nu - \nu_0)^2 + (\frac{\Gamma}{2\pi})^2]} \quad (2.4)$$

after normalization such that

$$\int_{-\infty}^{\infty} \phi(\nu) d\nu = 1. \quad (2.5)$$

Equation (2.4) is then the spectrum of the oscillator known as a *Lorentz profile*. Figure 2.2 shows its shape. The full width of the Lorentz profile at half-intensity, $\Delta\nu_L$, is $\Gamma/(2\pi)$.

The Lorentz profile can also be derived from quantum mechanics. The uncertainty principle of Heisenberg states that the energy E of a system can be known only to an accuracy ΔE within a time interval Δt , where

$$\Delta E \Delta t \approx \frac{h}{2\pi}. \quad (2.6)$$

Accordingly, in the absence of collisions, the energy uncertainty of a quantum level n is given by

$$\begin{aligned} \Delta E_n &\approx \frac{h}{2\pi} \frac{1}{\Delta t} \\ &\approx \frac{h}{2\pi} \Sigma R_{nm}, \end{aligned} \quad (2.7)$$

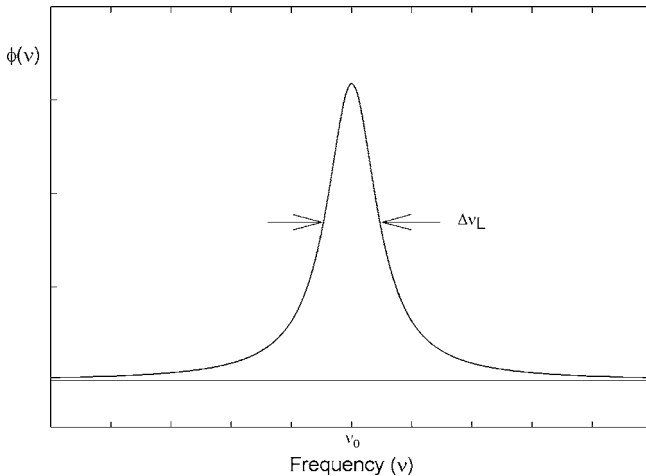


Fig. 2.2 A hypothetical Lorentz line profile described by (2.4). Note the characteristic narrow core, the steep shoulders near line center, and the broad sloping wings

where the sum is taken over all of the transition coefficients R_{nm} depopulating the level, from $n \rightarrow m$. This relationship shows that the width ($\Delta\nu \equiv \Delta E_n/h$) of a spectral line must be related to the inverse lifetime of its quantum levels, i.e., to the net transition rate out of its levels.

2.2.1.2 Natural Width of RRLs

In 1930, Weisskopf and Wigner derived the Lorentz line profile from quantum mechanics. They found the line width $\Delta\nu_L$ of (2.4) to equal the sum of the damping constants for the upper and lower quantum levels n_2 and n_1 , respectively:

$$\Gamma \equiv \Gamma_{n_2} + \Gamma_{n_1} \quad (2.8)$$

$$= \frac{1}{\tau_{n_2}} + \frac{1}{\tau_{n_1}}. \quad (2.9)$$

The total spontaneous rate out of level n_2 , or Γ_{n_2} , is

$$\Gamma_{n_2} = \sum_{n_1=1}^{n_2-1} A_{n_2, n_1}. \quad (2.10)$$

One writes Γ_{n_1} in a similar way.

The full width of the line at half-maximum then is

$$\frac{\Delta\nu}{\nu_0} = \frac{1}{2\pi\nu_0} (\Gamma_{n_2} + \Gamma_{n_1}) \quad (2.11)$$

$$= \frac{1}{2\pi\nu_0} \left(\sum_{n_1=1}^{n_2-1} A_{n_2, n_1} + \sum_{n_0=1}^{n_1-1} A_{n_1, n_0} \right) \quad (2.12)$$

$$\approx \frac{1}{\pi\nu_0} \sum_{n_1=1}^{n_2-1} A_{n_2, n_1}, \quad (2.13)$$

where (2.12) results from the quantum mechanical form of the Lorentz profile found by Weisskopf and Wigner (1930). Equation (2.13) obtains because the damping constants Γ_n are about the same for both upper and lower levels of normal RRLs involving large values of n and when $n_2 - n_1 \ll n_2, n_1$.

Another simplification is possible for most RRLs. Section 5.4.1 of Sobelman et al. (1995) gives a useful approximation to estimate the sum A_n of the spontaneous transition probabilities out of level n :

$$A_n \approx 2.4 \times 10^{10} \frac{\ln n}{n^5}, \quad n > 20. \quad (2.14)$$

Substituting the approximation for the RRL frequencies given by (1.22) and the summation given by (2.14) into (2.13) gives a useful expression for the natural width of RRLs:

$$\frac{\Delta\nu}{\nu_0} \approx \frac{1.2 \times 10^{-6} \ln n}{n^2}, \quad n > 20, \quad (2.15)$$

which gives a fractional natural width of 4.5×10^{-10} or $1.3 \times 10^{-4} \text{ km s}^{-1}$ for the H109 α line, for example. This natural width is negligibly small compared to other types of broadening, as will be seen below.

2.2.2 Doppler Broadening

Observations of thermal gases in the cosmos such as H II regions involve measurements of Maxwell–Boltzmann velocity distributions if we exclude large-scale turbulence. In the absence of magnetic fields, an H II region with a kinetic temperature of 10^4 K and an electron density of 10^2 cm^{-3} will thermalize in minutes following any perturbation in the velocity distribution because of the high collision rate between the electrons and the ions. For higher densities, the thermalization proceeds even faster.

For a gas with a Maxwell–Boltzmann velocity distribution, the probability of an atom having a velocity component¹ between v_x and $v_x + dv_x$ along a line of sight through the nebula is then

$$N(v_x) dv_x = N \left(\frac{M}{2\pi kT} \right)^{1/2} \exp \left(-\frac{Mv_x^2}{2kT} \right) dv_x, \quad (2.16)$$

where N is the total number of atoms contributing photons to the line and M is the mass of the atoms of that species. Using the classical Doppler formula² to relate the frequency observed to the line-of-sight velocity

$$\nu = \nu_0 \left(1 - \frac{v_x}{c} \right), \quad (2.17)$$

and differentiating to relate the intervals of the two domains

$$dv_x = -\frac{c}{\nu_0} d\nu, \quad (2.18)$$

¹ N.B. One axis only. See Sect. 4.11 of Chapman and Cowling (1960) for a full discussion of Maxwell–Boltzmann statistics.

² This formula works well when $v_x \ll c$. Gordon et al. (1992) discuss the relationship between spectral red shift and velocity for Euclidean (Special Relativity) and cosmological (General Relativity) models.

we convert (2.16) into the intensity of the line $I(\nu)$ with the assumption that the total intensity I in the line is proportional to the number of emitters N in the antenna beam and that the intensity interval $dI(\nu)$ is proportional to $dN(v_x)$, i.e., that the gas is optically thin to obtain

$$I(\nu) = I\phi_G(\nu), \quad (2.19)$$

where the line profile $\phi_G(\nu)$ of the Doppler-broadened line is

$$\phi_G(\nu) = \left(\frac{4 \ln 2}{\pi}\right)^{1/2} \frac{1}{\Delta\nu_G} \exp \left[-4 \ln 2 \left(\frac{\nu_0 - \nu}{\Delta\nu_G} \right)^2 \right], \quad (2.20)$$

where $\Delta\nu_G$ is the full width of the thermally broadened, Gaussian line at half-intensity and is obtained by equating the exponential arguments of (2.16) and (2.20). Using (2.17) to relate the frequency term to velocity, we obtain (Fig. 2.3)

$$\Delta\nu_G = \left(4 \ln 2 \frac{2kT}{Mc^2} \right)^{1/2} \nu_0, \quad \text{or,} \quad (2.21)$$

$$\approx 7.16233 \times 10^{-7} \left(\frac{T}{M} \right)^{1/2} \nu_0, \quad (2.22)$$

where the mass M is in units of amu and T is in K.³

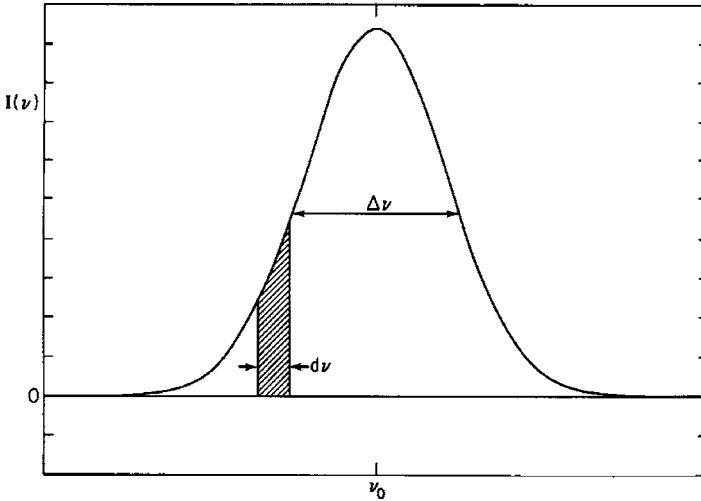


Fig. 2.3 The hypothetical Gaussian line profile described by (2.20). Note the broader core and the narrower extent of the wings with respect to the Lorentz profile

³ A useful expression is that the area I of a Gaussian line is

Substituting a typical H II region temperature of 8,000 K and the mass of hydrogen into (2.22) gives a value of $\Delta\nu/\nu_0 = 6.4 \times 10^{-5}$ for the H109 α RRL – 10^5 times greater than the natural width estimated above. Based upon (2.18), this ratio is equivalent to a full width at half-intensity of about 19 km s^{-1} .

Microturbulence also contributes to the Doppler profile of the RRL. In many astronomical objects, emitting RRLs, such as H II regions, are cells of gas moving with respect to each other. Often, these cells are unresolved by the radio telescope; they fall within its comparatively large beam and, hence, merit the name “microturbulence.” A characteristic of Doppler broadening is that the line width in units of velocity is a constant because $\Delta v = (c/\nu)\Delta\nu$. Because the velocity distribution of these cells is usually Gaussian, the observed width of the RRL results from the convolution of the thermal and turbulence Gaussians, a process (see Bracewell (1965)) that results in a new Gaussian profile with a width Δv_Σ given by

$$(\Delta v_{\Sigma G})^2 = (\Delta v_{G-\text{thermal}})^2 + (\Delta v_{G-\text{turbulence}})^2, \quad (2.25)$$

such that the Doppler line width can be written

$$\Delta V_G \equiv \Delta v_{\Sigma G} = (4 \ln 2)^{1/2} \sqrt{\frac{2kT}{M} + V_T^2} \quad (2.26)$$

in terms of gas temperature and the turbulence velocity V_T .

Figure 2.4 shows how the turbulence widths $\Delta v_{G-\text{turbulence}}$ – labeled here as V_t – of RRLs increase with the angular size of the telescope beam, as seen in the Orion nebula. The width reaches a limit when the H II region becomes unresolved. The illustration shows that even at very high resolution, the turbulence component can be comparable to the thermal broadening.

2.2.3 Stark Broadening of RRLs

2.2.3.1 Early Theory

The third and, at times the most important mechanism for broadening RRLs, is a form of “pressure broadening” called the *Stark effect*. It consists of the splitting and displacement of atomic energy levels by the superposition of

$$I = \left(\frac{\pi}{4 \ln 2} \right)^{1/2} I(\nu_0) \Delta\nu_G \quad (2.23)$$

$$\approx 1.064 I(\nu_0) \Delta\nu_G, \quad (2.24)$$

in terms of the intensity measured at line center $I(\nu_0)$ and the full width of the line at half-intensity, the expression resulting from the integration of (2.19).

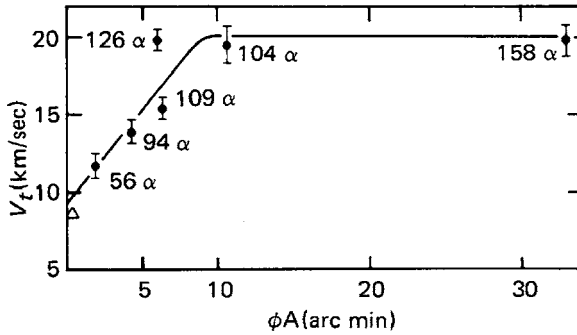


Fig. 2.4 The turbulence width of RRLs from the Orion nebula plotted against the beam size of the radio telescope used for observation (Sorochenko and Berulis, 1969). The *triangle* marks the width of a line seen at high angular resolution at optical wavelengths (Smith and Weedman, 1970)

an external electric field. When the electron is in a noncircular orbit, there is an electric dipole moment that responds to the application of an electric field. Early laboratory measurements referred to two kinds:⁴ “linear” in which the displacement of the emission line is linearly proportional to the strength of the electric field and “quadratic” which has a square-law dependency on the electric field strength. The linear type occurs for weak electric fields, the quadratic type for strong fields. The effect was discovered by Stark (1913) not long before the quantum theory of atom was created.

Bohr (1923) explained the effect in the following way. If (1.14) gives the energy of an atom with a principal quantum number n , then the application of an external electric field \mathcal{E} to that atom should cause a change in its energy of

$$\Delta E = \frac{3h^2\mathcal{E}}{8\pi^2em}nn_f, \quad (2.27)$$

where the n_f -quantum number can take the values of $0, \pm 1, \pm 2, \dots, \pm n$. Therefore, the applied field splits the principal quantum number into $2n - 1$ sublevels because there is only one 0 in the series. From (2.27), the maximum width of each broadened energy level E_n is then $\propto n^2$.

Stark-shifted line components arise from transitions from the split upper energy levels $E_{n_2} \pm \Delta E$ to the split lower levels $E_{n_1} \pm \Delta E$. The distribution of intensities of the Stark components is complex – even for the simple hydrogen atom, as described in Sect. 65 of Bethe and Salpeter (1957). In general, the outermost line components have unobservable intensities.

⁴ A tertiary Stark broadening can also occur.

2.2.3.2 Stark Broadening in Astronomical Plasmas

In astronomy, Stark broadening is more complicated. Astronomical plasmas include a wide range of temperatures and densities difficult to reproduce in a terrestrial laboratory. The principal characteristics of plasmas associated with the interstellar medium are low densities and low temperatures compared with, say, stellar cores and envelopes. The electric field experienced by an emitting atom might consist of a series of brief, weak, time-spaced transient fields induced by a series of successive colliding charged particles rather than by the steady-state field of a laboratory experiment.

Stark broadening in astronomical plasmas can be understood by examining the wave train emitted by the atom. Figure 2.5 illustrates the effect of sudden small phase shifts upon a sinusoidal wave train. The Fourier transform relationship between the phase shift time-line and frequency dispersion produces the profile of the spectral line. Consequently, theorists considered how charged particles could induce phase shifts in the emitted wave train. When the such encounters are transitory, they are called “collisions” or “impacts.”

For example, we can calculate a Lorentz line profile by considering the effect of collisions on an oscillator with a constant amplitude. If the oscillation has the form $f(t) = \exp(i2\pi\nu_0 t)$ over the duration T between successive collisions that stop the oscillation abruptly, the frequency spectrum of the oscillation will be the Fourier transform of the wave train:

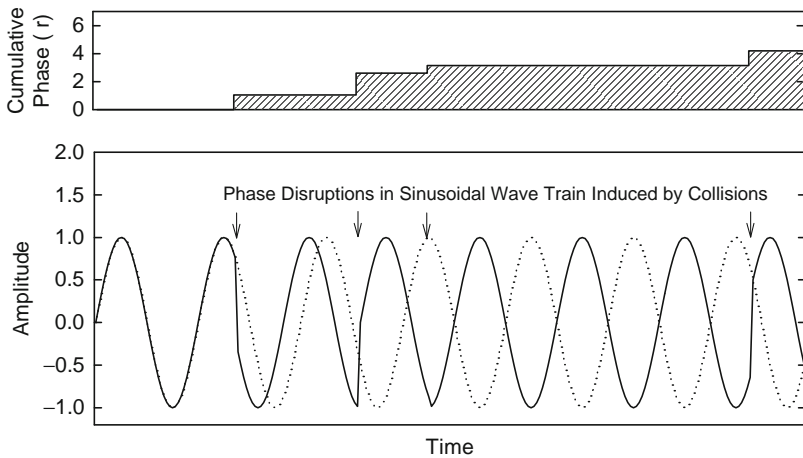


Fig. 2.5 *Top:* cumulative small phase shifts as a function of time in radians. *Bottom:* the *solid line* shows the resulting distortion of a sinusoidal wave train, the *dotted line* the undistorted wave train. The Fourier transform of the distorted wave train is a Lorentz profile

$$\begin{aligned}
F(\nu) &= \int_{-\infty}^{\infty} f(t)e^{-i2\pi\nu t} dt \\
&= \int_0^T f(t)e^{-i2\pi\nu t} dt \\
&= \frac{\exp[i2\pi(\nu_0 - \nu)T] - 1}{i2\pi(\nu_0 - \nu)}, \tag{2.28}
\end{aligned}$$

where F is the complex spectrum of the truncated oscillator. If the collisions are distributed in time T with a mean time between collisions of τ , then the probability of a collision is $P(T) = (\exp -T/\tau)/\tau$. The spectral profile $\phi(\nu)$ results from the average of the power spectrum $|F^2(\nu, T)P(T)|$ over the distribution of T :

$$\phi(\nu) \propto \int_0^{\infty} F^*(\nu, T)F(\nu, T) \frac{e^{-T/\tau}}{\tau} dT \tag{2.29}$$

$$= \frac{2\Delta\nu_L}{\pi [4(\nu - \nu_0)^2 + (\Delta\nu_L)^2]} \tag{2.30}$$

after the normalization described by (2.5).

Equation (2.30) is a Lorentz profile like that shown in Fig. 2.2. As with the damped oscillator, the full width of the line profile at half-intensity is $\Gamma/(2\pi)$. It demonstrates that disruption of the wave train of an emitting atom also produces a Lorentz profile.

2.2.3.3 Weisskopf Radius

One way (Weisskopf, 1932) to characterize the size of the phase disruptions induced by individual perturbers is in terms of an “impact parameter,” the closest distance between a perturber and an emitting atom.

Figure 2.6 shows a charged particle moving in a straight line past an emitting atom at a constant speed v . The impact parameter is ρ . Assume

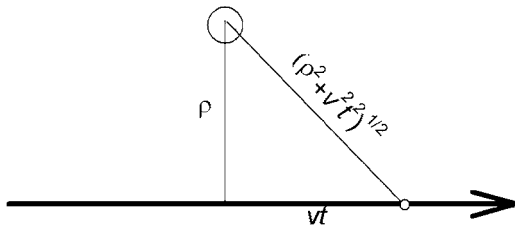


Fig. 2.6 The geometry of a perturber of speed v passing within a distance ρ (impact parameter) of an emitting atom

that, at any instant of time, the resulting shift in the angular frequency of emission $\Delta\omega$ can be modeled by a simple power law of the form

$$\Delta\omega = \frac{C_n}{R^n}, \quad (2.31)$$

where R is the instantaneous distance between the perturber and the atom, n expresses the power of the relationship between distance and frequency shift, and C_n is a constant of proportionality appropriate for the exponent n . In our rectilinear system, $R^2 = (\rho^2 + v^2 t^2)$, where ρ is the closest distance (the impact parameter) between the straight-line trajectory and the atom, and vt is the distance traveled by the perturber along the trajectory. The total shift in the phase of the wave train caused by the perturbation is the integral over the entire trajectory:

$$\Delta\phi = \int_{-\infty}^{+\infty} \Delta\omega dt \quad (2.32)$$

$$= \int_{-\infty}^{+\infty} \frac{C_n}{(\rho^2 + v^2 t^2)^{n/2}} dt \quad (2.33)$$

$$= \alpha_n \frac{C_n}{v\rho^{n-1}}, \quad (2.34)$$

where the coefficient

$$\begin{aligned} \alpha_n &= \frac{\Gamma\left(\frac{1}{2}\right) \Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)} \\ &= \pi, 2, \pi/2, 4/3, 3\pi/8, \dots \quad \text{for } n = 2, 3, 4, 5, 6, \dots \end{aligned} \quad (2.35)$$

If we ask what value of the impact parameter will result in a phase shift of 1 rad, (2.34) gives

$$\rho_0 = \left(\frac{\alpha_n C_n}{v} \right)^{1/(n-1)}, \quad (2.36)$$

a parameter known as the “Weisskopf radius.” The parameters $n = 2, 3, 4$, and 6 correspond to the interaction laws of the linear Stark effect, resonance broadening, quadratic Stark effect, and van der Waals broadening, respectively.

While the Weisskopf radius holds no particular physical meaning other than to characterize an arbitrarily chosen phase shift of 1 rad in the emitted wave train, it is useful for distinguishing quantitatively between “strong” and “weak” interactions of the perturbers and the wave train as we will see below.

2.2.3.4 The Impact and Quasistatic Models of Stark Broadening

Because of widely varying conditions within interstellar plasmas and stellar atmospheres, no unified theory of Stark broadening exists for astronomical applications. Instead, astronomers select an approximation appropriate to the particular environment they are considering.

Mathematical models for astronomical Stark broadening developed up to now fall into either of two extreme approximations: “impact” or “quasistatic.” We describe these briefly below, generally following the discussion in the monograph by Mihalas (1978) but see the monographs by Jefferies (1968), Griem (1974), and Sobelman (1992) for more details.

2.2.3.5 The Impact Approximation

If the total phase shift in the wave train is the result of an accumulation of discrete phase shifts within the time the atom is radiating, then the perturbation model is called “impact” because each perturbation represents a collision with the emitting atom. The simple illustration calculated early in Sect. 2.2.3 is an impact model. Its effect upon the line profile resulted from summing the results of all impacts, distributed as a probability function in that particular model. Figure 2.5 illustrates an impact situation where each discrete impact induces a rapid phase shift of less than 1 rad but the cumulative phase change in the wave train is about 4 rad.

Although the details of the impact approximation have evolved a great deal since its introduction in the early part of the twentieth century by Lorentz and others with regard to the spectrum of a damped, interrupted oscillator, its requirements are generally considered to be:

1. Each impact involves only one perturber and one atom at a time – a binary interaction. Simultaneous triple and multiparticle interactions are not considered.
2. The approximation requires a series of discrete impacts while the atom is radiating, where the effective duration τ_D of each impact must be much less than the interval τ_I between them.

To illustrate, Mihalas (1978) defines the effective duration τ_D of an impact such that the product $2\pi\Delta\nu \times \tau_D$ is the phase shift in the wave train induced by each impact, where $\Delta\nu$ is the frequency shift in the line produced by the collision. The separation interval $\tau_I \equiv (N\pi r_0^2 v)^{-1}$; i.e., it is the reciprocal of the collision rate. The variable N is the volume density of the perturbers and r_0 is the effective interparticle distance (mean free path). The “discreteness” criterion can then be written as $\tau_D/\tau_I \ll 1$. Because $\tau_D/\tau_I \propto (\rho_0/r_0)^3$, these conditions will generally occur when the interparticle distance r_0 is much larger than Weisskopf radius, i.e., when the distance parameters can be described by the inequality $r_0 \gg \rho_0$.

3. Each weak collision causes a nearly instantaneous phase shift in the wave train. Between these phase interruptions, the oscillation of the wave train continues unperturbed, as illustrated in Fig. 2.5.
In the rare case of strong collisions, i.e., with impact parameters $\rho < \rho_0$, a huge disruption of the wave train occurs. The impact model assumes that the oscillation stops and then restarts, with a complete loss of coherence. However, the contribution of strong collisions is small.⁵ The principal contribution to the broadening comes from collisions in which the impact parameters are greater than the Weisskopf radius.
4. The time distribution of these collisions may be described by a probability function.

2.2.3.6 The Quasistatic Approximation

This model represents the other extreme situation from the impact model:

1. It assumes each atom to lie in an electric field produced by a chaotic, statistical distribution of the perturbing (charged) particles surrounding it. Appropriate corrections due to electrical shielding (Debye shielding) of the negative (electron) charges by the positive (ion) charges may be required to calculate the field accurately.
2. The charged perturbers are considered to be at rest with respect to the atom over the duration of the emission, and the perturbation model is accordingly called “quasistatic.”
3. The shift in the oscillator frequency for each transition corresponds to the magnitude of the effective electrical field experienced by the radiating atom.
4. The line profile results from the probability distribution of the individual line frequency shifts, i.e., to the probability distribution of the electric fields for the radiating atoms. The amplitude at each frequency shift with respect to the center of the line is proportional to the number of atoms experiencing that particular field strength.

2.2.3.7 Application of the Stark Models to RRLs

Impact and quasistatic theories of Stark broadening give significantly different relations for the line profiles.

For a given physical situation, only one approximation can be used for a species of perturbers. *For a given kind of perturbers, the approximation must be either impact or quasistatic – not both.* Therefore, to calculate the linear Stark broadening of an RRL, one first needs to determine which of the two mathematical approximations is appropriate for a particular situation.

⁵ Griem (1967) shows that they can be as large as 20% in some circumstances.

This test depends upon both the temperature (relative particle speeds) and density (relative interparticle distances) that determine the mean free path of the ionized particles within the plasma.

The principle of energy equipartition complicates the situation further. If the constituents of the plasma are well thermalized, the ions and electrons will have the same kinetic energy. The higher mass and, therefore, slower moving ions may require a different approximation for Stark broadening than the lighter and faster moving electrons, so that in principle *both quasistatic and impact broadening approximations may be appropriate for the same plasma*: the former describing collisions with the atoms by the ions, the latter by the electrons.

2.2.3.8 Derivation of the Weisskopf Radius for RRLs from H II Regions

To use the Weisskopf radius to distinguish between the mathematical quasistatic and impact regimes of Stark broadening, we first need to derive the coefficient C_2 in its definition given by (2.36).

As described earlier, in a weak electric field, each energy level n is linearly split into $2n - 1$ separate components. Equation (7.36) of Sobelman (1992) shows that the energy splitting ΔE of a quantum level associated with a principal quantum number n due to the linear Stark effect of an electric field \mathcal{E} is

$$\Delta E = \frac{3}{2}n(p_1 - p_2)ea_0\mathcal{E}, \quad (2.37)$$

where e is the electronic charge and a_0 is the Bohr radius given by (1.7). The term $(p_1 - p_2)$ contains the parabolic quantum numbers⁶ p_1 and p_2 and is equivalent to n_f in (2.27). At a given n , $(p_1 - p_2)$ may be $n, n - 1, \dots, -n$ to create $2n - 1$ levels because there is only one 0 in the series.

For elastic perturbations in which no energy is exchanged between perturber and the emitting atom, we calculate the difference between the frequency shift of components of the upper (n') and lower (n) principal quantum levels of an RRL transition. Equation (2.37) gives these as

$$\Delta\nu = \frac{3h\mathcal{E}}{8\pi^2m_e} [n'(p'_1 - p'_2) - n(p_1 - p_2)], \quad (2.38)$$

where we have substituted the definition of the Bohr radius a_0 and where $p'_{1,2}$ and $p_{1,2}$ are the parabolic quantum numbers of the levels n' and n , respectively.

⁶ For calculation of perturbations induced by an external field, parabolic coordinates can be more useful than spherical coordinates because of the asymmetrical nature of the charge distribution within the atom. See Sect. 6 of Bethe and Salpeter (1957) or similar texts.

For most of the stronger Stark components of RRLs where $n \gg \Delta n \geq 1$, the differences in parabolic terms are small and $(p'_1 - p'_2) \approx n'$ and $(p_1 - p_2) \approx n$. Substitution then gives

$$\begin{aligned} \Delta\nu &\approx \frac{3h\mathcal{E}}{8\pi^2me} [(n')^2 - n^2] \\ &\approx \frac{3h\mathcal{E}}{4\pi^2me} n \Delta n, \quad n \gg \Delta n \geq 1. \end{aligned} \quad (2.39)$$

For convenience, we now parameterize this frequency shift into a Weisskopf radius for linear Stark broadening. The frequency form of (2.31) is

$$2\pi\Delta\nu \equiv \Delta\omega = \frac{C_2}{r^2}. \quad (2.40)$$

Substituting the definition of the electric field

$$\mathcal{E} \equiv \frac{Ze}{r^2} \quad (2.41)$$

into (2.39), substituting the resulting expression for $\Delta\nu$ into (2.40), and rearranging gives

$$C_2 = \frac{3Zh}{2\pi m} n \Delta n, \quad n \gg \Delta n \geq 1. \quad (2.42)$$

In turn, substituting C_2 into the definition of the Weisskopf radius for linear Stark broadening derived from (2.36) gives

$$\rho_0 = \frac{3Zh}{2mv} n \Delta n, \quad n \gg \Delta n \geq 1. \quad (2.43)$$

2.2.3.9 Which Approximation for the Stark Broadening of RRLs?

Applying (2.43) to RRLs from astronomical H II regions guides us to the appropriate mathematic model for calculating Stark broadening. We compare the interparticle distance ($r_0 \approx N^{-1/3}$) of the ionized gas with the Weisskopf radius derived above. If

$$N^{-1/3} \gg \rho_0, \quad (2.44)$$

the collisions will likely be discrete and well separated, i.e., the impact approximation would be the appropriate mathematical model for calculating the line broadening.

To apply this test quantitatively, it is first necessary to adapt the Weisskopf radius of (2.43) to the specific environment of the H II region. This is done by substituting an appropriate value for the ion velocity v_i . In a Maxwell-Boltzmann gas,

$$\left\langle \frac{1}{v_i} \right\rangle = \frac{4}{\pi} \frac{1}{\langle v_i \rangle} = \left(\frac{2M}{\pi kT} \right)^{1/2}, \quad (2.45)$$

where M is the mass of the ion and $\langle v_i \rangle$ is the mean value of the ion speed.⁷ The relative velocity between the hydrogen atoms and these ionized perturbers (protons) will be a factor of $\sqrt{2}$ larger because of the equal masses (via the principal of energy equipartition), and the appropriate substitution should then be

$$\left\langle \frac{1}{v_i} \right\rangle = \left(\frac{M}{\pi kT} \right)^{1/2}. \quad (2.46)$$

Combining (2.43), (2.44), and (2.46) and solving for the principal quantum number n gives the regime where the impact approximation for ion broadening will be valid:

$$n \Delta n \ll \frac{2m}{3ZhN_i^{1/3}} \left(\frac{\pi kT}{M} \right)^{1/2}, \quad n \gg \Delta n \geq 1, \quad (2.47)$$

$$\ll 6,850 \left(\frac{T}{10^4 \text{ K}} \right)^{1/2} \left(\frac{10^4 \text{ cm}^{-3}}{N_i} \right)^{1/3}, \quad n \gg \Delta n \geq 1. \quad (2.48)$$

For electrons, the expression will be similar except that the relative speed between electrons and atoms is

$$\left\langle \frac{1}{v_e} \right\rangle = \left(\frac{2m}{\pi kT} \right)^{1/2}, \quad (2.49)$$

and, hence, the validity region for impact broadening by electrons is

$$n \ll 208,000 \left(\frac{T}{10^4 \text{ K}} \right)^{1/2} \left(\frac{10^4 \text{ cm}^{-3}}{N_e} \right)^{1/3}, \quad n \gg \Delta n \geq 1, \quad (2.50)$$

because the ratio of the relative speeds is 30.3, as can be verified by dividing (2.46) by (2.49).

Substituting appropriate values of T and N_e , we conclude that the impact approximation is the appropriate one for calculating Stark broadening for RRLs from most H II regions for either ions or electrons.

There is an alternative way of determining which approximation to use. When the frequency shifts of the line become large, the impact approximation fails and the QS approximation must be used. In fact, one can calculate a frequency on the line profile $\Delta\nu_W$, corresponding to a collision with an impact

⁷ See, e.g., Sect. 4.11 of Chapman and Cowling (1960) or any other text on statistical mechanics for the general technique for deriving weighted speeds of a Maxwell-Boltzmann gas.

parameter equal to the Weisskopf radius, which roughly marks the transition from the validity region of the impact approximation (discrete, separated impacts) to that of the quasistatic approximation (statistically continuous electric field).⁸

2.2.3.10 The Effect of RRL Observations on Stark Broadening Theory

The theory of impact broadening applicable to spectral lines in plasmas had been developed since the late 1950s, principally for transitions in the optical part of the spectrum (Griem, Kolb and Shen, 1959; Griem, 1960). This research showed for the first time that, at high levels of excitation, collisions with electrons might be the main cause of line broadening, even for low electron densities (Griem, 1960). These calculations served as the basis for estimating the Stark broadening of RRLs just before they were detected. They showed that Stark broadening would be important for RRLs with transitions with $n > 100$ (Sorochenko, 1965).

As we have described in Sect. 2.1, the early observations of RRLs disagreed with the broadening theory. Stark broadening theory available at that time predicted line widths many times what was actually observed. For example, Lilley et al. (1966) in their announcement of their detections of the H156 α and H158 α lines at 1.7 GHz stated, “If the Stark equations as given by Kardashev were actually valid, ... for the source M17... we obtain [electron densities] of 10 cm^{-3} ” which, of course, would be ridiculously low for that H II region by about two or three orders of magnitude. They suggested that the Stark broadening theory developed for high densities may be responsible for the overestimate of the Stark effect. Mezger (1965) wrote to Sorochenko: “the line widths of your, our, and the Harvard observations plotted against line frequency can be fitted with a straight line. . . This means that the line broadening is entirely due to Doppler broadening and that no Stark broadening is effective. . . This contradicts all theories of Stark broadening.”

These observations indicated the necessity of revising Stark broadening theory for hydrogenic atoms in an astrophysical plasma. Examination of the available impact theory showed that, while it had been tested for low- n transitions, it did not consider an important characteristic of interaction with atoms at high levels of excitation. During elastic interactions with highly excited atoms, a compensation mechanism occurs. The distortions of the

⁸ Griem (1967) used this method by comparing the threshold QS frequency shift from the center of an RRL with the Doppler width. If this ratio $\gg 1$, then the validity region of the QS approximation falls far from the Doppler core, and the QS approximation cannot be used. Substitution of $T = 10^4 \text{ K}$ – typical for an astronomical H II region – into his expression gave a ratio of several thousand for both ions and electrons. Griem thereby concluded that the QS approximation was an inappropriate model for the Stark broadening of RRLs by either ions or electrons. This is the same conclusion we reach above with (2.48) and (2.50).

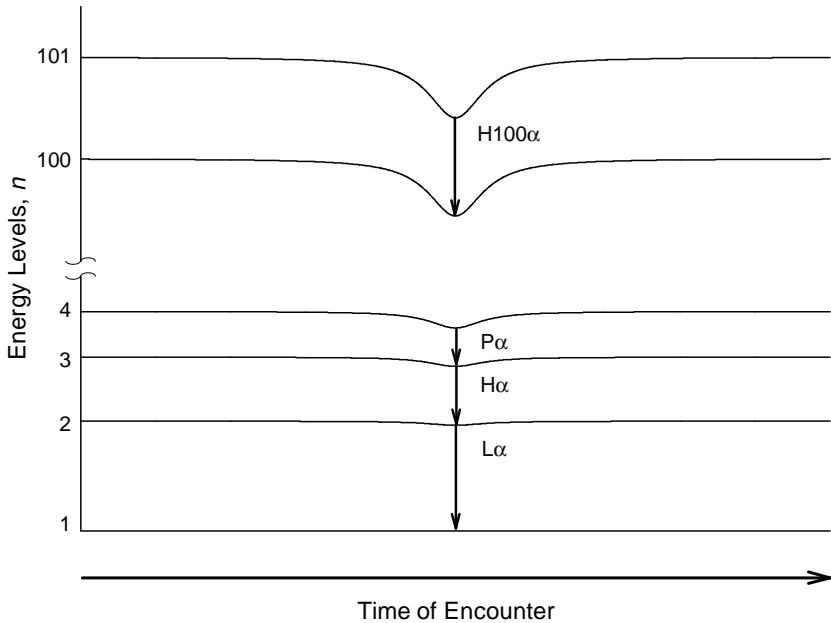


Fig. 2.7 Distortions in the energy of Stark components of quantum levels n caused by elastic collisions. This sketch – not to scale – illustrates the situation for the H100 α RRL and the optical lines: Paschen α , H α , and Lyman α lines. Unlike the optical lines, similar distortions of high- n levels compensate for each other to reduce the Stark broadening of RRLs enormously

energy levels produced by the Stark effect occurred in the same sense for closely neighboring levels as shown in Fig. 2.7. The *difference* between the upper and lower energy levels thereby changes much less than the energy levels themselves. As a result, the frequencies and the widths of the RRL line profiles, determined just by this difference, are not changed very much (Griem, 1967; Minaeva, Sobelman and Sorochenko, 1967).

For Stark broadening of the Balmer and Paschen series in the optical range, the compensation mechanism does not play a significant role. The Stark perturbation of the upper energy level substantially exceeds the perturbation of the lower one, the compensation mechanism is insignificant, and the theoretical calculations agree with experimental data within the measurement errors. Through experiments with the Balmer and Paschen series, some authors noticed that the theory needed to be improved to apply to the higher-order lines of hydrogen (Ferguson and Shlüter, 1963). Vidal (1964; 1965) noticed the slight difference between the Stark theory and the measured broadening of the optical lines. In spite of this, most laboratory researchers at that time were happy that the agreement of theory and data was so good. In fact, the puzzling RRL observations provided the impetus for a re-examination of the impact theory in astronomical plasmas.

2.2.3.11 Elastic and Inelastic Impact Broadening: A Closer Look

The collision of a charged particle with an atom can be either elastic or inelastic. “Elastic” means the interaction between perturber and the emitting atom does involve an exchange of energy; i.e., the interaction is adiabatic. The interaction splits and shifts the energy levels but does not change the n values. The collision efficiency is inversely proportional to the relative velocity of the atom and the charged particle. Because of their higher masses and correspondingly lower velocities, ions interact mainly through elastic collisions.

Inelastic collisions have a very different physical nature. They can induce nonradiative atomic transitions and thereby change the principal quantum number n . Therefore, they can be nonadiabatic. RRL broadening for these kinds of collisions involves not just through the splitting and shifting of the n levels but by decreasing the lifetime of the atomic quantum levels.

For inelastic collisions, the minimum impact radius ρ_{min} (or effective distance ρ_{eff}) becomes the critical parameter rather than the Weisskopf radius, which is used in elastic collisions. According to Griem (1967), this minimum impact radius is

$$\rho_{min} \approx \sqrt{\frac{5}{6}} \frac{hn^2}{2\pi m v_e} \approx \frac{n^2}{v_e}. \quad (2.51)$$

Substituting ρ_{min} into the inverse of (2.44), we obtain a criterion for the impact approximation in case of inelastic collisions:

$$n \ll \sqrt{\frac{1}{N_e^{1/3} < v_e^{-1} >}}. \quad (2.52)$$

For the same conditions ($Te = 10^4$ K, $Ne \leq 10^4$ cm $^{-3}$), the impact approximation is valid for inelastic collisions with electrons if $n \ll 1,500$ and for inelastic collisions with ions if $n \ll 300$.

Therefore, both for elastic and inelastic collisions of highly excited atoms with electrons in the conditions of H II regions, the impact approximation is appropriate through the entire range of RRLs. The question is the relative contribution of a given type of collisions to the RRL broadening. For collisions with ions, the impact approximation is valid only to moderate values of n . But, as will be shown below for interactions with ions, more significant is the question of whether inelastic collisions are possible at all.

The criterion that determines the division between elastic and inelastic collisions depends on the relationship of the collision time τ_c with the angular frequency $\omega_{n'n}$ of the transition $n' \rightarrow n$, where n' is the upper principal quantum number. Inelastic collisions are possible if (Griem, 1967; Kogan, Lisitsa and Sholin, 1987)

$$\omega_{n'n} \tau_c = \frac{\omega_{n'n}}{v/\rho_{min}} \ll 1, \quad (2.53)$$

or, in other words, when the transition frequency is significantly lower than the perturbation frequency (the reciprocal of the time of flight of the particle at the distance ρ_{min}).

Substitution into (2.53) of ρ_{min} from (2.51) and the frequency of the hydrogen α line from (1.22) gives the principal quantum number n of hydrogen atom where inelastic transitions between neighboring levels can occur:

$$n \gg \frac{1.8 R_H c h}{m v^2} \approx \frac{4.5 \times 10^{16}}{v^2}, \quad (2.54)$$

where R_H is the Rydberg constant for hydrogen. At the average electron velocity of $v_e \approx 6 \times 10^7 \text{ cm s}^{-1}$ (for $T = 10^4 \text{ K}$), inelastic transitions are possible for $n \gg 12$. A parallel situation occurs for interactions with ions. Because the relative atom-ion velocity is approximately 30 times lower than the electron velocity, the inequality of (2.54) changes to $n \gg 10^4$ and, accordingly, the threshold value of n for inelastic collisions increases by a factor of about 900.

Therefore, for Stark broadening of RRLs in H II regions, the impact approximation is the appropriate one. In collisions of excited atoms with electrons with transitions $n' \rightarrow n$, both elastic and inelastic types can occur. In contrast, only elastic collisions can occur for ion-atom collisions. Inelastic collisions with ions are unlikely and are negligible for Stark broadening of RRLs.

Note that the inelastic collisions with electrons cannot occur for atoms with low- n values and, therefore, cannot broaden recombination lines in the optical range.

2.2.4 Elastic and Inelastic Impact Broadening: Calculated Line Widths

Griem (1967) produced a revised theory of impact broadening specifically directed toward RRLs. For impact broadening by ions, Griem (1967) neglects the inelastic collisions to obtain an expression for the “ion” width of an RRL:

$$\begin{aligned} \Delta\nu_L^i = & \frac{3}{\sqrt{\pi}} \left(\frac{h}{2\pi m} \right)^2 \sqrt{\frac{M}{kT_e}} N_i n^2 \left[\frac{3}{2} + \frac{2}{e^2} \ln \left(\frac{2n}{3} \right) \right] \\ & \times \left[\frac{1}{2} + \ln \left(\frac{\lambda}{n} \frac{m}{h} \sqrt{\frac{kT_e}{M}} \right) \right], \end{aligned} \quad (2.55)$$

where $\Delta\nu_L^i$ is the full line width at half-intensity, λ is the spectral line wavelength, N_i is the volume density of the ions, e is the base of the natural logarithm, and M is the mass of the ion. After substitution of the numerical

values of the constants into (2.55), the expression simplifies, indicating dependence of Stark broadening on level number, ion density and temperature only:

$$\Delta\nu_L^i = \frac{6.7 \times 10^{-5}}{\sqrt{T_e}} N_i n^2 \ln(172n) [\ln(9.4 \times 10^{-3} n^2 \sqrt{T_e})] \quad (2.56)$$

in units of Hz when N_i is in units of cm^{-3} and T_e is in K.

For the interaction of the highly excited atoms with electrons, the inelastic collisions are the most important. The cancellation of the perturbations of the upper and lower states by elastic collisions greatly reduces their contributions to line broadening – as mentioned earlier. The inelastic contribution (Griem, 1967) is

$$\Delta\nu_L^e \approx \frac{10}{3} (2\pi)^{-5/2} \left(\frac{h^4}{m^3 k T_e} \right)^{1/2} N_e n^4 \left[\frac{1}{2} + \ln \left(\frac{2\pi k T_e \lambda}{3 h c n^2} \right) \right], \quad (2.57)$$

which Griem (1974) evaluates as

$$\Delta\nu_L^e \approx \frac{5.16^{-6} N_e n^4}{\sqrt{T_e}} \ln(8.25 \times 10^{-6} T_e n) \quad (2.58)$$

by substitution of numerical constants. Figure 2.8 shows these predictions for the inelastic broadening by ions and electrons. The ratio of (2.56) to (2.58) is $\ll 1$ for $20 < n < 200$ for $N_i = N_e$ and $T_e = 10^4 \text{ K}$, indicating that only inelastic electron collisions are important for Stark broadening of RRLs from H II regions – as can be seen in the figure.

Unlike Griem (1967; 1974), who utilized a modified classical approach of perturbers interacting inelastically with the emitting atoms, Brocklehurst and Leeman (1971) calculated RRL broadening with quantum theory applicable to the inelastic collisions of excited atoms with electrons. In this case, the line width for the transition $n_2 \rightarrow n_1$ is determined by the total cross sections of inelastic collisions:

$$\Delta\nu_L^e = \frac{1}{2\pi} N_e [\langle \sigma(n_1) v_e \rangle + \langle \sigma(n_2) v_e \rangle], \quad (2.59)$$

where $\langle \dots \rangle$ denotes the average over a Maxwellian distribution of the electron velocities and cross sections determined by summing over all possible transitions $\sigma_n = \sum_{\Delta n > 0} \sigma_{n, n \pm \Delta n}$.

Later, various authors defined the cross sections for inelastic collisions more exactly. The most exact are probably the cross sections calculated through semiempirical formulae by Gee et al. (1976). The rates of collision transitions from level n to level $n \pm \Delta n$ are $\alpha(n, n + \Delta n) = \langle \sigma(n, n + \Delta n) v_e \rangle$, obtained by averaging over the Maxwellian distribution of electron velocities, have an error less than 20% in the region of temperatures $100(100/n)^2 < T_e \ll 3 \times 10^9$ and quantum numbers $n, n + \Delta n \geq 5$.

Using the transition rates of Gee et al. (1976), Smirnov (1985) obtained the rather simple approximate expression for the RRL broadening by electron collisions in Hz when N_e is in units of cm^{-3} :

$$\Delta\nu_L^e = 8.2N_e \left(\frac{n}{100} \right)^\gamma \left(1 + \frac{\gamma}{2} \frac{\Delta n}{n} \right). \quad (2.60)$$

The factor γ is the growth rate of RRL collisional broadening as a function of n . In the range $n = 100 - 200$, γ increases while temperature decreases, but the value of its variation $\Delta\gamma$ is small. For example, $\Delta\gamma \leq 0.16$ when temperature decreases from 10^4 to 5×10^3 K and the average value of γ is 4.5.

Because the probability of a collisional transition is a sharply decreasing function of Δn , it is adequate to sum only over $\Delta n \leq 10$ to calculate the rates $\alpha(n, n + \Delta n)$.

Figure 2.8 shows the broadening by electrons calculated from (2.60). For completeness, the figure also shows broadening by inelastic ion collisions. These ion calculations utilized a classical approach for the collision of ions

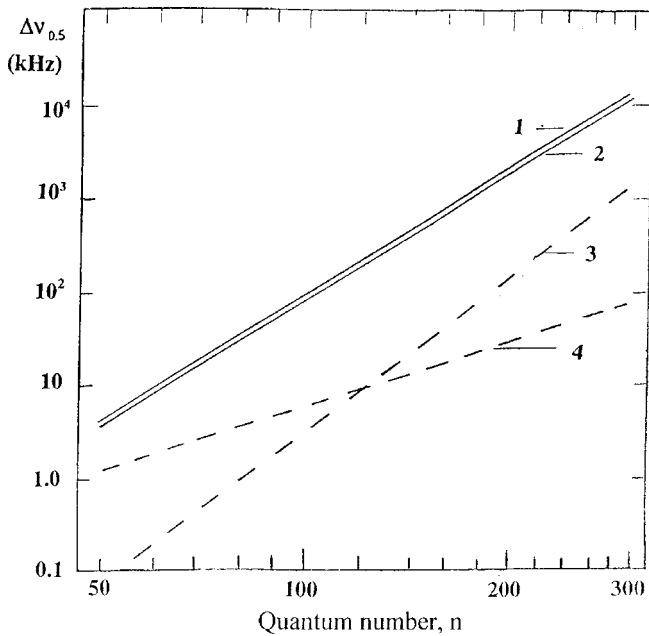


Fig. 2.8 Stark broadening of hydrogen RRL in plasmas as a function of quantum number n for $N_e = N_i = 10^4 \text{ cm}^{-3}$ and $T_e = 10^4 \text{ K}$. Solid lines show the broadening by electron collisions, dashed ones by ion collisions. Curve 1 is from inelastic electron collisions calculated by a classical approach from (2.58). Curve 2 is the same, but calculated with an aid of inelastic collisions with cross sections from (2.60). Curve 3 is from inelastic ion collisions from (2.61). Curve 4 is from elastic ion collisions from (2.56)

with excited atoms and appropriate cross sections (Beigman, 1977). The complete expression for the ion broadening is complex but can be simplified to a more compact form obtained by Smirnov (1985):

$$\Delta\nu_L^i = (0.06 + 0.25 \times 10^{-4}T_e) \left(\frac{n}{100}\right)^{\gamma_i} \left(1 + \frac{2.8 \Delta n}{n}\right) N_i, \quad (2.61)$$

where $\gamma_i = 6 - 2.7 \times 10^{-5}T_e - 0.13(n/100)$. The value of $\Delta\nu_L^i$ obtained from (2.61) differs no more than 5% from the exact values (Beigman, 1977) in the range of temperatures $5,000 \text{ K} < T_e < 15,000 \text{ K}$ and quantum numbers $50 < n < 300$.

Inspection of Fig. 2.8 shows that the dominant mechanism for the Stark broadening of RRLs is inelastic electron collisions by a large margin. The similarity of curves 1 and 2 also demonstrates that the broadening calculations made with classical physics agree well with those made with quantum theory.

The Stark broadening by ions is significantly less important for RRLs than broadening by electrons for two reasons. For elastic collisions, the energy levels shift in the same direction, thereby compensating each other and reducing the contribution to line broadening from this mechanism. The result is that the dependence of the broadening on principal quantum changes from $\Delta\nu \propto n^4$ to $\Delta\nu \propto n^2$, the lower exponent becoming significant at $n \sim 100$ and more so for $n > 100$. For inelastic collisions, the larger ion mass means smaller velocities and a correspondingly smaller probability of collision with the atoms than the electrons. The overall result is that inelastic electron collisions dominate the Stark broadening of RRLs and, in subsequent theoretical calculations and comparison with observations, we shall use (2.60) to estimate Stark broadening of RRLs.

In (2.30), $\Delta\nu_L$ is the full width at half-maximum of the line profile $I(\nu)$ and ν_0 is the center frequency of the line. Strictly speaking, since line profile is a summation over individual Stark components, the sum of Lorentzian profiles does not have a Lorentzian line shape. But, according to Smirnov (1985), the Stark-broadened profile of the RRLs may be considered to be Lorentzian with an accuracy sufficient for practical use. Compared to the “pure” Lorentzian, the summed profile is about 2–3% higher at maximum and about 1–2% lower at half-maximum.

2.2.5 Combining Profiles: The Voigt Profile

In general, RRLs will have line profiles influenced by both thermal effects (Gaussian profiles) and linear Stark effects (Lorentz profiles). If the two mechanisms are independent, the two-component profiles can be convolved to produce a composite line profile. The thermal broadening involves only the

velocities of the atoms. The impact Stark broadening involves only the velocities of the colliding electrons because, in energy equipartition, their masses are much smaller and, hence, their velocities are much greater than the emitting atoms. Therefore, the two velocity distributions are independent, and we can convolve the two-line profiles to obtain the composite line profile:

$$\Psi(\nu, \eta) \propto \int_{-\infty}^{\infty} \phi_G(\nu') \phi_L(\nu - \nu') d\nu', \quad (2.62)$$

where η indicates the relative weights of the line profiles G and L in the mix. The proportionality sign signifies that the resulting function $\Psi(\nu)$ will be normalized such that

$$\int_{-\infty}^{\infty} \Psi(\nu) d\nu = 1, \quad (2.63)$$

so that the resulting composite profile can be written

$$I_V(\nu) = I_0 \Psi(\nu). \quad (2.64)$$

Combining by substitution and convolution equations (2.19), (2.20), (2.30), and (2.60) and normalizing produces the composite line profile:

$$\begin{aligned} I_V(\nu) &= \left(\frac{4 \ln 2}{\pi} \right)^{1/2} \frac{1}{\Delta\nu_G} H(a, v) \\ &\approx \frac{1.665}{\Delta\nu_G \sqrt{\pi}} H(a, v), \end{aligned} \quad (2.65)$$

where the a parameterizes⁹ the mix of Gaussian and Lorentzian profiles:

$$\begin{aligned} a &= \frac{(\ln 2)^{1/2} \Delta\nu_L}{\Delta\nu_G} \\ &\approx \frac{0.833 \Delta\nu_L}{\Delta\nu_G}, \end{aligned} \quad (2.66)$$

and the parameter v parameterizes¹⁰ the distance from line center:

$$\begin{aligned} v &= \frac{(4 \ln 2)^{1/2} (\nu - \nu_0)}{\Delta\nu_G} \\ &\approx \frac{1.665 (\nu - \nu_0)}{\Delta\nu_G}, \end{aligned} \quad (2.67)$$

and the function H is

$$H(a, v) \equiv \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2} dt}{a^2 + (v - t)^2} dt. \quad (2.68)$$

⁹ Some authors define a differently.

¹⁰ The parameter v is sometimes listed as b .

In spectroscopy, this composite profile $I_V(v)$ is called a *Voigt profile* because it uses the function $H(a, v)$ named after the German spectroscopist Voigt (1913). Hjerting (1938) describes this function in detail, and Davis and Vaughan (1963) and Finn and Mugglestone (1965) tabulate it in convenient forms.

Examination of (2.65) shows the characteristics of the Voigt profile. Near the Doppler core, i.e., where $v \approx t$, the function H behaves like an exponential as one would expect from a Gaussian profile. In fact, when the collisional broadening becomes very small such that $a \Rightarrow 0$, the function $H(a, v) \Rightarrow 1$, and the intensity at the center of the line ($\nu_0 = 0$) is

$$I_V(\nu_0) = \frac{1.665}{\Delta\nu_G\sqrt{\pi}}, \quad (2.69)$$

which is the peak intensity of a purely Doppler-broadened line as can be seen from (2.20).

On the other hand, well away from the line core where a^2 is small with respect to $(v-t)^2$, the function $H \propto v^{-2}$ and falls rapidly with increasing $|v|$ and, hence, with increasing distance ν from line center as is characteristic of the wings of a Lorentz profile. In fact, when $\Delta\nu_L \gg \Delta\nu_G$, the broadening is a pure Lorentzian one, with the intensity at line center:

$$I_V(\nu_0) = \frac{2}{\pi\Delta\nu_L}, \quad (2.70)$$

as shown by (2.30). When the two-line widths are the same, i.e., $\Delta\nu_L = \Delta\nu_G$, the ratio of the intensities at line center show the Gaussian to be the stronger

$$\frac{I(\nu_0)_G}{I(\nu_0)_L} = 1.476, \quad (2.71)$$

because the normalized Lorentzian line profile has more emission in the line wings than the normalized Doppler profile.

Smirnov (1985) gives a useful simple approximation for the width of the Voigt profile described by (2.65):

$$\Delta\nu_V = 0.5343 \Delta\nu_L + \sqrt{\Delta\nu_G^2 + (0.4657 \Delta\nu_L)^2}, \quad (2.72)$$

where $\Delta\nu_V$, $\Delta\nu_G$, and $\Delta\nu_L$ are the full widths at half-intensity of the Voigt, Gaussian (Doppler), and Lorentz (impact) profiles, respectively. This expression gives an error of less than 0.08% for $0.1 < a < 10$.

Figure 2.9 compares three profiles of equal area: Gaussian, Lorentz, and Voigt profiles. Both Gaussian and Lorentz profiles have full widths at half-intensity of 20. The Voigt profile results from the convolution of these two profiles, as given by (2.65). Its corresponding width is 35, as given by (2.72). The convolution process of the Gaussian and Lorentz profiles shifts radiant

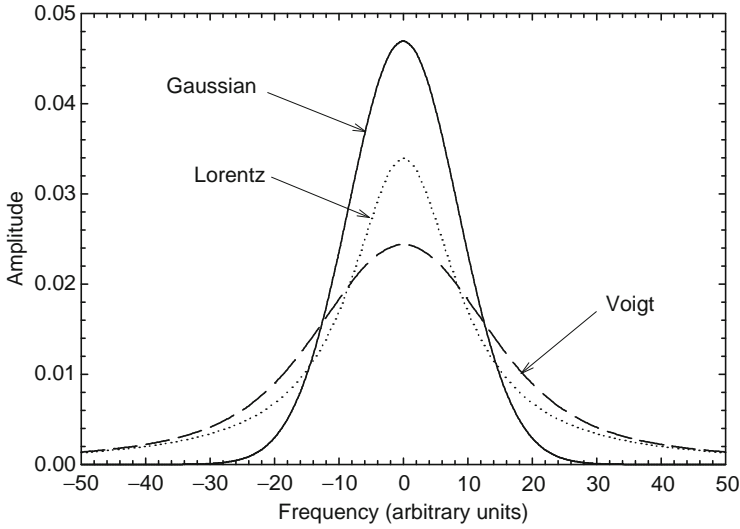


Fig. 2.9 Comparison of Gaussian, Lorentz, and their convolution (Voigt) line profiles. Each profile has the same area. The full widths at half-intensity are 20 for the Gaussian and Lorentz profiles, and 35 for the resulting Voigt profile

energy from their line cores to the near wings of the composite profile, thereby decreasing the intensity of the composite core near the line center and widening the core. The far wings of the Voigt profile are the same as those of the Lorentz profile.

2.2.6 Observational Test of the Revised Theory

The revised theory of RRL broadening in plasma decreased the calculated values of the line widths, thereby explaining the first observations of RRLs with n up to 166 from H II regions.

However, there were still problems. The revised theory conflicted with observations. Transitions up to $n = 220$ were observed in Orion nebula – the most studied H II region – but no Stark broadening was detected (Pedlar and Davies, 1972). Figure 2.10 shows the measured line widths in the Orion nebula plotted from data in the RRL catalogue of Gulyaev and Sorochenko (1983). The observations agreed with theory only for $N_e = 100 \text{ cm}^{-3}$, which is much smaller than the electron density of $N_e = 10^4 \text{ cm}^{-3}$ determined from the optical forbidden lines (Osterbrock and Flather, 1959).

Something fundamental was being overlooked. As discussed earlier, theoretical calculations of Stark broadening by various methods produced similar results. Differences in the estimates of cross sections for electron impact broadening agreed within 30% (Griem, 1974) even though the calculations

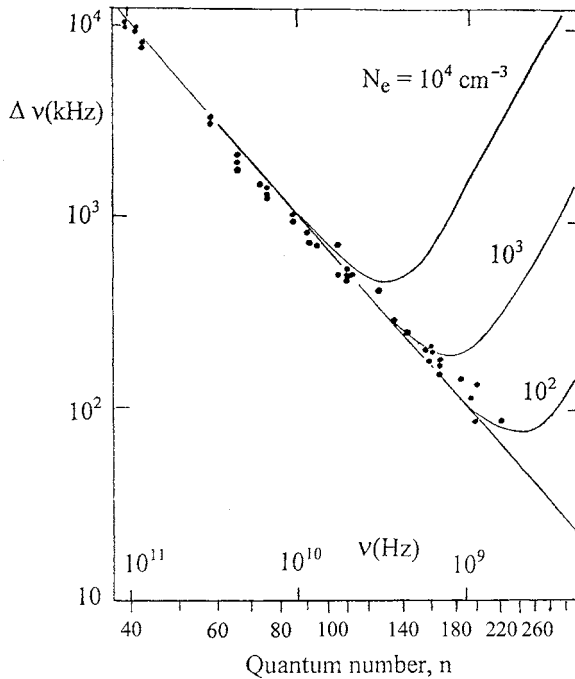


Fig. 2.10 The full widths of $Hn\alpha$ RRLs from the Orion nebula plotted as a function of principal quantum number n . The straight line shows pure Doppler broadening with $\Delta\nu_G/\nu = 10^{-4}$. The curved lines show the expected line widths $\Delta\nu$ calculated for different values of N_e from (2.26), (2.60), and (2.72)

were made by different researchers. Yet, the observations of Stark broadening of the RRLs disagreed with theory by an order of magnitude.

The explanation of the discrepancy of theory and observation was found by closely examining the characteristics of the astronomical targets. The theory of spectral line broadening in plasmas was based upon the *homogeneous* densities, corresponding to laboratory conditions. However, in the interstellar medium, such as H II regions, this density is almost always inhomogeneous.

Some hints had already appeared in the literature but many in the radio astronomical community seemed not to be paying attention. Hoang-Binh (1972) specifically suggested that the low-density gas within the H II regions was determining the line shapes such that Stark broadening would not be seen. Brocklehurst and Seaton (1972), using a spherically symmetrical model of the Orion nebula with the density decreasing from the center outward toward the borders, were able to fit the widths and intensities of all RRL observations up to $n = 220$. Simpson (1973a; 1973b) suggested a mix of low and high electron densities from her radio and optical studies of the Orion nebula. Gulyaev and Sorochenko (1974) came to similar conclusions as Brocklehurst and Seaton after modeling the Orion nebula as a dense central core ($N_e = 10^4 \text{ cm}^{-3}$)

surrounded by a rarefied ($N_e = 200 \rightarrow 700 \text{ cm}^{-3}$) extended envelope, based upon earlier continuum observations at a wavelength of $\lambda = 8 \text{ mm}$ (Berulis and Sorochenko, 1973).

Still, at that time, not all astronomers were convinced that the explanation for the enigmatic Stark broadening – or rather, the absence of – lay solely in the density structure. Lockman and Brown (1975a) suggested that the densest parts of the Orion nebula must be somehow cooler than the surrounding lower density gas. They considered the nebula essentially to consist of three regions, each with a unique temperature, density, and size. And, Shaver (1975) also worked out a model involved varying densities but with spatially varying temperatures as well.

The physical reasons for the absence of Stark broadening in the observed profiles in the case of an inhomogeneous distribution of electron density in the nebula are rather evident. They are a consequence of the fact that absorption in a plasma increases as frequency decreases, gradually leading to complete opaqueness. Figure 2.10 suggests this effect. The decrease in the observed line widths with increasing n is correspondingly a decrease with frequency. Here, the increase in gas opacity means that the lines increasingly reflect conditions in the more transparent, lower density outer regions of the H II region. For the high- n RRLs that occur at lower frequencies, the core of the H II region becomes opaque. The more rarified envelope gas would contribute much less if any Stark broadening even at the same electron temperature.

To complicate analysis further, the observations shown in Fig. 2.10 were made with radio telescopes of differing physical diameters and, accordingly, differing beamwidths – even for the same lines. Moreover, the beamwidths of these telescopes increased with decreasing frequency and, hence, with increasing n . The result was that the data involved different regions of the H II region except for the small telescopes whose large beamwidths included the entire nebula.

The changing opacity as a function of frequency and location in most H II regions and the variations in the beamwidths of the radio telescopes prevented the detection of Stark broadening as a function of principal quantum number n .

The new models of H II regions incorporating density inhomogeneities explained the absence of Stark broadening in astronomical RRLs. But, these “negative” results were insufficient to verify the refined theories of Stark broadening. Such verification had to await specially designed observations.

Minaeva et al. (1967) suggested observing RRLs of increasing order. Here, as Δn increases from 1 to 2, 3, and 4, the corresponding $\text{H}\alpha$, $\text{H}\beta$, $\text{H}\gamma$, and $\text{H}\delta$ RRLs would occur near the same frequency with suitable choices of n and, therefore, with the same beamwidth. If the observations are made at sufficiently high frequency where the gas is optically thin, the lines will be emitted by the same volume of gas. The idea was that the higher-order lines in the series would exhibit increasing amounts of Stark broadening compared with the pure Doppler broadening of the $n\alpha$ line.

The basic Bohr theory predicts the series. Rewriting (1.17) for $\Delta n \equiv n_2 - n_1 \ll n$ gives

$$\nu \cong \frac{2cR_H \Delta n}{n^3}, \quad n_1, n_2 \gg 1. \quad (2.73)$$

If $\Delta n = 1, 2, 3, 4, \dots$ and values of n are selected to make $\Delta n/n^3 \approx \text{constant}$, then (2.73) will give a sequence of RRLs of increasing order but with similar rest frequencies. For example, the frequencies of the H110 α , H138 β , H158 γ , and H173 δ RRLs all lie between 4.87 and 4.91 GHz.¹¹

The problem with this technique is that the intensities of the RRLs weaken with increasing order, varying approximately as $(\Delta n)^{-2}$. The decreasing intensities make it impossible to observe a series with a single telescope to detect with adequate precision a long sequence of higher-order lines with increasing amounts of Stark broadening.

A special observing technique overcame this difficulty by using two large radio telescopes of different diameters (Smirnov, Sorochenko and Pankonin, 1984). Rather than one telescope observing a complete series of higher-order lines, the different diameters of two telescopes observed at different frequencies such that a large range of high-order lines were observed at the same beamwidths. Combining the line observations from both telescopes as a function of beamwidth allowed a wide range of quantum numbers to be sampled at a single beamwidth.

In this experiment (Smirnov et al., 1984), the smaller telescope, the 22-m telescope of the Lebedev Physical Institute in Pushchino, Russia, shown in Fig. 1.4, observed RRLs at 22 and 36.5 GHz. The larger telescope, the 100-m telescope of the Max-Planck-Institut für Radioastronomie at Effelsberg, Germany, shown in Fig. 2.11, observed RRLs at 5 and 9 GHz. The RRLs observed at 5 GHz with the 100-m telescope used the same beamwidth (2'6) as those observed at 22 GHz with the 22-m telescope. Similarly, those observed at 9 GHz with the 100-m telescope used the same beamwidth ($\approx 1'7$) as those observed at 36.5 GHz with the 22-m telescope. It was then possible to assemble two series of line profiles for the same volume of gas over a wide range of principal quantum numbers – with signal-to-noise ratios adequate to search for Stark broadening.

In each beamwidth series, the smallest- n H α line was assumed to be dominated by Doppler broadening and, hence, Gaussian except for small deviations due to possible large-scale gas flows in the nebular gas subtended by the telescope beams. These transitions were the H56 α and H66 α lines for the series associated with the small and large beamwidths, respectively. Fitting each line with a Gaussian established the Doppler component assumed to be part of the Voigt profile for each of the higher- n RRLs in the series for that particular beamwidth and, hence, for that particular volume of nebular gas. Fitting Voigt profiles to each spectrum composed of the now known Doppler

¹¹ See also very recent observations described at the end of this section.



Fig. 2.11 The 100-m radio telescope at Effelsberg, Germany

component would determine the residual Lorentz profile created by the Stark broadening.

The expected small size of the Lorentz residual required that the RRL line observations be processed carefully. Averaging individual spectrograms increased the signal-to-noise ratios of the composite line profile. Particular attention was paid to removing the inevitable instrumental baselines from the composite spectra, including the weak sinusoidal ripples generated by multiple reflections (standing waves) between parts of the telescope surfaces.

The results clearly showed Stark broadening in RRLs. Figure 2.12 shows the variation in line widths as a function of frequency for the smaller beamwidth observations. The line widths increase systematically with principal quantum number n as expected. Figure 2.13 shows the widths of the Lorentz component of the fitted Voigt profiles, after extraction of the Gaussian component contributed by Doppler effects.

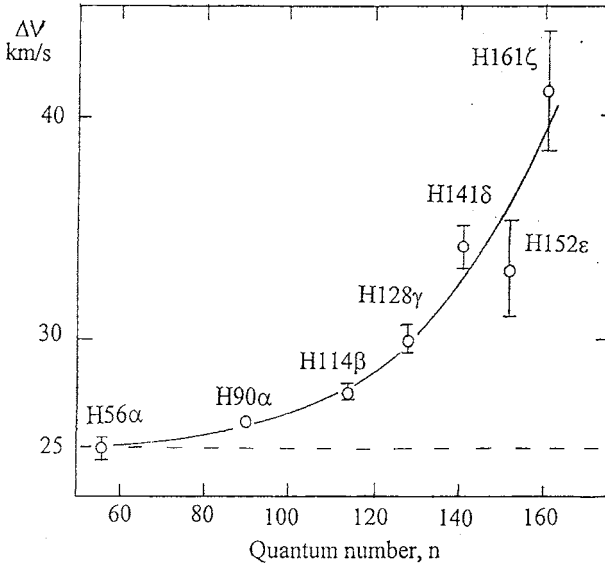


Fig. 2.12 The full width at half-intensity of the Voigt profiles plotted against principal quantum number. The width of the H56 α line is presumed to be entirely Doppler. The *solid line* is an approximation fitted to the observations obtained by Smirnov et al. (1984)

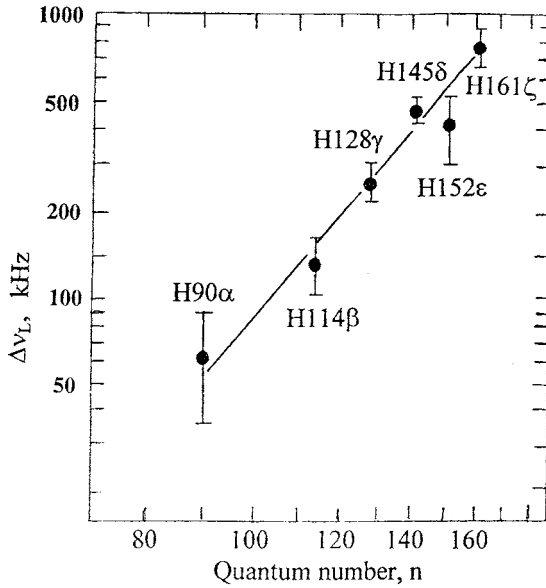


Fig. 2.13 The full widths at half-intensity of the Lorentz component of Voigt profiles of RRLs observed at 9 GHz from the Orion nebula (Smirnov et al., 1984). The *slope of the regression line* corresponds well to theoretical predictions of Stark broadening theory

The power law dependence of principal quantum number n derived from the data agreed well with theory. Fitting the observed Lorentz widths to the simple expression

$$\Delta\nu_L = A N_e \left(\frac{n}{100} \right)^\gamma \left(1 + \frac{\gamma}{2} \frac{\Delta n}{n} \right) \quad (2.74)$$

over the observed range of n and Δn gives a γ of 4.4 ± 0.6 . The constant $A = 8.2 \text{ Hz cm}^3$ if the electron density is approximately 10^4 cm^{-3} as suggested from observations of the O III optical lines from the core of the nebula (Osterbrock and Flather, 1959). Similar electron densities have been derived from more recent observations of the O II, O III, and Cl III emission lines from the Orion nebula (Peimbert and Torres-Peimbert, 1977). This value of γ agrees well with the value of 4.4 derived theoretically by Brocklehurst and Leeman (1971).

The second series of RRLs corresponding to the larger 2'6 beamwidths obtained the smaller value of $\gamma = 3.8$. This value may result from the inclusion of more rarified gas in the telescope beam, thereby contributing less Stark broadening to the line profiles. This de-emphasis of Stark broadening in the large-beam observations could explain why Churchwell (1971), Davies (1971), Simpson (1973a), and Lang and Lord (1976) were unable to detect the broadening.

An example of new information is a report of observations of line broadening inconsistent with current theory. Bell et al. (2000) used an innovative observing technique (Bell, 1997) to observe many orders of hydrogen recombination lines within a single spectral window. Observations near 6 GHz showed RRLs over the range $1 \leq \Delta n \leq 21$ from the Orion and W51 H II regions the measured line width to increase with the principal quantum number n , reaching a maximum near $n = 200$ before decreasing. This result conflicts with theoretical predictions of Stark broadening.

Two explanations were considered and rejected. Watson (Watson, 2007) examined electron impact broadening for $\Delta n \leq 70$ and found that the widths of lines with different n and Δn must increase up to $n \geq 300$. Oks (Oks, 2004) suggested that increasing collisions with ions at higher values of n might explain the Bell's observations. However, Griem (2005) found the contributions of ion collisions too small to create a maximum broadening at $n \approx 200$.

What could be wrong? Central to these results is Bell's technique of producing radio spectra by rapidly switching the frequency window by frequency intervals large compared with the expected line widths but small with respect to sinusoidal "ripples" in the baseline produced by variations in impedance match as a function of frequency. Essentially, this process is a Fourier convolution of the spectrum with the function $F(\nu) = \delta(\nu) - \pi(\nu)$, using nomenclature developed by Bracewell (1965). Recovering the astronomical spectrum requires a Fourier deconvolution. However, a rigorous deconvolution is not possible because the process would involve division by 0 at points within the transform domain. Therefore, Bell (1997) developed an iterative, guess-and-subtract technique he calls LINECLEAN. As with similar deconvolution

algorithms used in radio astronomy, the accuracy of the deconvolved spectra is sensitive to the distribution of noise (signal-to-noise ratio) in the observations, which is unknown a priori.

The Bell et al. (2000) results are so different from what had been expected, and the observing technique is so new, that we suggest waiting for an independent confirmation of the observations before accepting a fault in the present theory of RRL Stark broadening.

We conclude this section by noting that observations of the line widths of RRLs not only provided new information regarding the density structure of H II regions, but also offer an opportunity for testing and refining theories of elastic and inelastic Stark broadening in low-density plasmas.

2.3 Intensity of Radio Recombination Lines

2.3.1 *Radiation Transfer*

Radiation from astronomical bodies is the primary source of information for astronomers. Not only does it carry spatial information that locates the position of object in the cosmos, but also the radiation itself carries characteristics that identify its origin and describe its environment. In addition, the interstellar medium (ISM) between its source and the telescope also impresses information on the radiation. Disentangling the characteristics of these natural messengers from the cosmos, identifying the nature of the radiation source, and sharing their findings through publication in journals is what astronomers do for a living.

The special physics used to analyze these characteristics is called “radiation transfer.” In its simplest form, radiation transfer involves two basic assumptions. It assumes symbolic loss and gain mechanisms for the radiation along the path of propagation without regard to the detailed processes involved – initially, at least. Usually, it assumes all processes associated with the radiation to be “stationary,” i.e., that the parameters do not change over the timescale of the observations.

Neither of these assumptions is realistic. Eventually, the astronomer needs to understand the detailed physics of the loss and gain mechanisms, which can change within the timescales of the observations. But, all analyses must have a beginning. It is better to see how much understanding can be achieved with a simple model before adding complexities.

Consider the situation sketched in Fig. 2.14. As the radiation intensity $I(0)$ moves through the cloud from back to front, it loses intensity to the cloud through absorption and by scattering photons into directions other than the observer. On the other hand, the radiation intensity increases from additional radiating elements dx within the cloud. The net change in the intensity dI is

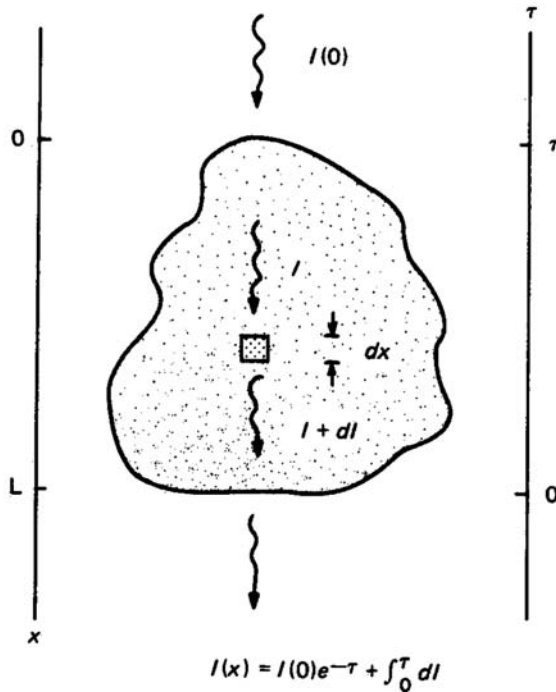


Fig. 2.14 The background radiation intensity $I(0)$ travels from the back of a cloud toward the observer where it is strengthened or weakened by the incremental amount dI in each distance interval dx . The equation shows the intensity seen by the observer. From Gordon (1988). Reproduced with permission of Springer-Verlag

$$dI = -I\kappa dx + j dx, \quad (2.75)$$

where x is the distance the radiation travels *toward* the observer, κ is called the *linear absorption coefficient* and accounts for *all* depletions from the radiation in the direction of the observer, and j is called the *linear emission coefficient* and accounts for *all* gains in intensity in the direction of the observer. The first term on the right accounts for the weakening in the intensity by extinction, i.e., by scattering or absorption within the differential distance element, the second for the strengthening by emission.

Integrating (2.75) from the far side of the nebula to the observer¹² gives the intensity seen by the observer as

$$\begin{aligned} I(\tau = 0) &= \int_{\text{cloudback}}^{\text{observer}} dI \\ &= I(\tau)e^{-\tau} + \int_0^{\tau} \frac{j}{\kappa} e^t dt, \end{aligned} \quad (2.76)$$

¹² The integration uses the integrating factor $e^{-\tau}$ and integration by parts.

where the parameter τ is called the “optical depth”:

$$\tau \equiv - \int_{x_1}^{x_2} \kappa(x) dx. \quad (2.77)$$

The first term of (2.76) is the background radiation attenuated as it passes through the medium. The second term describes the contribution to I by the medium itself, including emission and extinction in the direction of the observer. Under some conditions, a third term might be appropriate to include a radiation source between the emitting “cloud” and the observer.

The parameter τ reduces the number of variables by concealing the spatial variation of the extinction coefficient $\kappa(x)$ along the sight line x that, in most circumstances, cannot be known. Because factor $e^{-\tau}$ is a direct measure of the fractional loss of photons propagating through a medium, the factor $(1 - e^{-\tau})$ is the fraction of photons surviving that passage.

For those unusual circumstances where the sight line conditions are known, (2.76) becomes

$$I(x = L) = I(0)e^{-\hat{\kappa}L} + \int_0^L \frac{j}{\kappa} e^{\kappa(x-L)} dx, \quad (2.78)$$

where $\hat{\kappa}$ is the mean value of the extinction along the sight line calculated as $[\int_L \kappa(x) dx]/L$.

The symbol I specifies a radiant quantity known as the “specific intensity” (Chandrasekhar, 1950; Jefferies, 1968). Its general definition is radiant energy per unit time per unit collecting area per unit bandwidth interval per unit solid angle. Sometimes also called “brightness,” here I means I_ν in units of ergs per second per Hertz per square centimeter per steradian.

Because of their different bandwidth dependencies, the relationship between the frequency (I_ν) and wavelength (I_λ) forms of I is

$$I_\nu d\nu = I_\lambda d\lambda, \quad (2.79)$$

and, hence,

$$I_\nu = I_\lambda \frac{c}{\nu^2}. \quad (2.80)$$

What does I mean physically? The specific intensity is a simple proportionality constant between the radiant energy E and all of the factors that affect its detected strength, such as the time length of the observations dt , the collecting area of the surface or telescope $d\sigma$, the bandwidth of the detector $d\nu$ or $d\lambda$, and the solid angle into which the energy is collected $d\Omega$. In this sense, I is the essence of radiation that is independent of the observing process. Figure 2.15 shows the geometry.

By design, the specific intensity has some peculiar properties. Through its definition, I is constant along any ray in free space. Even if the radiation cone diverges from source to observer, I will not weaken between source

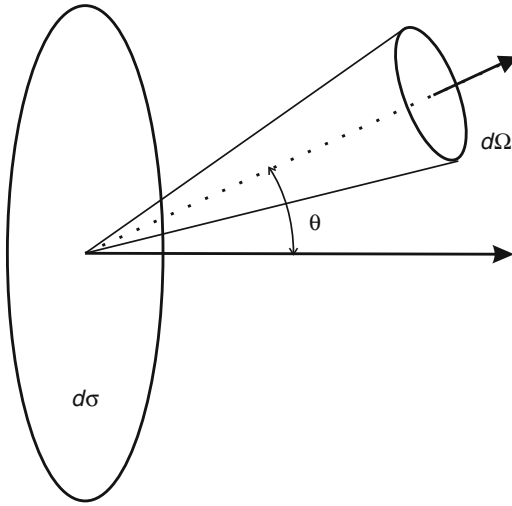


Fig. 2.15 The geometry of specific intensity I passing through a surface element $d\sigma$ and an angle θ to the surface normal n

and observer in the absence of absorption or scattering. Also, I does not change by *perfect* reflections at any mirror or combination of mirrors. Using a mirror to focus the Sun's rays to heat an object does not change I but, rather, increases the solid angle of the cone containing I and, correspondingly, the angular energy falling on the object. In fact, the spectral flux density of radiation S_ν is “energy/unit collecting area/unit frequency interval/unit time,” defined as

$$S_\nu \equiv \int_{\text{solid angle}} I_\nu d\Omega, \quad (2.81)$$

and showing that S_ν can be varied by changing the solid angle range of the integral without changing I . Appendix F discusses the relationship of S_ν to parameters directly observed by radio telescopes.

2.3.1.1 Source Function

Both (2.76) and (2.78) contain the ratio j/κ . These linear absorption and emission coefficients, κ and j , are physically related. Their ratio j/κ defines the ratio of photons being emitted to those being lost from I to each point along the radiation path. Consequently, this ratio is called the “source function,” usually symbolized by S . The source function is an intrinsic property of the emitting medium. Its specification as a function of location and frequency (or wavelength) allows the solution of the equation of radiation transfer.

In some circumstances, the source function is easy to specify without detailed calculations of the linear coefficients j and κ . In an enclosure from which no photons can escape, the radiation field will reach a state called “thermodynamic equilibrium” that is fully specified by the *single parameter* T called “temperature.” Here,

$$j = \kappa B(T) \quad (2.82)$$

from Kirchhoff’s law of thermodynamics. The function $B(T)$ is known as the *Planck function* and is written in the frequency form

$$B_\nu(T) \equiv \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \quad (2.83)$$

$$= \frac{1.4743 \times 10^{-47} \nu^3}{e^{4.7993 \times 10^{-11} \nu/T} - 1} \quad (2.84)$$

or the wavelength form (in CGS)

$$B_\lambda(T) \equiv \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \quad (2.85)$$

$$= \frac{1.1909 \times 10^{-5}}{\lambda^5} \frac{1}{e^{1.4388/\lambda T} - 1} \quad (2.86)$$

where h and k are Planck’s and Boltzmann’s constants, respectively. Equations (2.83) and (2.85) are not the same but relate to each other by

$$B_\nu d\nu = B_\lambda d\lambda \quad (2.87)$$

and completely specify radiation fields known as “black-body radiation.” Here, the word “black” means that all emitted photons are subsequently absorbed by the medium, i.e., none escape from the enclosure. True black-body radiation does not exist in nature although some environments closely approximate it in certain frequency or wavelength regions.

If $h\nu \ll kT$, which in astronomy usually occurs in the radio range,

$$B_\nu(T) \approx \frac{2\nu^2 kT}{c^2} = \frac{2kT}{\lambda^2}, \quad (2.88)$$

and is known as the “Rayleigh–Jeans approximation” to the Planck function. If $h\nu \gg kT$, which in astronomy usually occurs in the optical range,

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2} \exp(-h\nu/kT), \quad (2.89)$$

and is known as the “Wien approximation” to the Planck function.

Note that neither approximation may be appropriate for the millimeter and submillimeter wavelength ranges in astronomical conditions. Usually, a test must be made for each situation in these ranges.

2.3.2 Continuum Emission

Before applying the transfer equation to predict the intensities of RRLs, we will examine the characteristics of the continuum emission from H II regions underlying these lines.

Continuum radiation from H II regions is a mixture of thermal emission from heated dust particles and “free–free” emission or Bremsstrahlung (braking radiation) from unbounded charged particles. Figure 2.16 shows the emission from the two components in the typical H II region W3 as a function of frequency. Ultraviolet radiation from embedded stars heats the dust and ionizes the gas. Because the ionized particles are free, their energy states are not quantized, and the free–free radiation is continuous over the spectrum.

In gaseous nebulae, the spectrum of the dust emission is similar to that of a black body. However, it differs somewhat. The short-wave side (Wien) of the peak emission is optically thick but the long-wave side (Rayleigh–Jeans) can be optically thin in the millimeter wavelength range. At very high frequencies, the dust emission can be more intense than the free–free emission and, because its effective temperature is about 100 K, peaks in the infrared part of the spectrum.

With regard to RRLs, the free–free emission is the more important of the two continuum components. This emission measures the amount of ionization in the H II region. As we shall see, the ratio of the line emission in the RRLs to the continuous free–free emission can be a good measure of the thermodynamic state of the gas. It indicates the ratio of bound to unbound electrons in the H II region. Because most H II regions include heated dust, Fig. 2.16 shows that observations of free–free emission are best made at frequencies $\leq 10^{11}$ Hz where the dust emission is small compared with the free–free emission.

2.3.2.1 Continuum Absorption Coefficient

Calculating the continuum absorption coefficient κ_C is challenging.¹³ The calculations require modeling not only the electrical interaction between two charged particles, but also the velocity distribution of the particles. Classically, the encounter of two moving charged particles involves changes in their directions – either toward each other for unlike charges or away from each

¹³ Oster (1961) discusses the history and problems of calculating the free–free emission coefficient in detail.

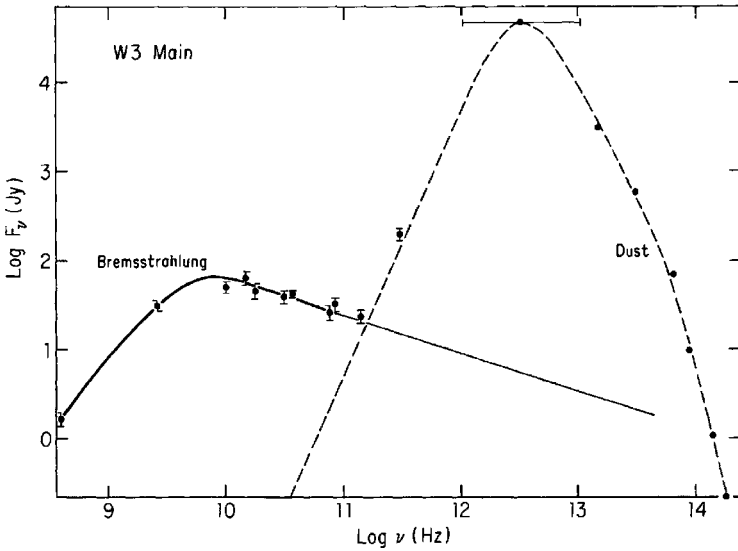


Fig. 2.16 The flux density in Janskys plotted against frequency from approximately 400 MHz ($\lambda = 75$ cm) through 178 THz ($\lambda = 1.7 \mu\text{m}$) for the H II region W3. The *solid curve* marks the free-free or Bremsstrahlung emission and the *broken curve* marks the thermal emission from warm dust embedded in the H II region. The *filled circles* show the observations. From Gordon (1988). Reproduced with permission of Springer-Verlag

other for like charges. The early work of Hertz showed that the acceleration from these direction changes causes radiation. Close encounters involve substantial Coulomb forces and large accelerations, leading to radiation in the X-ray range. Correspondingly, emission in the radio range involves more distant encounters where the Coulomb forces are weaker, the accelerations are much smaller, and the particles can be considered to continue moving in almost a straight line. In any case, the free-free absorption coefficient is determined by integrating the emission produced during each encounter over the velocity distribution of the particles, which is usually taken to be Maxwellian.

Some approximations are required to perform the integration. The free electrons tend to shield the electric field of the ions, and thus the force field is effective only over some finite distance that depends upon the density. Usually, the calculations assume (1) that the radiated energy is small compared with the kinetic energy of the electrons moving past the ions and (2) that the reciprocal of the radiated frequency is small compared with the time for the electron to undergo a 90° deflection. Physically, these assumptions imply that the electron-ion encounter is nearly adiabatic and that the period of the emitted wave train is short compared with the duration of the encounter.

Equation (162) of Oster (1961) gives an expression for the free-free absorption coefficient valid for the Rayleigh-Jeans – usually the radio – domain ($h\nu \ll kT$):

$$\kappa_C = \left(\frac{N_e N_i}{\nu^2} \right) \left(\frac{8Z^2 e^6}{3\sqrt{3} m^3 c} \right) \left(\frac{\pi}{2} \right)^{1/2} \left(\frac{m}{kT} \right)^{3/2} \langle g \rangle, \quad (2.90)$$

where N_e and N_i are the volume densities of electrons and ions, respectively. The factor $\langle g \rangle$ is the Gaunt factor averaged over a Maxwellian velocity distribution characterized by the kinetic temperature¹⁴ T . Equation (2.90) results from the fundamental relationship between the absorption and emission coefficients:

$$\kappa_C = \frac{j_C}{B_\nu(T)} = j_C \frac{\lambda^2}{2kT}, \quad (2.91)$$

allowing recovery of j_C for the general frequency domain. All units are CGS, giving units of κ_C in cm^{-1} .

The choice of the Gaunt factor depends upon the environment. For temperatures less than 550,000 K where classical physics approximations obtain,

$$\langle g \rangle \approx \frac{\sqrt{3}}{\pi} \ln \left[\left(\frac{2kT}{\gamma m} \right)^{3/2} \frac{m}{\pi \gamma Z e^2 \nu} \right] \quad (2.92)$$

and for temperatures greater than 550,000 K where quantum effects obtain,

$$\langle g \rangle \approx \frac{\sqrt{3}}{\pi} \ln \left(\frac{4kT}{\nu h \gamma} \right), \quad (2.93)$$

where γ is an Euler's constant in the form $\exp(0.557) = 1.781$. Both approximations for the Gaunt factor apply only when the wave frequency greatly exceeds the plasma frequency, or $\nu \gg 10^4 \sqrt{N_e}$ Hz, which is the usual situation for radio waves from H II regions.

Combining (2.90) and the Gaunt factor appropriate for H II regions (2.92) gives an expression for the free-free absorption coefficient that can be evaluated numerically as

$$\kappa_C = 9.770 \times 10^{-3} \frac{N_e N_i}{\nu^2 T^{3/2}} \left[17.72 + \ln \frac{T_e^{3/2}}{\nu} \right]. \quad (2.94)$$

in CGS units, which gives κ_c in units of cm^{-1} when the densities are in units of cm^{-3} , T in K, and ν in Hz.

Altenhoff et al. (1960) suggested a simple approximation for (2.94) in units appropriate for many radio astronomical observations:

$$\kappa_C \approx \frac{0.08235 N_e N_i}{\nu^{2.1} T^{1.35}}, \quad (2.95)$$

¹⁴ In thermodynamic equilibrium, all temperatures are the same. In reality, various temperatures can differ from each other.

where ν is in units of GHz, N_e in units of cm^{-3} , T_e in units of K, and κ_C in units of pc^{-1} . Its simplicity makes this formula often used in the analysis of observational data from H II regions. The numerical version of (2.94) in CGS units is

$$\kappa_C \approx \frac{0.2120 N_e N_i}{\nu^{2.1} T^{1.35}} \quad (2.96)$$

to give κ_C in units of cm^{-1} when ν is in units of Hz.

Any approximation involves some loss of accuracy. Altenhoff et al. claimed an accuracy within 5% for the traditional radio range, which is better than the usual accuracy of observations. Other approximations exist with higher accuracies but often are restricted to specific circumstances. For example, Hjellming et al. (1979) give an alternative approximation but this is restricted to $8 < \nu < 11$ GHz, a narrow range of frequency.

Under conditions of low radio frequencies and low temperatures, (2.94) and (2.95) no longer hold because the basic assumptions (1) and (2) are violated. Here, the Gaunt factors must be evaluated for each particular case. Oster (1970) gives expressions appropriate for a range of low temperatures and wave frequencies.

2.3.2.2 Continuum Emission Coefficient

As was the case for the line emission coefficient, Kirchhoff's radiation law gives the emission coefficient for the continuum emission per unit volume as

$$j_C = \kappa_C B(T), \quad (2.97)$$

where $B(T)$ is the Planck radiation function that obtains in thermodynamic equilibrium as discussed in Sect. 2.3.1.

2.3.3 Transfer Equation for Continuum Radiation

From the basic relationship between κ_C and j_C given by the Rayleigh–Jeans form of Kirchhoff's law of (2.91) and from the general form of the transfer equation given by (2.78), we can model the intensity of the free–free radiation from an H II region at a frequency ν as

$$I_C(x) = I(0)e^{-\tau_C(0)} + \int_0^{\tau_C(0)} \frac{2kT\nu^2}{c^2} e^{-t} dt + I(x > L), \quad (2.98)$$

with reference to Fig. 2.14. The continuum optical depth $\tau_c \equiv \int \kappa_C dx$. The terms $I(0)$ and $I(x > L)$ refer to emission on the far side of the H II region and between the H II region and the observer, respectively.

If the H II region is homogeneous in density and temperature, and if the foreground emission is zero, (2.98) becomes

$$I_C(x) = I(0)e^{-\tau_C(0)} + \frac{2kT\nu^2}{c^2} (1 - e^{-\tau_C(0)}) \quad (2.99)$$

to describe the free-free continuum emission observed at the radio telescope.

In practice, astronomers use units of temperature to measure the intensity of radiation. At radio wavelengths where the Rayleigh–Jeans approximation usually holds, the brightness temperature corresponding to the observed specific intensity at the telescope is $T = I c^2 / 2k\nu^2$, where T is the temperature of the emitting gas such that the transfer equation for the continuum emission from the nebula becomes

$$T(x) = T(0)e^{-\tau_C(0)} + T (1 - e^{-\tau_C}) , \quad (2.100)$$

where $T(0)$ is background emission attenuated by passage through the nebula.

The actual antenna temperature¹⁵ T_A is less than the brightness temperature by an efficiency factor η and a beam dilution factor W , defined as the ratio of the solid angle subtended by the radio source to that of the radio beam. If we think of the telescope as a thermometer, this temperature is the effective rise in temperature of the antenna as it points to the H II region. Specifically,

$$T_A = T \eta W. \quad (2.101)$$

2.3.4 Comparison with Continuum Observations

Because RRLs are measured with respect to the free-free emission underlying their spectra, it is necessary to understand the characteristics of the continuum emission to interpret the observations of the spectral lines.

Over a wide range of frequencies, the continuum emission from an H II region has a unique spectrum. Using the spectral flux density defined by (2.81) because it is integrated over a solid angle and ignoring the background term, we rewrite the transfer equation for continuum emission as

$$S_\nu \equiv S (1 - e^{-\tau_C}) , \quad (2.102)$$

where S_ν is the total flux density emitted by the nebula and S is the source function of the radiation. At high frequencies, the gas is optically thin, and

¹⁵ At millimeter and submillimeter wavelengths where calibration procedures involve measurements of atmospheric emission, the units of antenna temperature can be quite different and unintuitive – often symbolized by T_A^* , T_R^* , or T_{mb}^* . Appendix F describes their relationship to astrophysical units.

$$S_\nu \approx S\tau_C \propto \nu^{-0.1} \quad (2.103)$$

because $S \propto \nu^2$ in the Rayleigh-Jeans range and $\tau_c \propto \nu^{-2.1}$ as shown in (2.95).

At low frequencies where the gas is optically thick, $\tau_c \gg 1$ and

$$S_\nu \approx S \propto \nu^2. \quad (2.104)$$

The range of middle frequencies is usually called the “turnover” range. One can describe the continuum spectrum from an H II region in terms of its “turnover frequency,” usually defined as the frequency where $\tau_c = 1$. A few authors use $\tau_c = 1.5$. Here,

$$S_\nu = S(1 - e^{-1}) = 0.632 S. \quad (2.105)$$

Identifying the frequency where $\tau_C = 1$ (or 1.5) and estimating the electron temperature allows an astronomer to determine from (2.95) an important characteristic of an H II region known as the “emission measure” (EM). Defined as $\int_{\text{source}} N_e^2 dx$, EM is a measure of the ionized mass of the nebula emitting the free-free radiation. Its usual units are $\text{cm}^{-6} \text{ pc}$.

Figure 2.17 illustrates how well these equations describe actual observations. The spectral flux density rises at low frequencies and falls very slightly at high frequencies. The broken vertical line marks the turnover frequency.

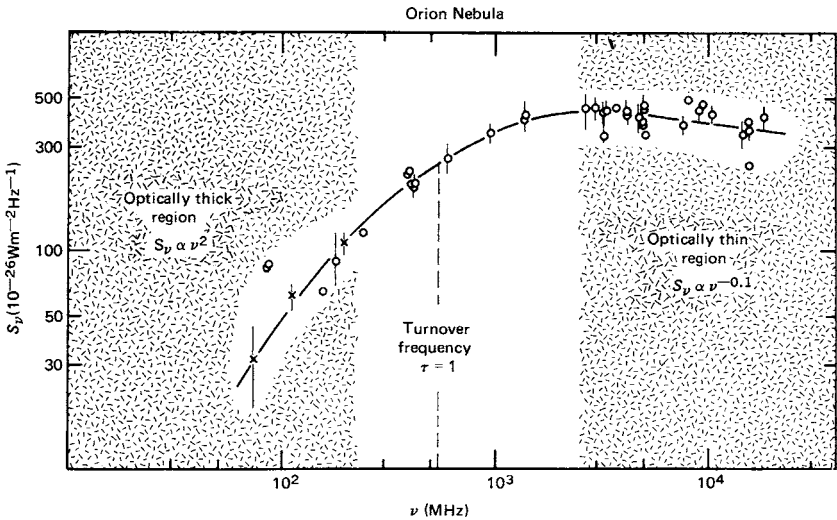


Fig. 2.17 The spectral flux density is plotted against frequency for observations of the Orion nebula (Terzian and Parrish, 1970). The *vertical broken line* marks the turnover frequency. *Points* indicate observations. The *solid line* marks the best fit of (2.102). Figure from Gordon (1988). Reproduced with permission of Springer-Verlag

The figure shows the optical thin and thick regions as shaded areas. Agreement between theory and observation is good.

Although the salient characteristics of the theory describe the continuum emission well, there are practical limitations. The analysis presumes an H II region to be homogeneous in density and isothermal. This is almost never the case. The distribution of newly formed stars that ionize the gas causes gradients in density and, probably, temperature as well. Furthermore, the nebular gas usually contains large-scale flows that appear as widenings or asymmetries in the spectral profiles of the RRLs. Optical studies of the Orion nebula made with high angular resolution show detailed structure on scales that would be within the beam of many radio telescopes (Osterbrock and Flather, 1959). As we shall see, these inhomogeneities can be important with regard to understanding RRLs.

2.3.5 Line Absorption and Emission Coefficients

Like free-free emission, calculating line emission begins with the linear emission and absorption coefficients, and the transfer equation.

2.3.5.1 Line Absorption Coefficient

The linear absorption coefficient for an RRL is

$$\kappa_L = \frac{h\nu}{4\pi} \phi_\nu (N_{n_1} B_{n_1, n_2} - N_{n_2} B_{n_2, n_1}), \quad (2.106)$$

where n_2 and n_1 are the principal quantum numbers of the upper and lower levels, respectively, N is the number density of atoms in the subscripted levels, and B_{n_1, n_2} and B_{n_2, n_1} are the Einstein coefficients for absorption and stimulated emission, respectively, in the indicated direction in units of inverse specific intensity per unit time.¹⁶ The units of κ_L are inverse length. As defined earlier, the factor ϕ_ν is the line profile in units of Hz^{-1} . Note that the term $h\nu/4\pi$ has units of energy per steradian – a part of the definition of I . The first term within the brackets is stimulated absorption, the second term is stimulated emission considered here as “negative absorption” because we are discussing an absorption coefficient.

¹⁶ Some authors (cf. Spitzer (1978)) define the Einstein coefficient B in terms of *energy density* rather than *specific intensity*, the latter being the form commonly used by astronomers and the form we use here (cf. Sect. 90.1 of Chandrasekhar (1950), Sect. 4.1 of Mihalas (1978), or Sect. 6.5.3 of Allen’s *Astrophysical Quantities* (Hjellming, 1999)). These two definitions are not interchangeable. In brief, for a transition of $n = 2 \rightarrow 1$, $B_{21}(\text{specific intensity}) = (4\pi/c) \times B_{21}(\text{energy density})$.

The Boltzmann formula gives the relative populations of two quantum levels in terms of volume densities:

$$\frac{N_{n_2}}{N_{n_1}} = \frac{\varpi_{n_2}}{\varpi_{n_1}} e^{-h\nu/kT}, \quad (2.107)$$

where ϖ is the statistical weight of the subscripted level. For hydrogen or hydrogenic ions, $\varpi = 2n^2$. Furthermore, $\varpi_m B_{m,n} = \varpi_n B_{n,m}$. Substituting into (2.106), we derive

$$\kappa_L = \frac{h\nu}{4\pi} \phi_\nu N_{n_1} B_{n_1,n_2} \left[1 - e^{-h\nu/kT} \right] \quad (2.108)$$

without approximations. The term in brackets of (2.108) is the correction for stimulated emission in thermodynamic equilibrium.

To obtain the Rayleigh–Jeans form that is usually appropriate for the radio range in astronomy, we expand the exponential term by a MacLaurin expansion to obtain

$$\kappa_L \approx \frac{h^2 \nu^2}{4\pi kT} \phi_\nu N_{n_1} B_{n_1,n_2}, \quad h\nu \ll kT. \quad (2.109)$$

Keeping the first two terms of the expansion series gives this form of κ_L , which underestimates the exponential term by an error of about $(h\nu/kT)^2/2$.

It is convenient to rewrite the stimulated transition coefficients in terms of the oscillator strength f_{n_1,n_2} .¹⁷ The absorption oscillator strength relates to the emission oscillator strength and to the specific intensity form of the B coefficient as

$$f_{n_1,n_2} = -\frac{\varpi_{n_2}}{\varpi_{n_1}} f_{n_2,n_1} \quad (2.110)$$

$$= \frac{mch\nu}{4\pi^2 e^2} B_{n_1,n_2}. \quad (2.111)$$

Note that emission oscillator strength is negative.

Goldwire (1968) and Menzel (1969) give oscillator strengths appropriate for RRLs from hydrogenic atoms. In addition, Menzel (1968) gives a useful approximation for the absorption oscillator strength:

$$f_{n_1,n_2} \approx n_1 M_{\Delta n} \left(1 + 1.5 \frac{\Delta n}{n_1} \right), \quad (2.112)$$

¹⁷ This term evolved from the “Ladenburg f,” a vestige of classical physics when the intensity of a spectral line was characterized in terms of the number of dispersion electrons per atom or, “oscillators.” See Appendix D for detailed information. Also, see the discussion relating the quantum mechanical to the classical form of the oscillator strength (Kardashev, 1959).

where $M_{\Delta n} = 0.190775$, 0.026332 , 0.0081056 , and 0.0034918 for $\Delta n = 1, 2, 3$, and 4 , respectively.

Using the temperature, we can relate the number density of atoms in the lower bound level N_{n_1} to the population of the electrons and ions of the unbound states of hydrogenic atoms, N_e and N_i , by means of the Saha–Boltzmann ionization equation:

$$N_{n_1} = \frac{N_e N_i}{T^{3/2}} \frac{n_1^2 h^3}{(2\pi m k)^{3/2}} \exp\left(\frac{Z^2 E_{n_1}}{kT}\right), \quad (2.113)$$

where E_{n_1} is the energy of level n_1 below the continuum. For hydrogen, dividing (1.17) by kT gives $E_n/kT = 1.579 \times 10^5/n^2/T$, where T is in K.

Substituting (2.111) and (2.113) into (2.108) gives a general expression for the line absorption coefficient for an RRL:

$$\begin{aligned} \kappa_L = & \frac{\pi h^3 e^2}{(2\pi m k)^{3/2} m c} n_1^2 f_{n_1, n_2} \phi_\nu \\ & \times \frac{N_e N_i}{T^{3/2}} \exp\left(\frac{Z^2 E_{n_1}}{kT}\right) \left(1 - e^{-h\nu/kT}\right). \end{aligned} \quad (2.114)$$

If $h\nu \ll kT$ (the Rayleigh–Jeans regime) and if the first two terms of (1.21) are substituted for the line frequency, the absorption coefficient becomes

$$\begin{aligned} \kappa_L \approx & \frac{2}{\sqrt{\pi}} \frac{e^2}{m} \left(\frac{h^2}{2mk}\right)^{3/2} \frac{h}{k} R \phi_\nu Z^2 \Delta n \frac{f_{n_1, n_2}}{n_1} \\ & \times \left(1 - \frac{3\Delta n}{2n_1}\right) \frac{N_e N_i}{T^{5/2}} \exp\left(\frac{Z^2 E_{n_1}}{kT}\right) \end{aligned} \quad (2.115)$$

$$\begin{aligned} \approx & 3.469 \times 10^{-12} \phi_\nu Z^2 \Delta n \frac{f_{n_1, n_2}}{n_1} \\ & \times \left(1 - \frac{3\Delta n}{2n_1}\right) \frac{N_e N_i}{T^{5/2}} \exp\left(\frac{Z^2 E_{n_1}}{kT}\right), \end{aligned} \quad (2.116)$$

where κ_L is in cm^{-1} if N_e and N_i are in cm^{-3} , T is in K, and E_{n_1} is in ergs.

Equation (2.116) – and (2.114) for the more general case – can be very useful. Because of the definition of the line profile given by (2.5), choose $\phi = 1 \text{ Hz}^{-1}$ for the total absorption over the line. To obtain the absorption coefficient for just the center ($\nu = \nu_0$) of a purely thermally broadened line, substitute the expression given by (2.20), $\phi_\nu = 1/(1.064 \Delta\nu_G)$ where the full width of the Gaussian line at half-intensity is in Hz.

Figure 2.18 compares the variation of the free–free absorption coefficient, given by (2.94), with the $\text{H}n\alpha$ line absorption coefficient at line center given by (2.116) with a Gaussian substitution for ϕ . For these particular conditions of $T = 10^4 \text{ K}$ and $N_i = N_e = 10^4 \text{ cm}^{-3}$, the peak line absorption exceeds the continuum absorption at $n > 33$, or $\nu > 192 \text{ GHz}$.

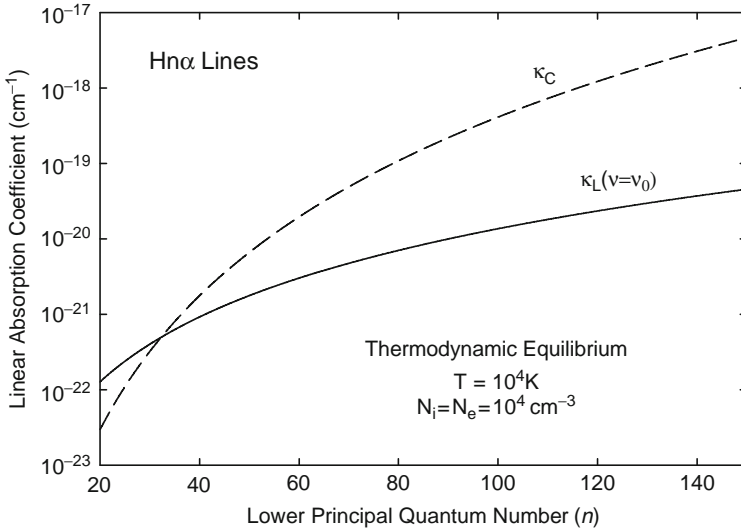


Fig. 2.18 Plots of the linear free-free absorption coefficient κ_C and the line absorption coefficient κ_L (at line center) as a function of the lower principal quantum number for $Hn\alpha$ lines. Here, the line broadening is entirely thermal; it contains neither turbulence nor Stark broadening

The line absorption coefficient increases approximately as n^3 but decreases inversely as $T^{5/2}$. This n -dependence results from $\kappa_L \propto \nu^{-1}$ at a given temperature, a variation principally due to the increase of the thermally broadened line width with the increasing line frequency and the corresponding decreasing of $\phi \propto \nu^{-1}$ (see (2.20) and (2.22)).

The absorption coefficient varies directly with $N_i N_e$ because, at any given temperature and principal quantum number n , the number of bound atoms available for absorption must be proportional to the ionization products $N_i N_e$ along any sight line through the plasma, i.e., matter is conserved in the plasma when summed over both bound and unbound constituents.

2.3.5.2 Line Emission Coefficient

In thermodynamic equilibrium, the emission coefficient per unit volume is defined as

$$j_L = \kappa_L B_\nu(T) \quad (2.117)$$

$$\approx \kappa_L \frac{2kT\nu^2}{c^2}, \quad h\nu \ll kT, \quad (2.118)$$

just as it was defined by (2.91) for free-free emission. Since we have defined κ_L earlier, this equation is sufficient to calculate the emission from the RRLs.

However, the line emission coefficient could also have been derived in terms of the spontaneous Einstein coefficient $A_{n_2 n_1}$:

$$j_L = N_{n_2} \frac{h\nu}{4\pi} A_{n_2 n_1} \phi, \quad (2.119)$$

where ϕ_ν is the normalized line profile. Applying the relationship¹⁸ between the A and B coefficients,

$$A_{n_2 n_1} \equiv \frac{2h\nu^3}{c^2} B_{n_2 n_1} \quad (2.120)$$

would allow us to work backward to derive the form of κ_L given by (2.118). *Note:* Whereas spontaneous emission occurs isotropically, stimulated emission occurs with the same angular distribution as I .

2.3.6 Transfer Equation for RRLs

Having the coefficients for both the line and continuum emission, we are now able to calculate the intensity of the RRL relative to the underlying free-free continuum. Because the linear opacities are additive, at any frequency within the RRL, the intensity of the emission will be

$$I = I_L + I_C = B_\nu(T) \left[1 - e^{-(\tau_C + \tau_L)} \right]. \quad (2.121)$$

The intensity of the line itself must then be

$$I_L = I - I_C = B_\nu(T) e^{-\tau_C} (1 - e^{-\tau_L}) \approx B_\nu \tau_L, \quad (2.122)$$

when $\tau_C, \tau_L \ll 1$ as is the usual case for H II region gas at centimeter wavelengths. Under the same conditions, $I_C \approx B_\nu \tau_C$, and the ratio of the line-to-continuum emission for the same object must be

$$\int_{\text{line}} \frac{I_L}{I_C} d\nu = \int_{\text{line}} \frac{\tau_L}{\tau_C} d\nu = \int_{\text{line}} \frac{\kappa_L}{\kappa_C} d\nu. \quad (2.123)$$

Equation (2.123) is particularly useful because beam efficiencies, dilution factors, and calibration factors apply more or less equally to numerator and denominator and, therefore, more or less cancel.

Direct substitution of the absorption coefficients derived in (2.95) and (2.116) gives an approximate expression¹⁹ relating the quantities observed

¹⁸ Here, B is again defined in terms of specific intensity rather than energy density. See the earlier footnote 16 for details.

¹⁹ Equation (2.124) does not include the second term of the expansion for the line frequency given by (1.21). This omission will lead to an overestimate of I_L by a few percent, depending

for *hydrogen* RRLs to the average physical conditions of the emitting gas in thermodynamic equilibrium:

$$\int_{\text{line}} \frac{I_L}{I_C} d\nu \approx 1.301 \times 10^5 \Delta n \frac{f_{n_1 n_2}}{n_1} \frac{\nu^{2.1}}{T^{1.15}} F \exp\left(\frac{1.579 \times 10^5}{n_1^2 T}\right). \quad (2.124)$$

The integration element $d\nu$ is measured in kHz and ν is in GHz. The ratio I_L/I_C allows intensities to be given in any convenient units, such as antenna temperature, because the units will cancel each other.

The continuum emission from gaseous nebulae will contain free-free emission from all ionized atoms, especially from hydrogen and helium because of their dominant abundance in terms of number density. Therefore, this observed emission will overestimate the contribution of hydrogen alone and underestimate the value of the integral. The factor F in (2.124) corrects the observed free-free emission for the contribution of ionized helium:

$$F \equiv \left(1 - \frac{N_{He}}{N_H}\right). \quad (2.125)$$

Observations have established that the cosmic ratio N_{He}/N_H is approximately 0.075 ± 0.006 (Gordon and Churchwell, 1970), giving a value for F of 0.925 that will compensate for the observed value of continuum.

In (2.124), note the relationship of the line-to-continuum ratio with temperature and frequency. The line-to-continuum ratio increases with frequency because of the decreasing intensity of the free-free emission. It decreases with temperature because fewer atoms occupy a particular principal quantum level – as described by the Boltzmann equation. It varies directly with line intensity as expected from quantum mechanics and inversely with n_1 as described by the Boltzmann equation.

Figure 2.19 illustrates the ratio of the line amplitude to the underlying free-free continuum as a function of principal quantum number for α -type RRLs. These calculations result from (2.124) when the line width is entirely thermal broadening from a gas with a temperature of 10^4 K, calculated from (2.22). In this example, the line amplitude exceeds the continuum emission for quantum numbers less than 43 (frequencies greater than 79 GHz).

2.3.7 The First Measurements of RRL Intensity

Early observations of RRLs led to surprising results regarding the line intensities. Figure 2.20 shows the temperatures calculated from the better detections reported through 1966 for the M17, Orion, and W51 nebulae

on the frequency. The reader can improve the accuracy by adding a multiplicative factor like $(1 - 3\Delta n/2n_1)$ to the right-hand side of (2.124).

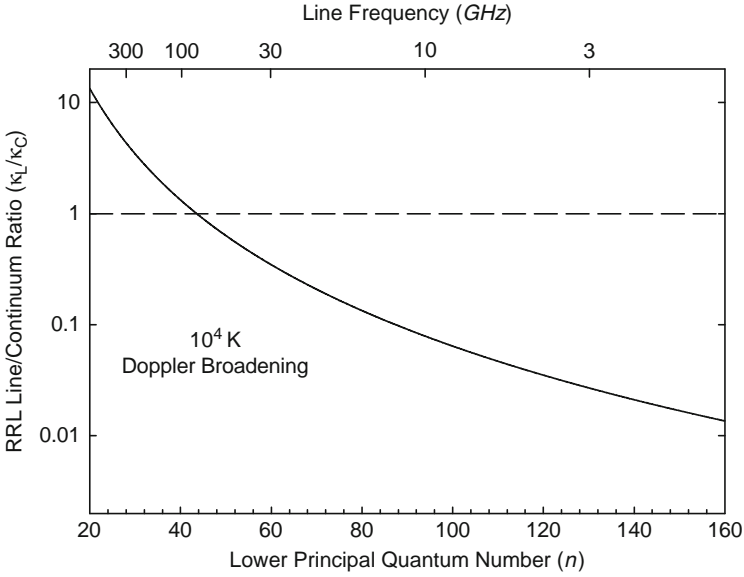


Fig. 2.19 The ratio of the amplitude of α -type RRLs to the underlying free-free emission as a function of n_1 . The gas temperature is 10^4 K and the line broadening is assumed to be entirely thermal. The *upper, nonlinear, abscissa* is marked with the corresponding frequency. *Note:* The slight difference from Fig. 2.18 in the value of n where $I_L/I_C = 1$ is due to the Altenhoff approximation for κ_C used in (2.124)

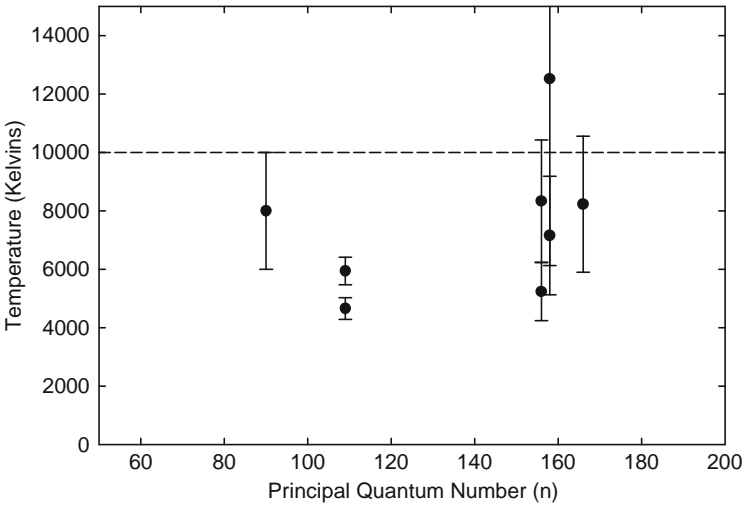


Fig. 2.20 Gas temperatures calculated from the initial detections of RRLs plotted against lower principal quantum number. The *broken line* marks the canonical 10,000-K temperature of H II regions accepted at that time on the basis of optical emission lines

(Sorochenko and Borodzich, 1965; Höglund and Mezger, 1965; Lilley, Menzel, Penfield and Zuckerman, 1966; Palmer and Zuckerman, 1966), calculated from (2.124). The temperature usually attributed to H II regions in our Galaxy on the basis of forbidden optical emission lines was 10,000 K. Yet, the temperatures derived from RRL observations with the best signal-to-noise ratios averaged to $\approx 5,000$ K – half of the value generally accepted for nebula gas.

Additional observations supported the surprisingly low gas temperatures. Mezger and Höglund (1967) obtained $T = 5,820$ K by averaging results from observations of the H109 α (5 GHz) line 16 H II regions. Dieter (1967) found a still lower value of 5,200 K from observations of the H158 α (1.6 GHz) line in 39 sources. At the same time, new observations of optical emission lines reaffirmed the characteristic gas temperatures of 10,000 K (O'Dell, 1966; Peimbert, 1967). Were these RRL temperatures really the correct ones? Or, was there something wrong with the RRL theory with regard to the line intensities?

2.3.8 Departures from LTE

The explanation came from the astrophysics developed decades earlier to explain anomalous intensities of optical lines from nebulae and stellar atmospheres. In a series of papers, Menzel and coworkers (cf. Baker and Menzel (1938)) had explained these intensities as consequences of departures from thermodynamic equilibrium. Using his experience interpreting similar line spectra from the Sun, Goldberg (1966) showed that the intensities of the newly detected RRLs were also a consequence of departures from thermodynamic equilibrium.

Physically, the term “thermodynamic equilibrium” (TE) describes a situation in which the energy exchange between the radiative and kinetic energy domains of a gas is so efficient that a single parameter, temperature, describes exactly the characteristics of both domains. While this situation cannot occur in the open systems found in astronomy, there are localized situations that are so close that the TE equations may be used. The term “local thermodynamic equilibrium” (LTE) describes these situations. Spectroscopically, LTE refers to circumstances in which one kind of rate into a level exactly balances a similar rate out of a level, i.e., the radiative rates into a level exactly balance the radiative rates out of that level, and, similarly, with the collisional rates.

Accordingly, the equations developed above, based upon thermodynamic equilibrium, require a correction factor. Goldberg noted that the “excitation temperature” T_{ex} that describes the relative population of the bound quantum levels was not the same as the temperature T_e of the ionized gas

in the nebula. Consequently, the factor $h\nu/kT_e$ in (2.108) is not the proper correction for stimulated emission in this environment.

To demonstrate this, Goldberg equated the exponential of the Boltzmann distribution of (2.107) to a corrected form involving T_e :

$$e^{-h\nu/kT_{ex}} = \frac{b_n}{b_{n-1}} e^{-h\nu/kT_e}, \quad (2.126)$$

where the factor b_n is the ratio of the actual number of atoms in a level n to the number which would be there if the population were in thermodynamic equilibrium at the temperature of the ionized gas. Choosing $T_e = 10^4$ K and estimating the ratio $b_{110}/b_{109} = 1.00072$ for the H109 α line at 5 GHz (Seaton, 1964) gives $T_{ex} = -360$ K. He concluded that the very small relative difference $(b_{110} - b_{109})/b_{109} = 7 \times 10^{-4}$ “takes on great importance at radio frequencies because it is large compared with $h\nu/kT_e = 2.40 \times 10^{-5}$.” In this way, ignoring the small departures from LTE in (2.108) significantly underestimates the amount of the stimulated emission, overestimates the absorption coefficient κ_L , overestimates the line intensity, and underestimates the equivalent T required to account for the intensity of the RRL relative to the underlying continuum. This effect would not occur for lines in the optical range because the energy difference between upper and lower quantum levels is much greater, i.e., the quantity $h\nu/kT_e$ is much larger than in the radio range.

Earlier, to develop the general theory appropriate to describe RRL line intensities in an LTE (or TE) environment, we used a single temperature, specifically T , to characterize the statistical state of bound *and* unbound levels of the atoms.

Now, to transform these LTE equations into non-LTE forms, we insert correction factors to account for the differences between the thermodynamic characteristics best described by an excitation temperature and those best described by the electron temperature. To emphasize this difference, we select T_e – the electron temperature that characterizes the free electrons – as *the* reference temperature for our calculations.

Because the non-LTE form of Boltzmann equation (see (2.107)) gives the relative level populations as

$$\frac{N_{n_2}}{N_{n_1}} = \frac{b_{n_2}}{b_{n_1}} \frac{\varpi_{n_2}}{\varpi_{n_1}} e^{-h\nu/kT_e} \left[= \frac{\varpi_{n_2}}{\varpi_{n_1}} e^{-h\nu/kT_{ex}} \right]. \quad (2.127)$$

The rightmost term in brackets shows how the population ratio can also be expressed in terms of an excitation temperature T_{ex} . Following Goldberg (1968), we multiply the line absorption coefficient given by (2.108) by b_{n_1} to correct for the population available to absorb photons and insert a second

factor, involving the ratio b_{n_2}/b_{n_1} , to correct for the non-LTE amount of stimulated emission:^{20,21}

$$\kappa_L = \kappa_L^* b_{n_1} \left[\frac{1 - (b_{n_2}/b_{n_1})e^{-h\nu/kT_e}}{1 - e^{-h\nu/kT_e}} \right] \quad (2.130)$$

$$= \kappa_L^* b_{n_1} \beta \quad (2.131)$$

$$\approx \kappa_L^* b_{n_1} \left(1 - \frac{kT_e}{h\nu} \frac{d \ln b_{n_2}}{dn} \Delta n \right), \quad h\nu \ll kT_e, \quad (2.132)$$

where κ_L^* refers to the LTE form developed earlier, given by (2.116).

The non-LTE emission coefficient is

$$j_L = \kappa_L^* b_{n_2} B_\nu(T_e) \quad (2.133)$$

by definition.

2.3.9 Non-LTE Line Intensities

As before, the intensity within the line is

$$I = I_L + I_C \quad (2.134)$$

$$= S \left[1 - e^{-(\tau_c + \tau_L)} \right], \quad (2.135)$$

where the non-LTE source function S for the RRL emission can be written

²⁰ The original mathematical expansion of β given by Goldberg (1968) as his equation (22) has

$$\beta \approx \frac{b_{n_2}}{b_{n_1}} \left(1 - \frac{kT_e}{h\nu} \frac{d \ln b_{n_2}}{dn} \Delta n \right), \quad (2.128)$$

which would create a cofactor of b_{n_2} in (2.132). Note the difference in subscript. Because that would be unphysical for an absorption coefficient, because $b_{n_1} \approx b_{n_2}$, and because it is an approximation in any case, we write (2.132) as it stands.

Also, the definition of the correction factor β has evidently evolved slightly. Brocklehurst and Seaton (1972) (BS) use

$$\beta \equiv \beta_{n_1, n_2} = \left(1 - \frac{kT_e}{h\nu} \frac{d \ln b_{n_1}}{dE_{n_1}} \right), \quad (2.129)$$

which differs in the argument of the logarithm. Because $n_2 - n_1 \equiv \Delta n \approx 1$ in many RRL observations of interest, it is a common practice to ignore the difference between b_{n_1} and b_{n_2} in applications of the corrective factors. Using the generic symbol n as a subscript usually involves no significant loss of accuracy and, in fact, BS adopt this simplification. Here, we include all subscripts for completeness, however.

²¹ Goldberg (1968) introduced the symbol $\gamma \equiv d \ln b_{n_2} \Delta n / dn$, which has now passed from common usage.

$$S = \frac{j_C + j_L}{\kappa_C + \kappa_L} \quad (2.136)$$

$$= \frac{\kappa_C + \kappa_L^* b_{n_2}}{\kappa_C + \kappa_L^* b_{n_1} \beta} B_\nu(T_e) \quad (2.137)$$

$$= \eta B_\nu(T_e), \quad (2.138)$$

where the factor η corrects the Planck function for departures from LTE and is

$$\eta = \frac{1 + b_{n_2}(\kappa_L^*/\kappa_C)}{1 + b_{n_1}(\kappa_L^*/\kappa_C)\beta}. \quad (2.139)$$

The non-LTE correction factors apply only to the line coefficients because, by definition, T_e describes the thermodynamic state of the ionized gas and, therefore, the continuum coefficient needs no correction.

The intensity of the line relative to the continuum can be written

$$\frac{I_L}{I_C} = \frac{I - I_C}{I_C} = \frac{I}{I_C} - 1 = \frac{\eta(1 - e^{-\tau_\nu})}{(1 - e^{-\tau_C})} - 1, \quad (2.140)$$

where τ_ν is the sum of the actual line and continuum opacities:

$$\tau_\nu \equiv \tau_C + \tau_L = \tau_C + \tau_L^* b_{n_1} \beta. \quad (2.141)$$

If $|\tau_L|$ and τ_C are much less than one²² as is the case for H II regions in the centimeter wave radio range:

$$I_L \approx I_L^* b_{n_2} \left(1 - \frac{\tau_C}{2} \beta\right), \quad h\nu \ll kT_e, \quad (2.142)$$

where we have expanded the exponentials to three terms. This approximation shows that line amplification involves competition between a weakening due to the depopulation of the lower level expressed by b_{n_1} and a strengthening due to the joint function of τ_C and β (usually negative) representing the enhancement of stimulated emission. In this approximation, the factor τ_C parameterizes the column density of material along the sight line.

Substitution of the approximation equation (2.142) into (2.123)) gives

$$\int_{\text{line}} \frac{I_L}{I_C} d\nu \approx \int_{\text{line}} \frac{I_L^* b_{n_2} (1 - \frac{\tau_C}{2} \beta) d\nu}{I_C}, \quad h\nu \ll kT_e, \quad (2.143)$$

and, in turn, this term into (2.124) and rearranging gives

$$T_e^{1.15} \exp\left(\frac{-1.579 \times 10^5}{n_1^2 T_e}\right) \approx 1.299 \times 10^5 \frac{T_C}{P} \Delta n \frac{f_{n_1 n_2}}{n_1} \nu^{2.1} F \times b_{n_2} \left(1 - \frac{\tau_C}{2} \beta\right), \quad (2.144)$$

²² Equation (2.132) shows that τ_L will be negative in non-LTE situations where $\beta < 0$. Hence, we use the absolute value of τ_L in the criteria for this approximation.

in “observational units” for the Rayleigh–Jeans regime where the integrated power in the line $P \equiv \int T_L d\nu$ is in units of K kHz and ν is in GHz.²³ For observations of H II regions in the centimeter wave range, the exponential term is effectively unity.

Inserting the departure coefficients estimated by Goldberg (1966) into (2.144) raises the derived LTE electron temperatures by approximately 32 and 122% for the observed H109 α and H165 α RRLs to approximately $6,500 \pm 550$ and $11,000 \pm 3,000$ K, respectively. These corrections moved the derived electron temperatures for H II regions closer – but not to – the canonical 10,000 K determined from optical emission lines. Rather than using the RRLs to determine accurate electron temperatures for H II regions, Goldberg suggested that astronomers accept the 10,000 K and use the RRLs to derive departure coefficients that, in turn, would determine the thermodynamic state of the nebular gas.

At this time, more needed to be done to interpret the intensities of the RRLs quantitatively. Astronomers had established that the line intensities were probably enhanced in the centimeter wavelength range because of the non-LTE environment of gaseous nebulae. The principal contribution to the line enhancement appeared to be stimulated emission, creating a “partial maser effect.” Further progress would require a wide range of improved observations and more accurate calculations of departure coefficients.

2.3.10 Calculating Departure Coefficients

2.3.10.1 Statistical Equilibrium

The departure coefficients result from solving a system of equations describing the equilibrium of the gas. All of the ways out of a quantum level n are equated to all of the ways into that level:

$$N_n \sum_{n \neq m} P_{nm} = \sum_{n \neq m} N_m P_{mn}, \quad (2.146)$$

where the rates P are in the directions indicated by the subscripts and include both radiative and collision processes. Such an environment is often called “statistical equilibrium.”

It is important to understand that this situation differs from thermodynamic equilibrium or even the more usual astrophysical situation known as

²³ A more accurate correction factor (the term following the \times symbol in (2.144)) is

$$\frac{b_{n2} \left(1 - \frac{\tau_C}{2} \beta\right)}{1 - \tau_C}, \quad \tau_C, |\tau_L|, (h\nu/kT_e) \ll 1. \quad (2.145)$$

local thermodynamic equilibrium that applies to a restricted locale or a restricted quantum mechanical environment. In TE or LTE, each rate into a quantum level must balance exactly with the *same kind* of rate out of that level, hence the term “detailed balance.” For example, TE would require

$$N_m N_p C_{mn} = N_n N_p C_{nm} \quad (2.147)$$

and

$$N_m J B_{mn} = N_n [A_{nm} + J B_{nm}], \quad (2.148)$$

where the collision (C) and radiative (A and B) rates between levels n and m exactly balance the inverse rates of their own kind. The parameter J is the mean intensity, i.e., the time-averaged integral of specific intensity I over all directions with units of $\text{ergs s}^{-1} \text{Hz}^{-1} \text{cm}^{-2}$. The volume density N_p refers to the number of particles available for collisions. Here, arbitrarily, n refers to the upper quantum level. These equations describe a thermodynamic environment where one temperature T is sufficient to characterize everything: the relative populations of bound levels of the atomic constituents of the gas through the Boltzmann equation, the intensity and spectral distribution of the ambient radiation field through the Planck equation, and the nature of the associated ionized gas through the Saha–Boltzmann equation. True TE does not occur in nature, which is why astronomers think in terms of LTE.

For most astronomical problems, both thermodynamic models assume time-invariant (often called “stationary”) models, i.e., they assume that the “equilibrium” achieved by the gas does not change over timescales long compared with the inverse of each rate. This assumption does not hold for all situations, however.

Specifically, the equations for statistical equilibrium are

$$\begin{aligned} N_n \left(\sum_{\substack{m=n_0 \\ m \neq n}}^{\infty} (C_{nm} + B_{nm}) + \sum_{m=n_0}^{n-1} A_{nm} + C_{ni} + B_{ni} \right) \\ = \sum_{\substack{m=n_0 \\ m \neq n}}^{\infty} N_m (C_{mn} + B_{mn}) + \sum_{m=n+1}^{\infty} N_m A_{mn} + N_e N_+ (\alpha_n^r + \alpha_n^3). \end{aligned} \quad (2.149)$$

The left-hand side includes all processes that depopulate level n . The first summation on the left accounts for collisional transitions up and down, and stimulated radiative decay. The second summation includes spontaneous transitions down, collisional ionization by electrons, and radiative ionization.

The right-hand side of (2.149) includes all processes that populate level n . The first term accounts for collisional and stimulated radiative transitions from other bound levels into n . The second term includes the spontaneous radiative terms and the three-body terms associated with radiative recombination into the bound level and collisional recombination. The level n_0 is the lowest level considered in the calculation.

The system of equations described by (2.149) is not closed. There is one more unknown variable than equations. The necessary extra equation for closure results from normalizing the level populations N_i to the populations N_i^* expected in thermodynamic equilibrium by using the Boltzmann equation. This process creates the dimensionless variables $b_i \equiv N_i/N_i^*$, referenced above as departure coefficients. Each resulting set of departure coefficients is a function of a specific electron temperature and volume density through the collision coefficients and the Boltzmann equation. For convenience, the reference temperature is taken to be the electron temperature T_e , which describes the characteristics of the ionized gas in the Saha–Boltzmann equation.

2.3.10.2 Evolution of the b_n Calculations

The first calculation of the population of hydrogen atomic levels was that of Baker and Menzel (1938) for the Balmer series. The solution was carried out for an infinite number of levels by the so-called n -method, which assumes that the population $N(\ell n)$ of the azimuthal sublevels ℓ is in equilibrium, i.e., proportional to their statistical weights:

$$N_{\ell n} = \frac{2\ell + 1}{n^2} N_n. \quad (2.150)$$

Only radiative transitions were considered, i.e., the quantum levels were assumed to be populated by recombination and cascade transitions and depopulated by cascade transitions to the lower levels. The object of these calculations was to determine the b_n coefficients. These calculations showed that the values of b_n for the first 30 levels differed significantly from unity. For $T_e = 10^4$ K and depending on the Case (see below), they found values ranging from $b_3 = 0.03 \rightarrow 0.1$ and $b_{30} = 0.45 \rightarrow 0.62$.

In general, calculations of departure coefficients are made for two situations: “Case B” in which the Lyman lines of the nebula are assumed to be optically thick (in LTE) but all other lines are optically thin and “Case A” in which all lines are presumed to be optically thin²⁴ (Baker and Menzel, 1938). Case B is the appropriate choice for H II regions from which RRLs have been detected.

In the years that followed, the accuracy of the calculations improved. For example, Seaton (1959) introduced a cascade matrix into the calculation technique, which enabled consideration of transitions into a given level by *all* possible routes. Such calculations showed that radiative processes were important only for small quantum levels (Seaton, 1964). For larger values of n , collisions dominate, causing the level population at, say, $n \geq 40$ (at a density of $N_e = 10^4 \text{ cm}^{-3}$) to move toward the Boltzmann distribution referenced

²⁴ Actually, Baker and Menzel (1938) subdivided Case A into a Case A₁ and a Case A₂, distinguished by the Gaunt factors that are used.

to the electron temperature, i.e., toward $b_n = 1$. These results showed that, in H II regions, collisions of atoms with electrons play a major role in the level populations, as well as radiation-less collisions, both of which excite and de-excite quantum levels.

These calculations were made before the detection of RRLs. Their purpose was to facilitate the understanding of the intensities of optical lines in stellar atmospheres and in other astronomical environments – the optical lines themselves involving only small quantum levels.

Following the detection of RRLs, the calculations resumed but now with emphasis on extending them to the much larger quantum numbers directly associated with the RRL transitions. Seaton's (1964) extant calculations had considered only transitions between adjacent levels, $n \rightarrow n \pm 1$. Now, processes between additional levels were necessary to determine departure coefficients sufficiently accurate for the interpretation of the RRL observations. For example, Hoang-Binh (1968) considered not only transitions between adjacent levels but also for $\Delta n = 2$ and 3. And, a year later, Sejnowski and Hjellming (1969) considered transitions up to $\Delta n = \pm 20$ using a matrix condensation technique.

The most important step in calculating the population of highly excited atoms was the one taken by Brocklehurst and Salem (1977) and Salem and Brocklehurst (1979) (SB). Unlike earlier calculations, theirs included the external radiation field, which causes stimulated transitions that profoundly influence the level populations. The influence of stimulated transitions increases with n and, hence, is very important for calculating the intensities of the high- n RRLs, especially for environments with low electron densities. The SB calculations consider collisional transitions using semiempirical values for the cross sections of excited atom–electron collisions (Gee, Percival, Lodge and Richards, 1976). The SB departure coefficients are the most generally accepted as well as the most often used for the interpretation of RRL observations from H II regions (emission nebulae) at wavelengths exceeding $\lambda = 1$ cm or principal quantum numbers $n > 40$ and at moderate to high electron densities.

Appendix E.1 lists FORTRAN code adapted from SB, converted to the FORTRAN 77 standard, and modified to extend to quantum numbers down to $n = 10$ (Walmsley, 1990). The results also include values of the β parameter. The departure coefficients in many of our figures (below) resulted from this code. This code executes in seconds on a modern PC when compiled under, e.g., the MS-Fortran Powerstation compiler.

Note: For transitions involving small quantum numbers $n < 40$ and low gas densities, Storey and Hummer (1995) have calculated even more accurate departure coefficients. Unlike the SB calculations, these calculations consider collisions with the angular momentum states of hydrogen, which can be important under conditions described below in more detail. However, under conditions common to many H II regions, these departure coefficients agree well with those of SB.

2.3.10.3 Tables of Departure Coefficients

Tables of departure coefficients appropriate for RRLs from H II regions exist in the literature (Seaton, 1964; Sejnowski and Hjellming, 1969; Brocklehurst and Salem, 1977; Salem and Brocklehurst, 1979; Walmsley, 1990; Storey and Hummer, 1995).²⁵ Departure coefficients are also available for the cold, partially ionized interstellar gas (Gulyaev and Nefedov, 1989; Ponomarev and Sorochenko, 1992).

Determining appropriate collision cross sections is a difficult part of the calculations. Approximations are generally required that cannot apply equally effectively to atoms in all levels. For example, visualize an electron orbiting the nucleus at a radius of $r = 0.529(n^2/Z) \text{ \AA}$, an equation derived from Bohr theory. An atom in a principal quantum level of 200 has a target area 10^4 times larger than one in level 20. In general, the most recent calculations of departure coefficients employ the most accurate cross sections.

2.3.10.4 Characteristics of Departure Coefficients

Figure 2.21 shows departure coefficients calculated from the modified SB code for $T_e = 10^4 \text{ K}$ and a range of electron densities. The figure plots the two new factors in (2.132) that represent a weakening factor (b_n) due to the depletion of the level population available for absorption and a factor (β) describing an enhancement of the stimulated emission because of the enhanced population gradient across the principal quantum levels.

The upper panel of the figure shows asymptotes to illustrate the physics involved with the radiation processes. In a dense medium where the collision rates dominate the level populations, the departure coefficients will be unity because the electron temperature T_e accurately characterizes the relative populations of the bound levels. In a tenuous medium, on the other hand, the radiative rates dominate, the temperature T_e no longer characterizes the populations well, and the departure coefficients correspondingly lie below 1. As the principal quantum number becomes smaller, the effective electronic radius decreases, the target area of the atom decreases, the influence of collisions wanes, and the departure coefficient decreases from unity toward the radiative asymptote. With respect to changes in density, the transition region between collision domination and radiative domination moves to lower principal quantum numbers as density increases and collision rates become increasingly effective.

The lower panel of the figure is also instructive. For conditions typical of H II regions, the largest correction for stimulated emission – and, correspond-

²⁵ The Storey and Hummer calculations of departure coefficients for a wide range of T_e and N_e are available from the Centre de Données Astronomique de Strasbourg via anonymous FTP to the directory /pub/cats/VI/64 of cdsarc.u-strasbg.fr. These files do not contain β_n values, however.

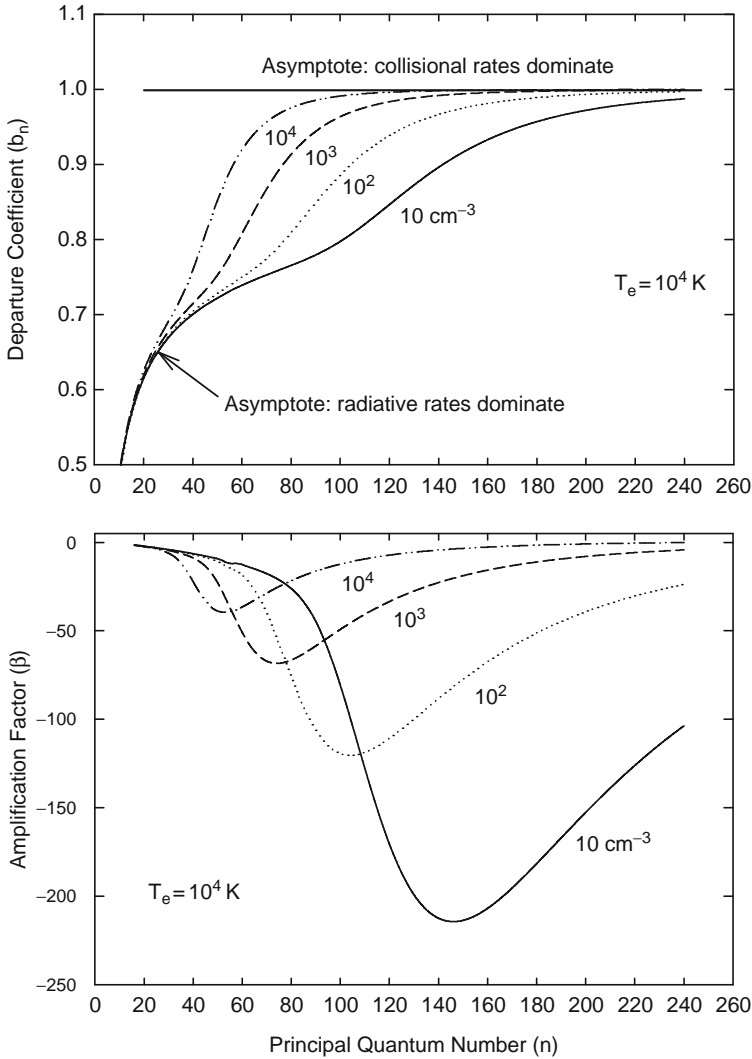


Fig. 2.21 *Top:* departure coefficients for the principal quantum levels of hydrogen for a range of electron densities, Case B. *Bottom:* the corresponding correction factor for stimulated emission for α -type RRLs plotted in the form β defined by (2.132)

ingly, the greatest line amplification – occurs for the lower densities. This results from the slope of the departure coefficient $d \ln(n)/dn$ being inversely weighted by the photon energy $h\nu$ in the correction term β . Perhaps, a better way of understanding the physics is to think of the derivative term as the relative (and normalized) population gradient with respect to the energy between levels, i.e., to think of the principal factor of β written as $d \ln(n)/dE_n$ because $h\nu dn \equiv dE_n$. This is why $|\beta|$ is largest at principal quantum numbers

larger than those where db_n/dn maximizes for a given density, as seen in the top panel. Very specifically, the denominator dE_n in the derivative becomes smaller with increasing n . In other words, the correction for stimulated emission involves the gradient of the normalized population across the quantum levels *and* the energy of the photons associated with those levels, peaking at principal quantum numbers larger than the region of the maximum population gradient $d\ln(n)/dn$.

2.3.10.5 Differences Between Calculations of b_n

Compared to the earlier calculations, the modern collision cross sections and the inclusion of many orders of transitions increased the effect of collisions with respect to radiative processes. Figure 2.22 compares departure coefficients (Case B) calculated for the canonical H II region conditions of $T_e = 10^4$ K and $N_e = 10^4$ cm⁻³ by Seaton (1964), by Sejnowski and Hjellming (1969), by the code contained in Appendix E.1 modified from Salem and Brocklehurst (1979), and by Storey and Hummer (1995). Not only do the most recent calculations extend the influence of collisions to lower quantum numbers, but also the slope of the transition region differs greatly from Seaton's 1964 calculation used by Goldberg (1966).

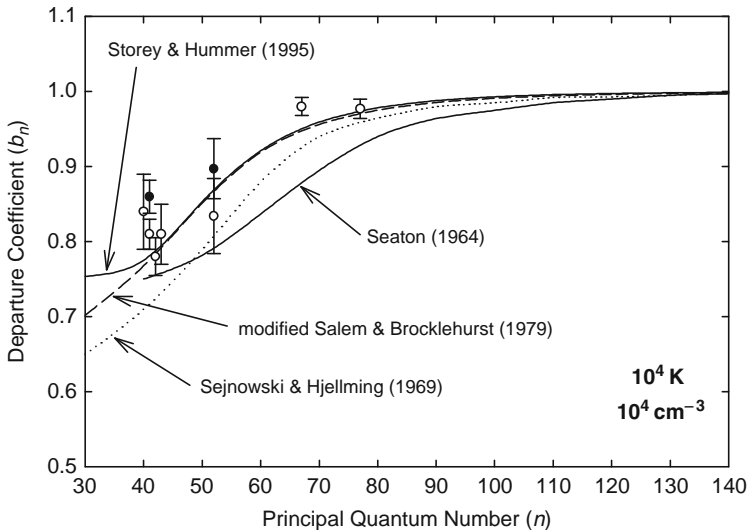


Fig. 2.22 Evolution of calculations of departure coefficients for hydrogen for $T_e = 10^4$ K and $N_e = 10^4$ cm⁻³ for Case B. Of the four, the Storey and Hummer calculation is the most sophisticated and, at $n \geq 40$ and this electron density, is nearly identical to the modified Salem and Brocklehurst calculation performed from the code listed in Appendix E.1. Juxtaposed are several departure coefficients determined from observations by Sorochenko et al. (1988) (*open circles*) and Gordon and Walmsley (1990) (*filled circles*)

Also shown in Fig. 2.22 are departure coefficients determined from observations of several RRLs made in the Orion nebula by Sorochenko et al. (1988) and Gordon and Walmsley (1990). Initially, to derive the departure coefficients observationally, the gas was assumed to have the 8,000-K temperature and 10^4-cm^{-3} density of Orion. Here, the values have been scaled to 10^4 K to allow comparison with the theoretical values plotted in Fig. 2.22. Agreement is good within the experimental errors but, unfortunately, not good enough to select the best theoretical calculations of b_n at $n < 40$. Nonetheless, the observationally determined points confirm the general correctness of the non-LTE transfer theory.

The difference between the Case B departure coefficients calculated by Storey and Hummer (1995) (SH) and the modified Salem and Brocklehurst (1979) code in Appendix E.1 is significant at small quantum numbers, as can be seen in Fig. 2.22. Although Salem and Brocklehurst limited their published tables of departure coefficients to $n \geq 50$, Walmsley (1990) extended their code to $n \geq 20$ by including collisions between low- n levels and collisional ionizations from low- n levels.

This difference results from the way the angular momentum sublevels are handled in the Storey and Hummer code, as described in detail by Strelitski, Ponomarev and Smith (1996). At densities appropriate to H II regions, the angular momentum sublevels ℓ are completely degenerate at moderate to high principal quantum numbers n because of proton collisions, and they can be neglected in the calculations of b_n . However, the collision cross section of the (Bohr) atom varies approximately as n^4 and, at small principal quantum numbers, the influence of the collisions becomes much less. The ℓ quantum levels emerge from degeneracy and become locations in quantum-mechanical space that the hydrogen atoms can occupy. Strelitski et al. refer to this transition as an “unblurring” of those angular momentum quantum levels as $n \rightarrow 1$. At a given density, the unblurring makes an increasing number of ℓ levels available as n decreases. Because the decay rates increase with decreasing ℓ , the low- ℓ sublevels of a given n are unblurred first (Strelitski, Ponomarev and Smith, 1996). Because the effective departure coefficient is calculated as

$$b_n = \sum_{\ell} \frac{2\ell + 1}{n^2} b_{n\ell}, \quad (2.151)$$

the result of unblurring is an increase in the b_n value of the entire n level at low quantum numbers, as is seen at $n \leq 40$ in the Storey and Hummer calculation plotted in Fig. 2.22. Of course, eventually, the departure coefficients return to the radiative asymptote as n continues to decrease, because (1) the proton collisions lose their influence as the diameter of the (Bohr) atom decreases and (2) the number of ℓ levels at any given value of n diminishes as $2n + 1$, i.e., with the statistical weight of the principal quantum level.

The n -location and b_n -range of the unblurring are also a function of density at a given electron temperature. Figure 2.23 plots hydrogen departure

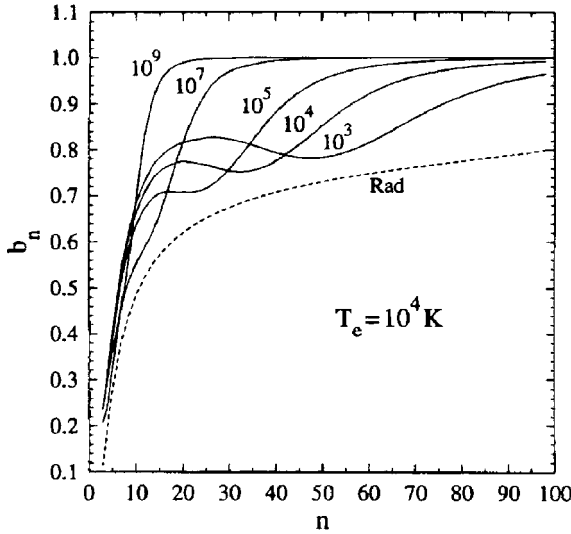


Fig. 2.23 Departure coefficients for hydrogen calculated by Storey and Hummer (1995) for $T_e = 10^4$ K and a range of electron densities N_e . The *broken line* marks the radiative asymptote calculated by V. Ponomarev. Figure from Strelitski, Ponomarev and Smith (1996)

coefficients for $n \leq 100$ for a range of N_e and a temperature of 10^4 K, calculated by Storey and Hummer (1995). Note that, at high densities, proton collisions blur the angular momentum states so there is no visible inflection in the b_n curve. As the density decreases, the blurring becomes increasingly important, moving toward higher- n as collisions become less effective.

We conclude by noting that the b_n code contained in Appendix E.1 is often adequate for centimeter and meter wave RRLs from H II regions; in fact, the resulting values of b_n agree well with the Storey and Hummer results. For the submillimeter and millimeter regimes where RRLs involve small principal quantum numbers, it is more accurate to use the departure coefficients calculated by Storey and Hummer.

2.3.11 Line Intensities in Terms of Transfer Theory

Figure 2.24 shows the change in line intensity described by the approximation given by (2.142). For comparison, it also shows the exact calculation to illustrate where the approximation fails. The calculations are for α -type RRLs from a fictitious H II region with a diameter of 0.025 pc, an electron temperature of 10^4 K, and an electron density of 10^4 cm^{-3} —a hypothetical small H II region not unlike the Orion nebula.

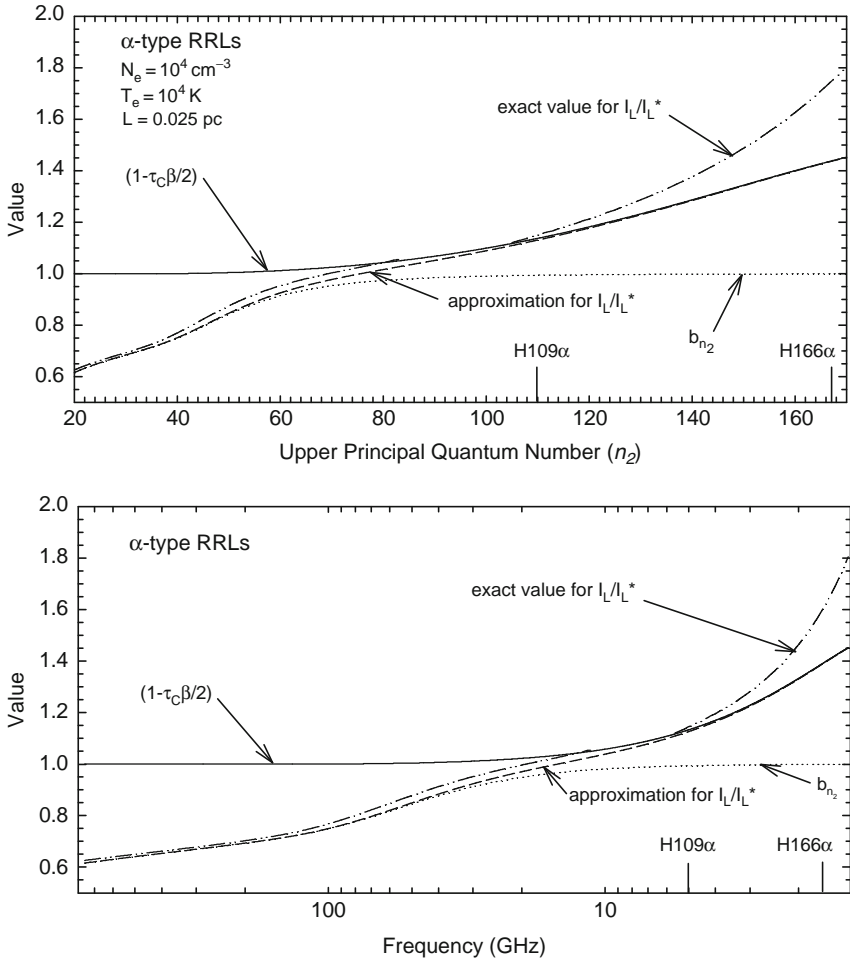


Fig. 2.24 *Top:* the simple approximation for line enhancement (2.142) for α -type RRLs plotted against the upper principal quantum number for conditions of a hypothetical H II region. For comparison, the exact enhancement is also shown, except for the range $n = 92 \rightarrow 100$ where the correction for stimulated emission goes through a discontinuity. The departure coefficient b_{n_2} is important at small quantum numbers and the factor $(1 - \tau_C \beta / 2)$ at large quantum numbers. *Bottom:* the same quantities plotted against frequency. The positions of the H109 α and H166 α RRLs are indicated. The frequency range corresponding to the discontinuity is not shown in curve for the exact calculation. Departure coefficients were calculated from the code of Appendix E.1

At small quantum numbers (high frequencies), the departure coefficient dominates the line intensity though the underpopulation of the upper level relative to that expected from a Boltzmann population with $T_e = 10^4$ K. Although Fig. 2.21 shows β to be significant in this range of quantum numbers, the amplification term $(1 - \tau_C \beta / 2)$ is unity because of the small value of the

free-free optical depth.²⁶ LTE temperatures calculated from RRLs in this region will be too high.

At large quantum numbers, collisions ensure that the departure coefficient b_{n_2} is near unity. However, the larger value of τ_C enhances the gradient term β , resulting in a large amplification of the RRL intensity. In this illustration, the line enhancement becomes quite significant at large quantum numbers even though there is no actual inversion of the level population. LTE temperatures calculated from RRLs in this region will be too low.

Although the H109 α and H166 α lines considered by Goldberg (1966) both fall in the enhancement region of Fig. 2.24, the sizes of the line enhancement are less than his estimates. He calculated amplification factors of 1.4 and 2.8 for the H109 α and H166 α lines, respectively, on the basis of departure coefficients calculated by Seaton (1964). The newer calculations (Salem and Brocklehurst, 1979) shown in Fig. 2.24 give amplification factors of only 1.15 and 1.73. Figure 2.22 shows why. The slope of the modern b_n -curve at $n = 109$ is significantly lower than the slope calculated by Seaton. Yet, the departure coefficients b_{109} are not very different for the two curves and the values of τ_C should be the same. Consequently, the line enhancement is less because the amplification factor $b_{n_2} (1 - \tau_C \beta / 2)$ in (2.142) is less, because $|\beta|$ is significantly lower in the modern calculations. The same is true at $n = 166$ although we have not plotted the calculations to this value of n .

2.3.12 Line Enhancement: A More General View

2.3.12.1 Non-LTE Line Intensities in Terms of Opacities

To understand the enhancement of RRL intensities more generally, it is useful to examine the detailed influence of the line and continuum opacities on the radiation transfer. The correction factor η to the Planck function must be considered at the same time. Equation (2.140) illustrates the relationship of these factors to the line intensity without simplifying assumptions, and we therefore repeat it below:

$$\frac{I_L}{I_C} = \frac{\eta(1 - e^{-\tau_\nu})}{(1 - e^{-\tau_C})} - 1. \quad (2.152)$$

Figure 2.25 plots the component optical depths τ_C and τ_L , the net optical depth $\tau_\nu = \tau_C + \tau_L$, and the correction factor η as functions of principal quantum number for the same fictitious Orion-like H II region described earlier. The departure coefficients for these parameters result from the code listed

²⁶ In the equations above, the free-free optical depth τ_C is also a measure of the free-free *emission* because of Kirchhoff's law of thermodynamics.

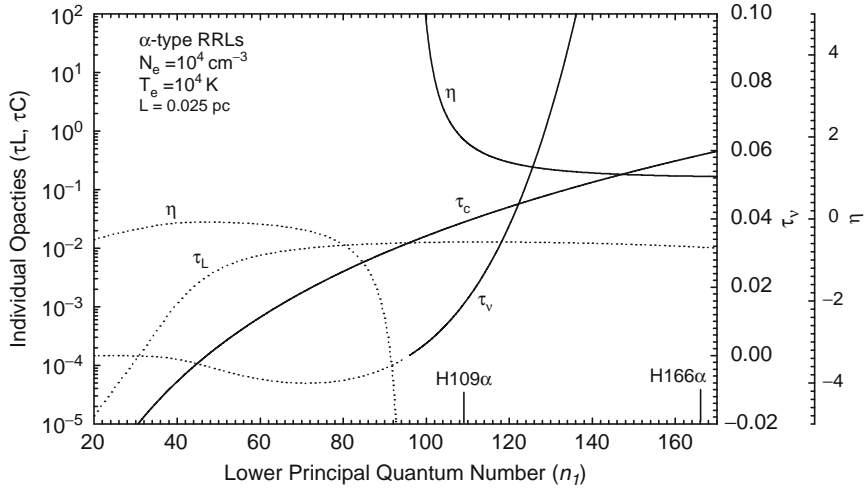


Fig. 2.25 Referenced to the *left ordinate* is the variation of the free-free (τ_C) and actual line (τ_L) opacities as a function of lower principal quantum number for the fictitious H II region. Referenced to the *inner right ordinate* is the sum of these opacities (τ_v) and, to the *outer right ordinate*, the correction to the Planck function for stimulated emission (η). Dotted segments of the lines indicate where the quantities are negative. The positions of the H109 α and H166 α RRLs are indicated

in Appendix E.1. In this figure, the solid curves mark the regions where the variables are positive and the dotted curves where they are negative.

At principal quantum numbers $n > 100$, the opacities and η are positive for our model H II region. The net optical depth in the line τ_v increases more slowly with n than the free-free optical depth τ_C while η approaches unity. The result is that the ratio I_L/I_C decreases *slowly* with increasing n .

At principal quantum numbers $n < 90$, the net line opacity and η are both negative, with both factors staying at small but – very, very roughly – constant values, thereby confining the $\eta(1 - e^{-\tau_v})$ term of (2.152) to a small positive value. However, in this regime of n , combining (1.22) and (2.95) shows the free-free absorption τ_C to vary roughly as n^6 , starting from a small value, thereby *sharply* decreasing the ratio I_L/I_C as n increases.

Figure 2.26 shows these results graphically for the hypothetical H II region. Note the logarithmic scale of the ordinate. The ratio I_L/I_C decreases²⁷ with n but at different rates in the low and high value regimes of n for the reasons we have just discussed above.

²⁷ In this particular model, because η and τ_v change sign at $n \approx 95$, our simple calculations of I_L/I_C exhibit a computational discontinuity due to insufficient significant figures, and we therefore show this region as a gap in the plot.

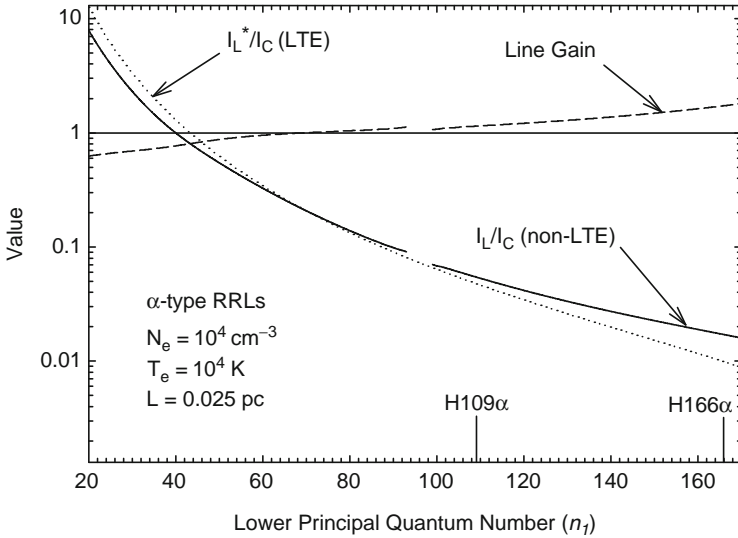


Fig. 2.26 The variation of the ratios I_L/I_C (non-LTE) and I_L^*/I_C (LTE) as a function of principal quantum number for a model H II region. A *broken line* indicates the ratio of I_L/I_L^* – the line gain – as a function of principal quantum number. The *horizontal line* is a reference for a line gain of 1. The positions of the H109 α and H166 α RRLs considered by Goldberg (1966) are marked. Stark broadening has not been included. *Gaps* indicate a region of insufficient computational precision

For comparison with the non-LTE value, Fig. 2.26 also shows the behavior of the LTE ratio I_L^*/I_C . For this model of an H II region, the line gain (defined here as I_L/I_L^*) is less than one at small values of n , increases to unity near $n = 66$, and then exceeds 1 as n increases further.

The definition of κ_L , given by (2.132), and the values of β in Fig. 2.21 show why. At small quantum numbers, the line intensity I_L lies below the LTE value because of the level depletion b_n , i.e., the correction factor $b_n|\beta| < 1$. The level depletion offsets the increased population gradient N_{n2}/N_{n1} parameterized by β , and the line intensity I_L falls below the LTE value of I_L^* . At larger quantum numbers, $b_n \rightarrow 1$ as $n \rightarrow \infty$ but $|\beta| > 1$ such that the line intensity I_L exceeds its LTE value. In other words, the population gradient increasingly dominates the effect of the level depopulation.

Figure 2.26 also illustrates that this model is close to what is required to explain observations of the H109 α and H166 α RRLs from the Orion nebula. The H109 α RRL would exhibit a small gain over the LTE value and the H166 α line would exhibit a somewhat larger gain. Goldberg referred to this as the “partial maser” effect because it does meet the maser gain criterion described below.

2.3.12.2 RRL Masers

Actual RRL masers²⁸ are possible.

Conditions for an RRL maser are stringent. To amplify, the populations of the principal quantum levels must be inverted such that $\tau_L < 0$. However, with respect to RRLs, this condition is necessary but insufficient for a maser. An intrinsic characteristic of the medium in which RRLs arise is continuum emission due to free-free radiation. For this reason, the line optical depth must not only be negative but its absolute value must exceed the optical depth of the free-free emission, which is always positive. The condition for an RRL maser is that the net absorption coefficient and, hence, the net optical depth in the line be (Ponomarev, 1994; Strelitski, Ponomarev and Smith, 1996)

$$\tau_\nu \equiv \tau_C + \tau_L < 0. \quad (2.153)$$

For example, if $\tau_L < 0$ but $\tau_\nu > 0$, the medium does not amplify – as masers are required to do by definition. This is the situation for the H109 α and H166 α lines considered by Goldberg (1966) as a “partial maser effect” and illustrated in Figs. 2.24–2.26. For these lines, τ_ν is positive. This situation is then not a “maser” but, instead, a line enhancement caused by a decrease in the net absorption with respect to the LTE values. For this particular physical model, Fig. 2.25 shows that the condition of $\tau_\nu < 0$ will only be fulfilled for $n < 95$. Only in this region of quantum numbers, the RRL will actually be amplified.

In special circumstances, great amplification can occur. Specifically, when $|\tau_\nu| \gg 1$, (2.152) becomes (Strelitski, Ponomarev and Smith, 1996)

$$\frac{I_L}{I_C} \approx \frac{|\eta|}{1 - e^{-\tau_C}} e^{|\tau_\nu|}, \quad \eta, \tau_\nu \ll 0. \quad (2.154)$$

Figures 2.25 and 2.26 do not show this regime.

Because (2.154) contains $|\tau_\nu|$ as an exponent, the ratio I_L/I_C can become very large if $\exp(|\tau_\nu|) \gg 1$. Because $\tau_\nu \equiv \tau_C + b_n \beta \tau_L^*$, the maser requires a physical environment where $\beta \ll 0$. Figure 2.21 shows that these conditions can occur when the density of an H II region is not very high. In general, $|\tau_\nu|$ does not reach maximum at the inflection of β because both τ_L^* and β have different dependencies on the electron density N_e . Section 2.4.1 discusses the conditions for RRL masers in more detail.

2.3.13 Classification of a Non-LTE Transition

Strelitski, Ponomarev and Smith (1996) give an interesting way to classify the effects of non-LTE on the population of RRL levels in terms of the exci-

²⁸ Microwave amplification by stimulated emission radiation.

tation temperature. From (2.127), we express the ratio of the weighted level populations N'_i s in terms of an excitation temperature T_{ex} :

$$\frac{N'_2}{N'_1} \equiv \frac{N_2/\varpi_2}{N_1/\varpi_1} = \exp\left(-\frac{h\nu_0}{kT_{ex}}\right), \quad (2.155)$$

where ϖ_i is the statistical weight of the principal quantum level i and ν_0 is the line frequency. The variable N'_i is then the population of level i per degenerate sublevel. We designate the upper level of the transition to be 2 and the lower to be 1. We can then write

$$T_{ex} = \frac{h\nu_0/k}{\ln(N'_1/N'_2)}. \quad (2.156)$$

The ratio β_{12} of the source function S_ν to the Planck function $B_\nu(T_e)$ is defined for this transition in the usual way:

$$\beta_{12} \equiv \frac{1 - \exp(-h\nu_0/kT_{ex})}{1 - \exp(-h\nu_0/kT_e)}. \quad (2.157)$$

Figure 2.27 shows the variation of T_{ex} and β_{12} as a function of the population ratio N'_1/N'_2 when $T_e = 10^4$ K and, for this illustration, $h\nu_0/k = 100$ K. As the population ratio N'_1/N'_2 decreases from ∞ (all the atoms in the lower

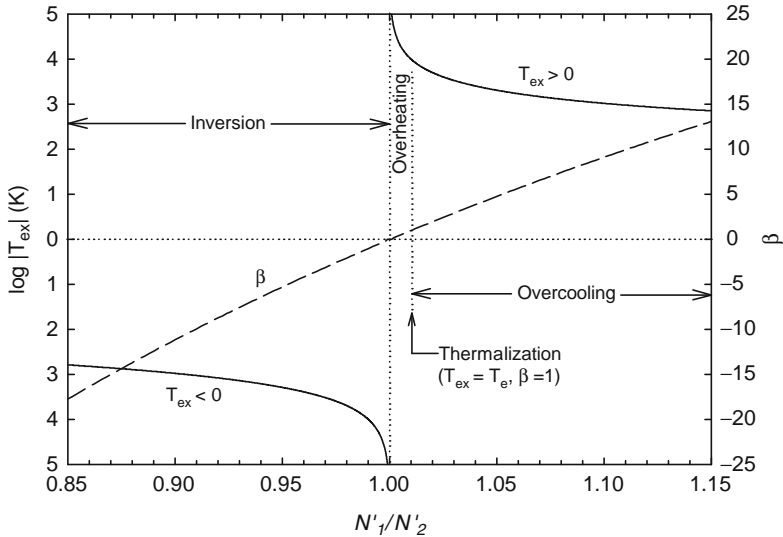


Fig. 2.27 Classification of non-LTE states of a quantum transition. After Strel'nitski, Ponomarev and Smith (1996). N'_1 is the population of the lower quantum level per degenerate sublevel, T_{ex} is the excitation temperature for the two levels, and β is the correction to the Planck function at ν_0

level) to 0 (all the atoms in level 2), the excitation temperature T_{ex} increases from 0 to ∞ , jumps to $-\infty$ at $N'_1 = N'_2$, and then increases to 0. Through this range of $N'_1/N'_2 = \infty \rightarrow -\infty$, β_{12} decreases monotonically from $[1 - \exp(-h\nu_0/kT_e)]^{-1}$ to $-\infty$, passing through 0 at $N'_1 = N'_2$.

As described by Strelitski et al., these excitation regimes may be easily understood in terms of temperature. The right side regime where most of the population lies in the lower level can be called the “overcooled” regime, and $T_{ex} > 0$; the left side where most of the population lies in the upper level can be called the “inversion” regime, and $T_{ex} < 0$. At $T_{ex} = T_e$, the levels are thermalized and $\beta_{12} = 1$. In the narrow regime where $T_e < T_{ex} < \infty$, the gas can be called “overheated” because the excitation temperature exceeds the kinetic temperature of the H II region. Here, the source function S_ν is positive but less than the Planck function.

Figure 2.28 shows these excitation regions with respect to specific b_n and β curves characteristic of a typical H II region. As noted earlier, the overcooled condition occurs where the b_n curve increases toward small values of n , an inflection that appears at low densities in the departure coefficients calculated by Storey and Hummer (1995) because of their consideration of the population of the degenerate levels. The inversion region occurs toward larger values of n , where the slope $d \ln b_n / dn$ is large and, hence, β is correspondingly large and negative. Thermalization corresponds to the region where $b_n \approx 1$, usually at high electron densities where collisions dominate the population of the upper and lower quantum levels.

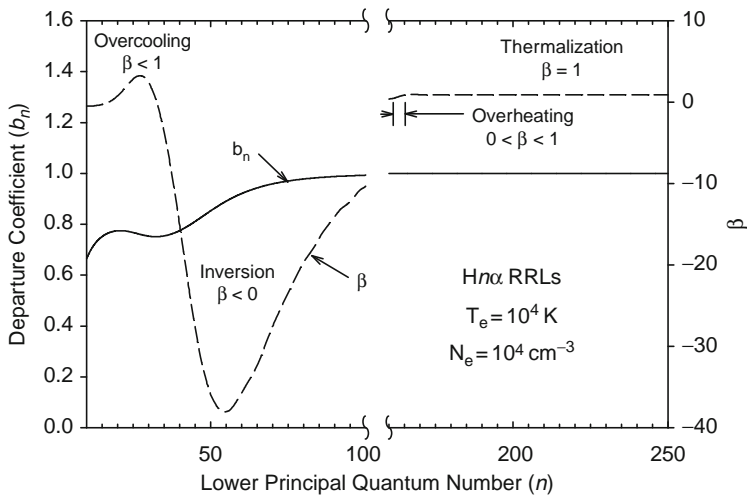


Fig. 2.28 Classification of the excitation regions of a quantum transition with respect to a b_n and β curve for hydrogen α -type RRLs. Departure coefficients from Storey and Hummer (1995). The *abscissa* is broken from $n = 100 \rightarrow 150$ to illustrate the distinct regions

The departure coefficients plotted in Fig. 2.23 illustrate these regions with respect to a larger range of electron densities. The overcooling regime disappears at high densities.

2.4 The Range of RRL Studies

2.4.1 High-Frequency RRLs

RRL observations began at centimeter and decimeter wavelengths but quickly spread into the millimeter wave range as receiver sensitivity improved. Observations in this range have several advantages:

1. The ratio of the line-to-continuum intensities grows with frequency because of the decreasing free-free emission, as shown by (2.124) and (2.143).
2. It is perhaps easier to interpret millimeter wave RRLs because one can neglect stimulated emission because of the decreasing free-free emission and Stark broadening because of the steep dependence of the line width on principal quantum number ($n^{4.4}$) shown by (2.74). Knowing the appropriate departure coefficient remains a problem, however.
3. For filled-aperture radio telescopes, the beamwidth decreases and, hence, angular resolution increases with increasing frequency. This is not a significant limitation for radio interferometers, however.

The first observations of an RRL in the millimeter wave range were carried out with the 22-m radio telescope in Pushchino, Russia, equipped with an 8-mm maser receiver. Figure 2.29 shows the H56 α line from the Omega nebula (Sorochenko, Puzanov, Salomonovich and Steinschleiger, 1969) at 36.5 GHz. This was the first spectral line of any kind detected in the Galaxy at millimeter wavelengths.

With improving technology, RRL detections moved to even shorter wavelengths. Waltman et al. (1973) detected the H42 α line at 85.7 GHz from the Orion nebula with the 36-ft. telescope of the National Radio Astronomy Observatory at Kitt Peak, AZ. Wilson and Pauls (1984) detected the H41 α and H39 α lines in Orion at 99.0 and 106.7 GHz, respectively, with the 7-m offset telescope of Bell Laboratories in New Jersey. With the resurfacing of the Kitt Peak telescope and a new 3-mm receiver, Gordon (1989) detected the H40 α line in seven Galactic sources.

To a large extent, further advancements toward shorter wavelengths involved observations of a single object, the strong RRL maser emission from MWC349 (Martín-Pintado, Bachiller, Thum and Walmsley, 1989). Figure 2.30 shows spectra from MWC349 at $\lambda \approx 1$ mm and $\lambda \approx 3$ mm. While the H41 α ($\lambda = 3.3$ mm) line has a Gaussian profile, the profiles of the H31 α ($\lambda = 1.42$ mm), H30 α ($\lambda = 1.29$ mm), and H29 α ($\lambda = 1.17$ mm) lines have double peaks. The dependence of intensity of observed lines on n was also

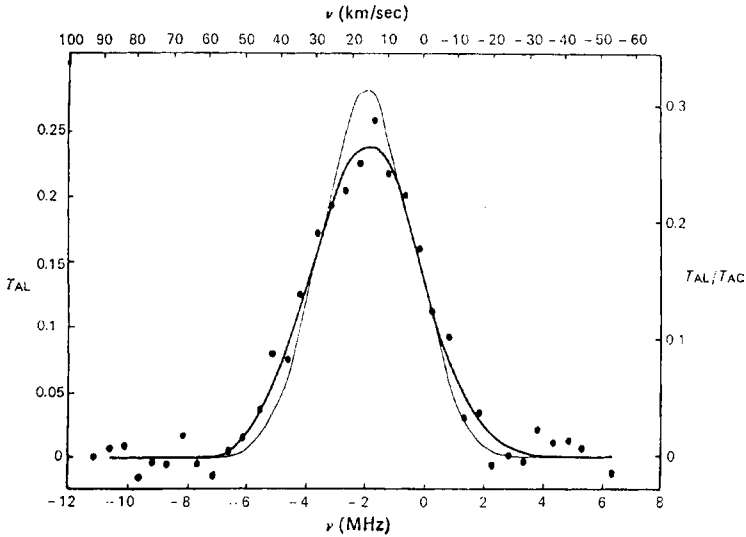


Fig. 2.29 Radio recombination line H56 α from the Omega nebula – the first RRL detected in the millimeter range. The *thick line* shows the observed profile. The *thin line* shows the profile corrected for the bandwidth of the spectrometer. The *left ordinate* is antenna temperature and the *right* is the ratio of the line-to-continuum intensity

unusual. Instead of the line intensity decreasing as the quantum number decreased as expected from thermodynamic equilibrium, the opposite occurred. The line intensity increased as the quantum number decreased. The line intensities of the $\lambda = 1$ mm lines were at least 50 times greater than those of the $\lambda = 3.3$ mm line.

Observations of RRLs at still shorter wavelengths soon followed. Thum et al. (1994) detected the H21 α ($\lambda = 0.45$ mm) line from MWC349 with the James Clerk Maxwell telescope on Mauna Kea, HI. This line had the same double-peaked profile as those observed in the $\lambda = 1$ mm range. The Earth's atmosphere prevents observations at shorter wavelengths from the ground, but Streltinski, Haas, Smith, Erickson, Colgan and Hollenbach (1996) overcame this limitation by observing from the Kuiper Airborne Observatory and detecting the H15 α , H12 α , and H10 α from MWC349 in the infrared portion of the spectrum. Technically, these were no longer “radio” recombination lines but “infrared” recombination lines (IRLs). Finally, Smith et al. (1997) made observations of the H6 α ($\lambda = 12.4 \mu\text{m}$), H7 α ($\lambda = 19.1 \mu\text{m}$), H7 β ($\lambda = 8.2 \mu\text{m}$), and H8 γ ($\lambda = 12.4 \mu\text{m}$) recombination lines from the same object in the middle infrared range (MIRLs). The spectral resolution was inadequate to determine the details of the line profiles but the line amplitudes showed MWC349 still to be masing in these lines.

The double-peaked profile of the RRLs from MWC349 seems to be consistent with a model of maser emission from the border of an edge-on, rotating

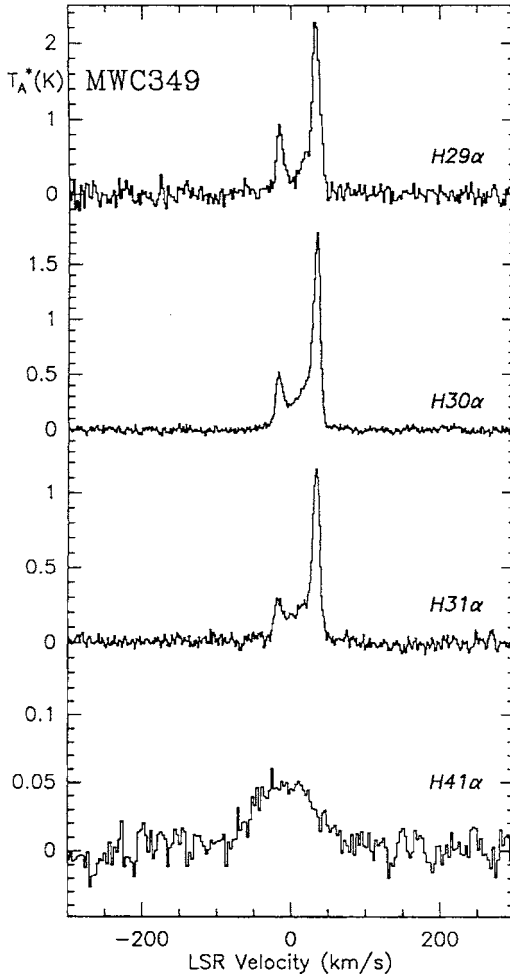


Fig. 2.30 Radio recombination lines from the binary star system MWC349 at millimeter wavelengths (Martín-Pintado et al., 1989). The $\lambda = 1$ mm lines are the first RRL maser ever detected. Note the double-peaked profile of the high-frequency lines

circumstellar disk. For this reason, the two components are believed to be shifted symmetrically relative to the system velocity (Gordon, 1992; Thum, Martín-Pintado and Bachiller, 1992). The next part of this book will consider the MWC349 emission in more detail.

The detection of the hydrogen RRL maser in the MWC349 source stimulated a closer look at the theory of masing RRLs.

Section 2.3.12 showed the necessary conditions for a maser in an H II region to be large negative values of the net optical depth (τ_ν). Because $\tau_\nu \equiv \kappa_\nu L$, we can examine the net absorption coefficient κ_ν independently of the path

length L through the nebula. The absolute value of k_ν determines the maser gain (Strel'nitski, Ponomarev and Smith, 1996). From (2.141),

$$\kappa_\nu = \kappa_C + b_{n_1}\beta k_L^*, \quad (2.158)$$

where n_1 is the lower level of the transition. The coefficient β is evaluated at the lower principal quantum number n_1 as defined by (2.132). Since all quantities in (2.158) except β are positive, β has to be sufficiently large and negative to fulfill the maser condition of $\kappa_\nu \ll 0$, i.e., $\beta \ll -\kappa_C/(b_{n_1}\kappa_L^*)$.

Within (2.158), each parameter depends on the physical conditions within the H II region – temperature and density – as well as upon the principal quantum number and the frequency of corresponding transition. These dependencies are different. For example, the density corresponding to maximum maser gain differs from the one corresponding to the maximum of the energy-weighted population gradient characterized by β .

Figure 2.31 shows the net absorption coefficient κ_ν as a function of electron density for a number of hydrogen α -lines at millimeter wavelengths (Strel'nitski, Ponomarev and Smith, 1996). The temperature for the calculations was taken to be 10^4 K, a canonical value for H II regions. The departure coefficients are those calculated by Storey and Hummer (Storey and Hummer, 1995).

The picture shows that there are optimal values of density for maximizing the amplification of each line, corresponding to minimum value of the net absorption coefficient. These minimums shift toward higher densities with decreasing quantum number, while absorption coefficient itself increases. Therefore, for each small group of adjacent lines, there is a relatively narrow interval of densities where the maximum maser gain can be realized. For example, the optimum density is $N_e = 5.6 \times 10^5 \text{ cm}^{-3}$ for the group of lines near H55 α , $N_e = 4 \times 10^7 \text{ cm}^{-3}$ near H30 α , and $N_e = 1.6 \times 10^9 \text{ cm}^{-3}$ near H15 α .

However, the density corresponding to minimum of the non-LTE line absorption coefficient does not coincide with the density of the maximum maser gain. Compare the dashed curve ($\kappa_L = b_{n_1}\beta\kappa_L^*$) for H36 α line shown at the upper right diagram with the solid line (κ_ν). The minima of the two curves occur at different densities. Because the total absorption coefficient (κ_ν) seen by the photons also includes free-free absorption that is also a function of density, the extrema of the line and total absorption coefficients occur at different densities for any given line – as shown by (2.158). The effect of κ_C is to decrease the absolute value of the negative line absorption and to shift its minimum toward lower densities.

With specific regard to the RRL maser in the MWC349 star system, the calculations (Strel'nitski, Ponomarev and Smith, 1996) show that the maximum gain should occur in the range H15 α to H34 α , which corresponds to the range of electron densities $N_e \approx 10^9 \rightarrow 10^6 \text{ cm}^{-3}$, respectively.

The existence of a high-gain hydrogen radio recombination maser requires a second condition in addition to the large negative value of κ_ν . The photons

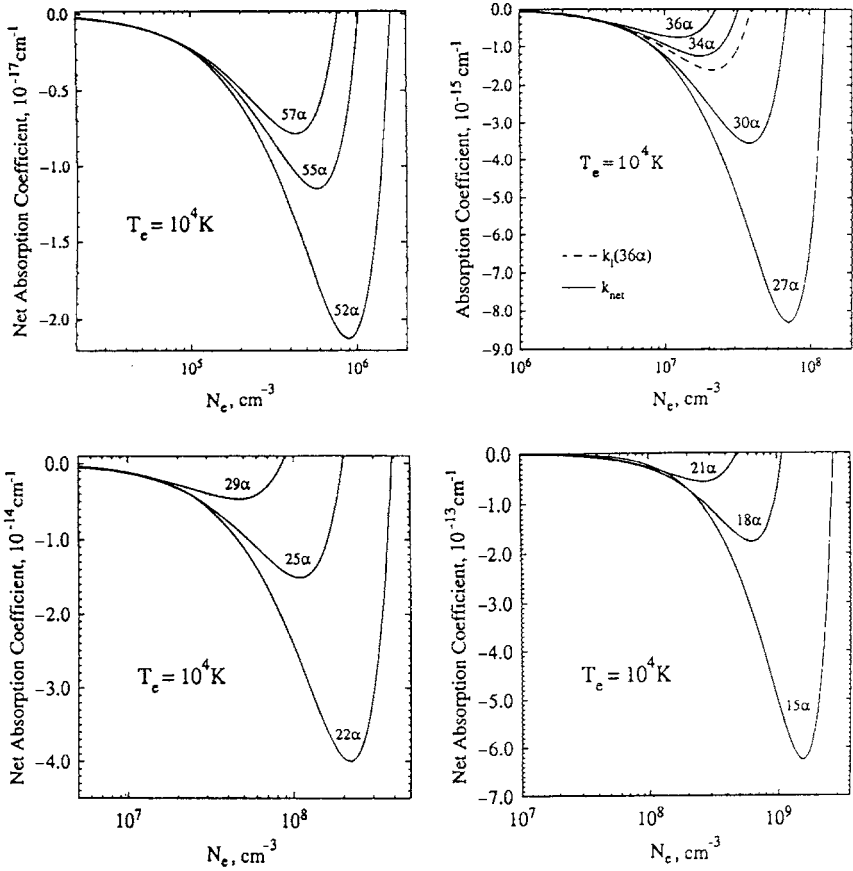


Fig. 2.31 The net absorption coefficient plotted as a function of electron density for a number of α -type RRLs (Strel'nitski, Ponomarev and Smith, 1996). In this figure, k_{net} and k_l correspond to our κ_ν and κ_L , respectively

along the path must maintain excellent phase coherence with each other, which puts stringent limits for homogeneity of density and hydrogen atoms velocity along the line of sight. This requirement usually means exceptionally narrowly focused maser beams in astronomical sources. For example, if the accumulated phase difference $\Delta\phi$ along two lines of sight differing by a small angle θ is

$$\Delta\phi = \frac{2\pi\ell}{\lambda}(1 - \cos\theta) \approx \frac{\ell}{\lambda}\pi\theta^2, \quad (2.159)$$

the path length in wavelengths through a hypothetical astronomical masing region, $\ell/\lambda \approx 10 \text{ AU}/1 \text{ mm} \approx 10^{15}$, would require a beam angle of $\theta < 2 \times 10^{-8}$ rad to maintain the phase difference of $\Delta\phi = 1$ rad. Perhaps, because of

this coherence requirement,²⁹ strong hydrogen RRL masers have only been detected in MWC349 up to present time; this discovery could have been a matter of luck for astronomers. The only other possible source of strongly masing RRLs is η Carina, where Cox et al. (1995) found peculiar millimeter recombination lines of hydrogen.

2.4.2 Low-Frequency RRLs

The solution to the Stark broadening problem explained why it was possible to observe RRLs at significantly higher atomic levels than originally presumed (see Sect. 2.2.4). In the early years, the highest atomic level associated with an observed RRL was $n = 301$. The H300 α line was detected from both the Sgr A region and W43 (Casse and Shaver, 1977; Pedlar, Davies, Hart and Shaver, 1978).

Can atoms with still higher levels of excitation exist in the cosmos? To answer this question, a number of observatories searched for RRLs at $n = 350 \rightarrow 650$ lying in the meter and decameter wavelength ranges.

At the Pushchino Radioastronomical Observatory after several years of attempts, and with continuous improvements of the equipment to increase the detection sensitivity to 4×10^{-4} of the background continuum (Ariskin, Kolotovkina, Lekht, Rudnitskij and Sorochenko, 1982), success was achieved. RRLs with $n > 400$ were detected, but from carbon rather than hydrogen. It has not been possible to detect hydrogen RRLs at these quantum levels. The C427 α ($\lambda = 3.56$ m), C486 α ($\lambda = 5.25$ m), C538 α ($\lambda = 7.12$ m), and C612 β ($\lambda = 3.56$ m) lines were observed with the North–South line 864×40 m size of the DKR-1000 cross-axis radio telescope. The lines were observed in absorption toward the powerful source of low-frequency radio emission Cassiopeia A (Ershov, Iljsov, Lekht, Smirnov, Solodkov and Sorochenko, 1984).

Carbon lines with still higher transition numbers were detected toward Cassiopeia A by Konovalenko and Sodin (Konovalenko and Sodin, 1980; Konovalenko and Sodin, 1981) of the Radioastronomical Institute in Kharkov. Observations of the C630 α ($\lambda = 11.4$ m), C631 α ($\lambda = 11.5$ m), and C640 α ($\lambda = 12$ m) were made with the decameter radio telescope UTR-2 – the world’s largest with a size of $1,800 \times 900$ m, shown in Fig. 2.32. It is worth noting that the decameter carbon lines were detected earlier than the meter ones, after unsuccessful attempts to detect excited hydrogen lines at long wavelengths (Konovalenko and Sodin, 1979).

²⁹ Moran (2002) notes that this argument can also be reversed. The equation illustrates that “the phase requirements are so stringent that no cosmic maser can operate as a spatially coherent amplifier. As a result, cosmic masers have virtually no intrinsic beaming properties ... [and] are essentially temporally incoherent and spatially incoherent, unlike laboratory lasers with parallel mirrors.”



Fig. 2.32 The $1,800 \times 900$ m UTR-2 radio telescope of the Radioastronomical Institute of the Ukraine at Kharkov. With this instrument, the longest wave carbon RRLs were detected at decameter wavelengths

The C631 α line, which was first detected in the decameter range, was initially identified by authors (Konovalenko and Sodin, 1980) as the hyperfine transition $F = 5/2 \rightarrow 3/2$ of atomic nitrogen ($\nu = 26.127$ MHz) predicted by Shklovsky (1956b). This interpretation, however, encountered large difficulties. The observed optical depth of the line would require more than an order of magnitude increase of the nitrogen abundance relative to the accepted value.

Blake et al. (1980) had shown that observed line could be identified as a C631 α line in absorption, whose frequency ($\nu = 26.126$ MHz) is very close to the frequency of nitrogen line. This explanation eliminated the necessity of revising the nitrogen abundance in interstellar medium.

Subsequent observations confirmed this interpretation. Konovalenko and Sodin (1981) detected two more lines toward Cassiopeia A with frequencies that corresponded exactly to the carbon RRL C630 α ($\nu = 26.250$ MHz) and C640 α ($\nu = 25.0396$ MHz). This frequency correspondence confirmed that all three lines were carbon RRLs.

During few next years in Kharkov (Konovalenko, 1984), in Green Bank (Anantharamaiah, Erickson and Radhakrishnan, 1985), and in Pushchino (Ershov et al., 1987), astronomers detected a number of carbon RRLs in the meter and decameter wavelength range up to C732 α ($\lambda = 18$ m) toward Cas A. Figure 2.33 shows the profiles of the low-frequency carbon lines obtained in Pushchino and Kharkov. These lines were observed in absorption with low ($\approx 10^{-3}$) line-to-continuum ratios. The integration time of each spectrogram was 20–40 h.

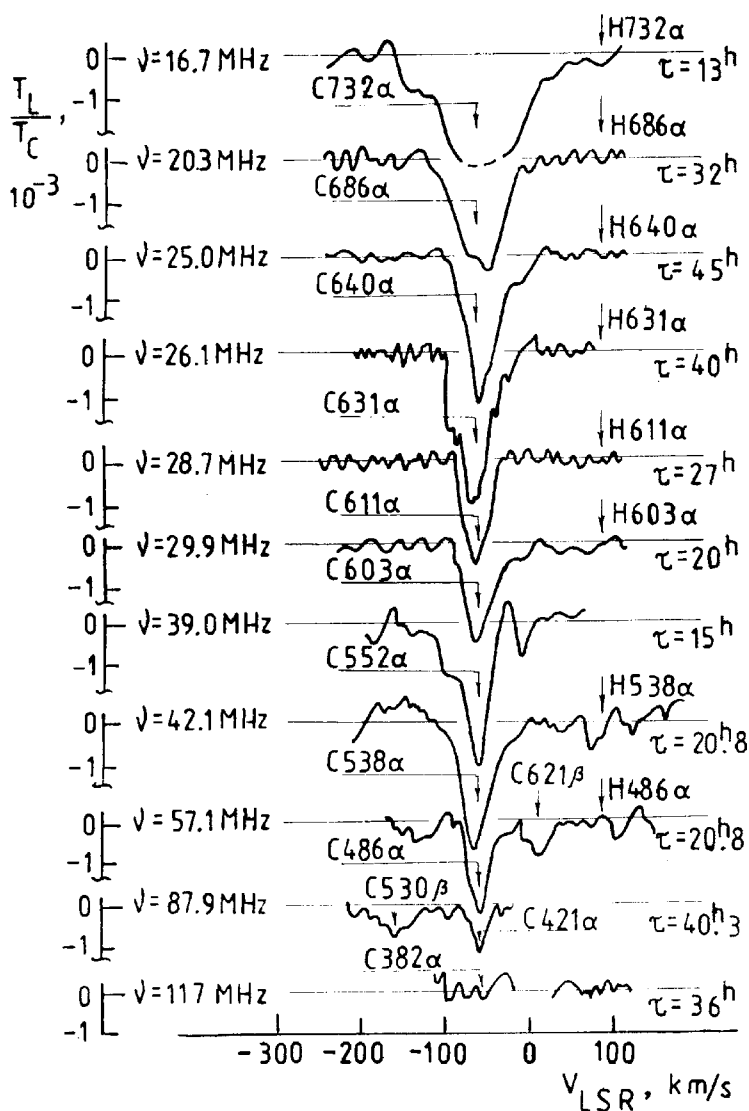


Fig. 2.33 Low-frequency carbon RRLs toward Cas A. $\nu < 30$ MHz, observations in Kharkov (Konovalenko, 1984); $\nu > 30$ MHz, observations in Pushchino (Ershov et al., 1984; Ershov et al., 1987). The *ordinate* is the amplitude of the line relative to the continuum emission. On the *left* is the frequency and on the *right* is the integration time. The *arrows* indicate the calculated positions of the carbon and hydrogen lines. From Sorochenko and Smirnov (1990)

The detection of the low-frequency lines was quite unexpected and extremely interesting both for physics and astronomy. A number of questions immediately arose:

1. Why are the carbon lines reliably observed at the highest excitation levels ($n > 400$) while the lines of the more abundant hydrogen are not detected? Why were all attempts to detect hydrogen lines with $n > 300$ toward Cas A and other sources unsuccessful (H352 α : Shaver et al. (1976); H400 α : Ariskin et al. (1979); H630 α –H650 α : Konovalenko and Sodin (1979); H351 α : Hart and Pedlar (1980); H392 α –H394 α : Ariskin et al. (1982))?
2. Why are carbon RRLs for the excitation levels $n = 530$ – 700 ($\nu = 44$ – 17 MHz) stronger than the lines of less excited atoms for $n = 420$ – 480 ($\nu = 88$ – 59 MHz), while the lines for $n = 380$ – 400 ($\nu \cong 100$ MHz) are in general impossible to detect?
3. If one can succeed in detecting lines corresponding to the 732th level and, in this case, their intensity does not fall with increasing n , then where is the limit to the formation of RRLs and what defines it?

At present, the answers to these questions appear to be quite clear. The answer to the first question – why carbon rather than hydrogen lines are observed at the highest excitation levels – is connected with the special nature of the regions where these lines originate. Analysis revealed that, to emit RRLs corresponding to transitions between extremely high excitation levels, the emitting region in the ISM must (a) have a low electron density and (b) be sufficiently cold.

The first requirement stems from the fact that the sensitivity of atoms to collisions with charged particles, mainly electrons, dramatically increases with excitation level. For H II regions with the temperature $T_e = 5 \rightarrow 10 \times 10^3$ K, the line width depends on such collisions as described by (2.74), i.e., $\Delta\nu_L \propto N_e n^{4.4}$.

For the lower temperatures found in H I regions, $T_e = 20 \rightarrow 200$ K. There, Stark broadening has a similar dependence on the principal quantum number of the level and the electron density (Ershov et al., 1984):

$$\Delta\nu_L = 1.16 N_e \left(\frac{n}{100} \right)^{5.1} \left(\frac{T_e}{100} \right)^{0.62} \text{ Hz.} \quad (2.160)$$

If we accept as a criterion that a line is observable when its width does not exceed 30% of the separation between adjacent RRLs, i.e., if

$$\Delta\nu_{\text{lim}} = \frac{6 \times 10^{15}}{n^4} \text{ Hz,} \quad (2.161)$$

then we can examine some general relationships between the density N_e and the principal quantum number associated with detectable low-frequency RRLs.

At these large quantum numbers, simplifications are possible. For example, Stark broadening dominates thermal broadening, i.e., $\Delta\nu_L \gg \Delta\nu_G$. Furthermore, at these large quantum numbers, carbon atoms can be considered to

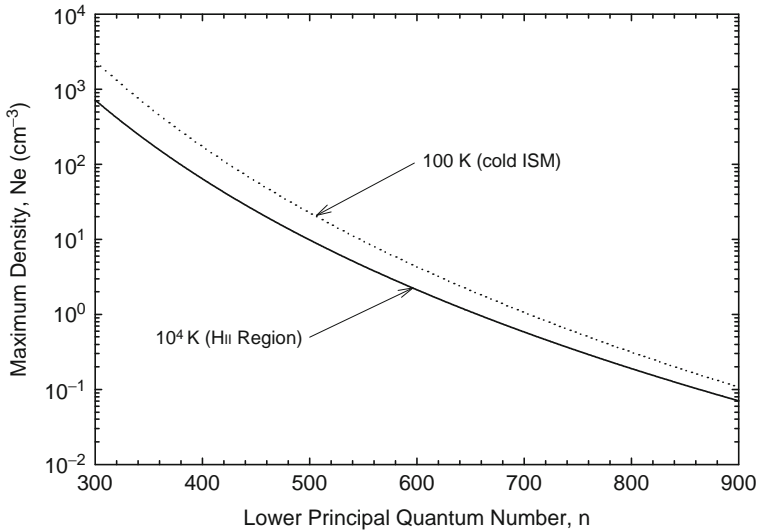


Fig. 2.34 Maximum values of the electron density N_e as a function of the lower principal quantum number n for detectable RRLs

be hydrogenic, i.e., these atoms have effectively the same electronic structure of a hydrogen atom – a single electron bound to a unitary, positively charged nucleus.

Therefore, combining (2.74) and (2.161) gives us a density limit of $N_e = 4.6 \times 10^{23} n^{-8.4}$ for detectable RRLs in H II regions with $T_e = 10^4$ K, and combining (2.160) and (2.161) gives a limit of $N_e = 8.2 \times 10^{25} n^{-9.1}$ for detectable RRLs in H I regions with $T_e = 100$ K. Figure 2.34 shows the detection limits for these two environments as a function of n , the “hot” environment of the H II region and the “cold” one of the ISM.

The figure shows the dependence of the maximum density for detectability of RRLs to be a strong function of principal quantum number. For example, the 400α line from the cold medium can be detected at densities up to 170 cm^{-3} , whereas the 800α line cannot be detected at densities exceeding 0.3 cm^{-3} . The limitation for the cold medium comes from the fact that the probability of recombination of ions and electrons increases as temperature decreases. Correspondingly, lower temperatures increase the likelihood of highly excited atoms existing and, also, increase the intensity of the line itself. Mathematically, (2.116) and (2.124) describe this behavior exactly, showing that T_L is approximately proportional to $N_e N_i / T_e^{3/2}$. In other words, in the ISM where the ionization products N_e and N_i are small – only partial ionization of the medium – detectable low-frequency RRLs can only occur when the temperature is very low.

In the ISM, the requirements of partial ionization and low temperatures are fulfilled in regions far from hot stars, in the so-called H I regions. Here, the

hydrogen is mostly neutral because of its high ionization potential of 13.6 eV and a weak ambient radiation field. In this environment, other elements with lower ionization potentials can be more easily ionized, as described by the Saha–Boltzmann equation (2.113). Note that the ionization energy enters as an exponential. Relatively, small differences in ionization potential can make enormous changes in $N - eN_i$, especially at low temperatures where the argument of the exponential term can be large.

Among the constituents of the ISM with ionization potentials lower than hydrogen is carbon, with an ionization potential of 11.3 eV. Of these candidates with lower ionization potentials, carbon is the most abundant. Consequently, the combination of a greater degree of ionization and a significant cosmic abundance is why carbon RRLs can be observed at very high excitation (n) levels but not hydrogen (Sorochenko and Smirnov, 1987).

The answer to the second question is connected to the specific dependence of the line intensities upon the level population. Since the lines form in a cool low-density environment, the level populations are not in thermodynamic equilibrium – even for $n > 400$. In the specific case of carbon, the level populations are influenced by a low-temperature dielectronic-like recombination (DR).

Watson et al. (1980) have shown that in the cold ISM ($T_e \approx 100$ K), the recombination of ionized carbon and free electrons to highly excited levels can occur simultaneously with the $^2P_{1/2} - ^2P_{3/2}$ fine-structure excitation of the C^+ core, where the energy (ΔE_{fs}) associated with this fine structure is 92 K, 1.27×10^{-14} erg, or $\lambda = 158 \mu\text{m}$ depending on your preference for units. In this case, the reverse process – autoionization – to a large extent is suppressed by rapid ℓ -changing collisions (ℓ is the quantum number for orbital angular momentum). For $\ell > 10$, autoionization is unlikely and the highly excited atom is stabilized.

This low-temperature dielectronic recombination³⁰ is an emission-free process. It occurs when the kinetic energy of the recombining electron is insufficient to excite the ion fine-structure level $C^+ \ ^2P_{3/2}$. This energy deficit exactly compensates the energy of the bound level n , i.e., the kinetic energy of the electron divides according to

$$\frac{mV^2}{2} = \Delta E_{fs} + E_n, \quad (2.162)$$

where $E_n = -2.18 \times 10^{-11}/n^2$ erg is the energy of the bound level n as described by (1.14) and V is the velocity of the recombining electron. Because the kinetic energy of the electron is always positive, (2.162) is executed if $n \geq 42$. For this reason, only highly excited levels of carbon experience can gain additional population through dielectronic recombination – a very difference situation than for hydrogen (Walmsley and Watson, 1982).

³⁰ Watson et al. (1980) suggested the term dielectronic “capture” for this low-temperature process.

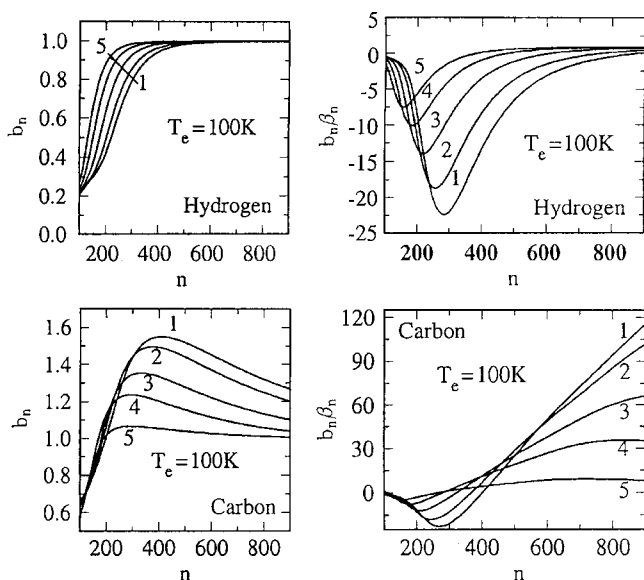


Fig. 2.35 b_n and b_n/β coefficients for hydrogen and carbon at $T_e = 100\text{ K}$ as a function of n . The numbers correspond to following values of electron densities N_e : 1–0.05, 2–0.1, 3–0.3, 4–1.0, 5–3.0 cm^{-3} . From Ponomarev and Sorochenko (1992)

Figure 2.35 shows values of b_n and b_n/β coefficients calculated for hydrogen and carbon for $T_e = 100\text{ K}$ and various electron densities (Ponomarev and Sorochenko, 1992). The curves for hydrogen are similar to those shown in Fig. 2.21. The b_n values smoothly increase to 1 with increasing quantum number, and the b_n/β terms are mostly negative for the large quantum numbers.

The carbon curves have a very different character. Unlike the case of hydrogen, the departure coefficients rise to values exceeding one and then decrease toward the LTE value of one, owing to the effects of dielectronic recombination. Consequently, the values of the term b_n/β cross into the positive domain in the range $300 < n < 500$, where their values considerably exceed their absolute values in the negative domain.

These characteristics explain the intensities of carbon RRLs in the meter and decameter wavelength ranges. In the region where $b_n/\beta_n = 0$, (2.132), (2.140), and (2.141) show that line optical depth (τ_L) will be zero, the total optical depth will be only that of the free-free continuum, and the carbon lines will not appear. This is why the C382 α line was not detected and why an earlier attempt to detect the C400 α line toward Cas A also failed (Ershov, Lekht, Rudnitskij and Sorochenko, 1982).

At $n > 420$, carbon lines are observed in absorption with the depth of the absorption line increasing with n – exactly as would be expected from the positive values of b_n/β_n shown in Fig. 2.35. In the range of negative values of b_n/β_n , the medium within the source amplifies the background radiation, and the lines appear in emission. Figure 2.36 shows C RRLs over a

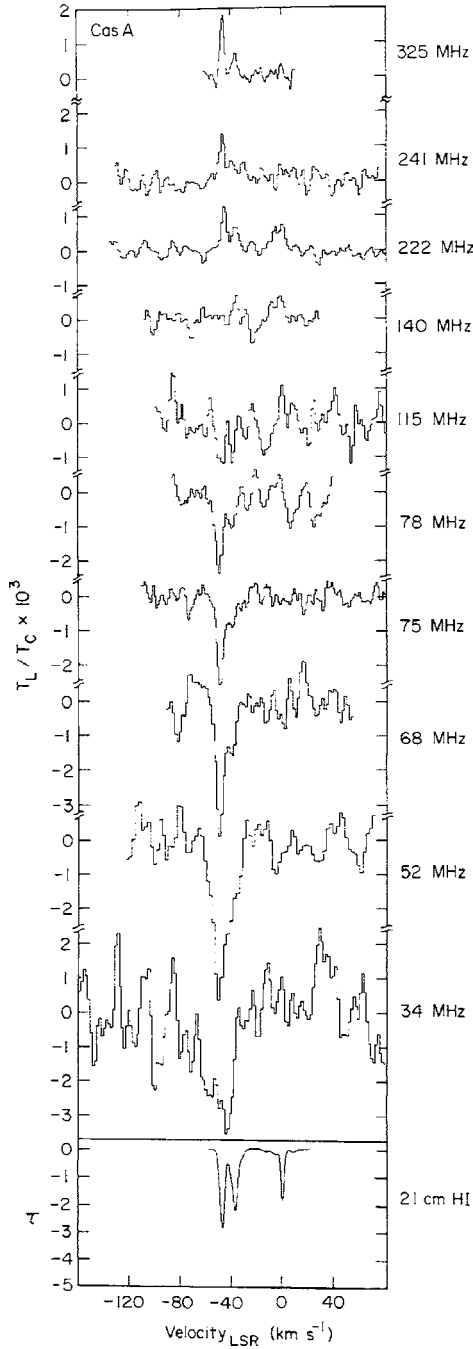


Fig. 2.36 Carbon α radio recombination lines observed toward Cas A at ten frequencies in the range 34–325 MHz. The quantum numbers corresponding to these frequencies are $n = 565, 502, 450, 446, 436, 385, 360, 310, 300$, and 272. The bottom spectrum is the HI absorption in units of τ . From Payne et al. (1989)

large frequency range observed at the NRAO observatory in Green Bank, WV (Payne, Anantharamaiah and Erickson, 1989). Note that the spectra change from absorption to emission with increasing frequency just as theory predicts. The quantum numbers $n = 272, 300$, and 310 involve carbon lines in emission; the numbers $n = 436, 446, 450, 502$, and 565 , absorption; and the intermediate numbers $n = 360$ and 385 , the transition region from emission to absorption where lines are not detected. The HI spectrum at the bottom shows the distribution of the line-of-sight interstellar gas as a function of radial velocity.

Therefore, the theoretical calculations of the population of the excited levels of carbon in the “cold” ISM, which take into account low-temperature dielectronic recombination, completely explain the dependence of the carbon RRL intensities in all wavelength ranges where they have been observed.

2.5 How Many Atomic Levels Can Exist?

The detection of RRLs in the meter and decameter wavelength ranges – particularly, the detection of carbon lines up to C732 α – prompts the questions: what is the maximum quantum number of stable atomic levels and what are the restrictions on this level? These questions are very interesting both for atomic and for elementary-particle physics. According to Bohr theory (see (1.6)), the diameter of an atom in a distinct quantum level n is $1.06 \times 10^{-8} n^2$ cm. Therefore, the carbon atom in the ISM with $n = 732$ has a diameter of $d \approx 50 \mu\text{m}$. Can atoms exist in the cosmos with an excitation level of 1,000, corresponding to a diameter of $100 \mu\text{m}$? Or, with $n = 3,000$, corresponding to $d \approx 1 \text{ mm}$?

Until the detection of meter and decameter RRLs, astronomers had assumed that the limit of highly excited atomic levels was determined by collisions with electrons, i.e., only by the electron density. It became evident that in interstellar medium, large-diameter atoms can occur at very low electron densities $N_e = 10^{-1}, 10^{-2}$, and 10^{-3} cm^{-3} . At these densities, electronic collisions would place a limit on highly excited atoms and, correspondingly on RRL emission, to values of n equal to many thousands.

The limiting effect of neutral particles is even weaker. Cross sections for the interactions of the excited atoms with atoms in ground state increase to a limit at $n = 20 \rightarrow 30$, after which this limit remains constant (Mazing and Wrubleska, 1966). As a result, for $n \approx 1,000$, these cross sections are almost ten orders of magnitude less than the ones for electronic collisions.

What determines the maximum number of distinguishable atomic levels and, hence, the limiting dimensions of cosmic atoms? It turns out to be the background Galactic, nonthermal radiation.

2.5.1 Radiation Broadening of RRLs

This background radiation stimulates transitions between highly excited levels, reducing the lifetimes of these levels, and consequently causing Lorentz line broadening described earlier by (2.9) and (2.10). In a thermalized medium, the stimulated emission is isotropic because the mean intensity J has no preferential direction; it is the time-averaged integral of I over all angles (see footnote 31). Accordingly, the full width of the Lorentz profile at half-intensity is

$$\Delta\nu_L \equiv \frac{\Gamma}{2\pi} = \frac{\Gamma_n + \Gamma_m}{2\pi}, \quad (2.163)$$

where the total rate out of level n , or Γ_n , is

$$\Gamma_n = \underbrace{\sum_{m=1}^{n-1} A_{n,m} + \sum_{m=1}^{n-1} J_\nu B_{n,m}}_{\text{emission}} + \underbrace{\sum_{k=n+1}^{\infty} J_{\nu'} B_{n,k}}_{\text{absorption}}. \quad (2.164)$$

In this environment, at large principal quantum numbers, the contribution of spontaneous emission (giving the natural width component of the line) will be small in comparison to the stimulated terms and can be neglected. The line width due to radiation broadening is then

$$\Delta\nu_L \approx \frac{1}{2\pi} \left(\underbrace{2 \sum_{m=1}^{n-1} J_{n,m} B_{n,m}}_{\text{depopulation of } n} + \underbrace{2 \sum_{l=1}^{m-1} J_{m,l} B_{m,l}}_{\text{depopulation of } m} \right) \quad (2.165)$$

$$\approx \frac{2}{\pi} \left(\sum_{m=1}^{n-1} J_{n,m} B_{n,m} \right), \quad n \gg \Delta n > 0, \quad (2.166)$$

for the emission transition from level n to level m . In (2.165), the terms within the large parentheses indicate the depopulation rates for levels n and m , respectively. For a level with a large n , depopulation is almost the same for emission and absorption and, hence, we insert cofactors of 2 for the emission term of each level to account for absorption processes out of that level. At large principal quantum numbers, the lifetime of the upper level is approximately equal to the lifetime of the lower level, and we sum the n and m terms to get (2.166).

2.5.1.1 Galactic Background Radiation

We now find the mean intensity J for the ambient radiation field in our Galaxy at long wavelengths. Cane (1978) gives the brightness temperature

of the isotropic nonthermal radiation to be $T_{NT} = 22.6 \times 10^3 \text{ K}$ at 30 MHz with a spectral index $\alpha = 2.55$. Therefore,

$$T_{NT} = 22.6 \times 10^3 \left(\frac{3 \times 10^7}{\nu} \right)^{2.55} = \frac{2.63 \times 10^{23}}{\nu^{2.55}} \text{ K}. \quad (2.167)$$

Because $h\nu = 10^{-19} \ll kT_{NT} = 10^{-12} \text{ ergs}$, we can use the Rayleigh-Jeans approximation to the Planck radiation function to calculate the mean intensity:

$$J_\nu \approx \frac{2kT_{NT}\nu^2}{c^2} \quad (2.168)$$

$$= \frac{2k\nu^2}{c^2} 22.6 \times 10^3 \left(\frac{3 \times 10^7}{\nu} \right)^{2.55} \quad (2.169)$$

$$= \frac{8.10 \times 10^{-14}}{\nu^{0.55}} \text{ erg s}^{-1} \text{ Hz}^{-1} \text{ cm}^{-2}. \quad (2.170)$$

The Einstein B terms result from their definition in terms of the A terms:³¹

$$B_{n,m} \equiv \frac{c^2}{2h\nu^3} A_{n,m} \quad (2.172)$$

$$= \left(\frac{c^2}{2h\nu^3} \right) \left(\frac{8\pi^2 e^2 \nu^2}{mc^3} \right) f_{m,n} \quad (2.173)$$

$$= 5.03 \times 10^{25} \frac{1}{\nu} \cdot \frac{0.19n}{(\Delta n)^3}. \quad (2.174)$$

Equation (2.174) uses an approximation for the absorption oscillator strength $f_{m,n}$ resulting from

$$f_{m,n} \approx nM(\Delta n) \left(1 + \frac{1.5 \Delta n}{n} \right) \quad (2.175)$$

$$\approx \frac{0.19n}{(\Delta n)^3}, \quad (2.176)$$

after evaluating the equal-order Bessel functions in the definition given by Menzel (1968).

Combining (2.170) and (2.174) gives

$$JB_{n,m} = 0.047 \left(\frac{n}{100} \right)^{5.65} \left(\frac{1}{\Delta n} \right)^{4.65} \text{ s}^{-1}, \quad (2.177)$$

³¹ This relationship is the specific intensity form. The relationship between $B_{n,m}$ and $A_{n,m}$ coefficients is a numerical rather than a directional one. The relationship results from the detailed balance requirement in TE that

$$n_1 JB_{1,2} = n_2 A_{2,1} + n_2 JB_{2,1} \quad (2.171)$$

such that the dimensions of $JB_{1,2}$ are the same as $A_{2,1}$, i.e., s^{-1} .

where we have used the approximation for the RRL frequencies given by (1.21). At large values of n , this expression³² gives sufficiently accurate frequencies.

2.5.1.2 Width of the Broadened Line

Summing the stimulated emission terms, we find

$$\Delta\nu_L = \frac{2}{\pi} \sum_{m=1}^{n-1} J_{n,m} B_{n,m} \quad (2.178)$$

$$\approx \frac{4.70 \times 10^{-13}}{\pi} \times n^{5.65} \left[\underbrace{\left(\frac{1}{1}\right)^{4.65}}_{\Delta n=1} + \underbrace{\left(\frac{1}{2}\right)^{4.65}}_{\Delta n=2} \right] \quad (2.179)$$

$$\approx \frac{4.70 \times 10^{-13}}{\pi} (1.04) n^{5.65} \quad n \gg 1, \quad (2.180)$$

$$= 0.031 \left(\frac{n}{100} \right)^{5.65} \text{ Hz}, \quad (2.181)$$

for the full width at half-intensity of radiation-broadened RRL. Note that two terms of the summation are sufficient; the term associated with $\Delta n = 2$ is nearly negligible.

Ershov et al. (1982) derived the same equation (their equation (9)) by considering the collisions of ambient photons with highly excited atoms in the nebular gas.

2.5.1.3 The Lowest-Frequency Detectable RRL

If we arbitrarily assume that an $n\alpha$ recombination line can be distinguished from its neighbor when its width is less than, say, $1/3$ of the interline spacing, we can use (1.22) and (2.181) to equate the frequency difference between adjacent lines to the spacing criterion:

$$6.58 \times 10^{15} \left[\frac{1}{n^3} - \frac{1}{(n+1)^3} \right] = 3 \Delta\nu_L \quad (2.182)$$

$$= 0.093 \left(\frac{n}{100} \right)^{5.65} \quad (2.183)$$

³² Using the approximate expression of (2.176) for the f_{mn} reduces the numerical accuracy to two significant figures, at most.

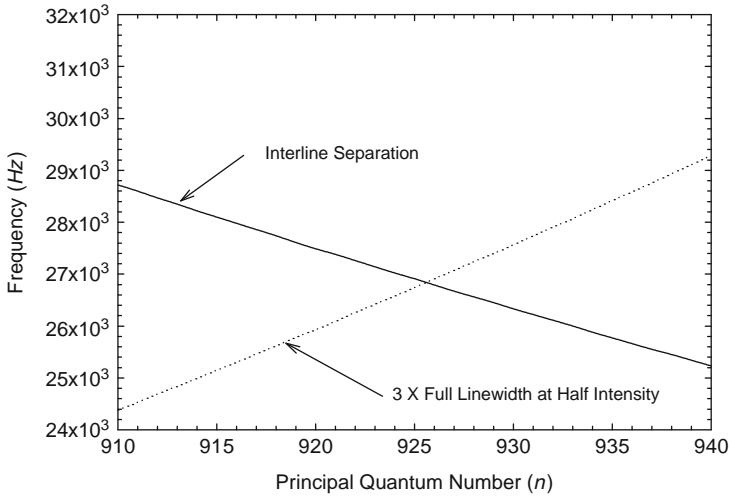


Fig. 2.37 The interline spacing and three times the line width plotted as a function of principal quantum number. The intersection gives the solution of (2.183)

and solve for n – the lower principal quantum number of the lowest-frequency, distinguishable line from the cold ISM of our Galaxy. Figure 2.37 shows the solution graphically. This equality is true for $n \approx 926$, which corresponds to a hydrogenic atom of diameter $\approx 91 \mu\text{m}$ – slightly larger than the thickness of this page.

This criterion for detectability is conservative, as can be seen from Fig. 2.38. If the criterion of the interline spacing is reduced to twice the line width, then the equality will give $n \approx 967$, corresponding to an even larger atom of diameter $\approx 0.1 \text{ mm}$.

2.5.2 *Existence as well as Detectability*

It is possible to consider the limit from another perspective. If one considers that a quantum state will not occur when the lifetime of that state is less than the rotation period of the bound electron around the nucleus, the nonthermal background radiation would limit the quantum states to $n \leq 1,600$ (Shaver, 1975), corresponding to an RRL with a frequency of about 1.6 MHz. This limit is less restrictive than the broadening discussed above but the two criteria may not be in conflict. The former refers to the *existence* of the quantum states and, therefore, the existence of RRLs in *any* form rather than just being detectable. Practically speaking, though, the detectability limit is the more important one for astronomical research.

Both of these calculations suggest that atoms as quantum systems in interstellar conditions can exist up to quantum levels of $\approx 1,600$, at the least.

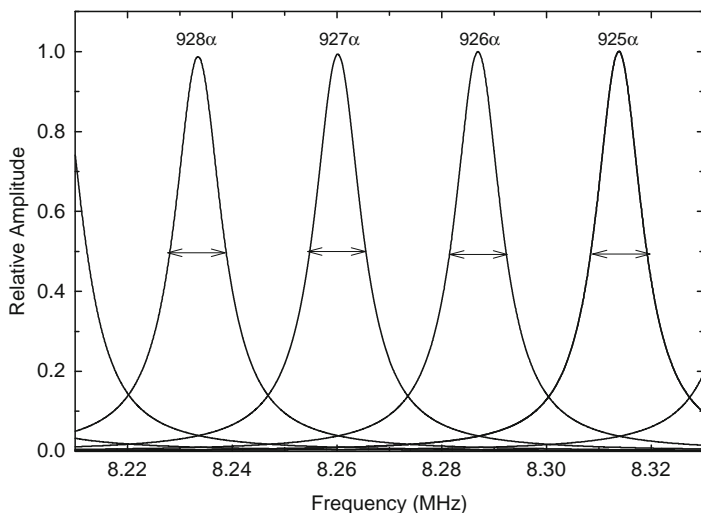


Fig. 2.38 What an observer might see: the superposition of radiation-broadened recombination lines plotted against rest frequency. The full widths at half-intensity are indicated. The separation between the H925 α and H926 α lines is approximately three line widths

These mean very large atoms. According to the Bohr model, such levels would correspond to atomic diameters of approximately 0.3 mm. Because ambient radiation limits their existence as well as their detectability, even larger atoms might exist and be detectable in cosmic environments with lower radiation than our Galaxy.

Despite the apparently fantastic nature of such huge atoms, their existence is real and natural. Let us imagine the rarified interstellar medium. Its most abundant element – hydrogen – is neutral. The ambient UV radiation from distant stars can only ionize atoms with ionization potentials less than that of hydrogen. Their lower – often, considerably lower – abundance means fewer electrons and ions. In some regions, the electron density is only a few cm^{-3} or so. Despite this sparseness, the free electrons and ions can recombine and produce highly excited atoms. These atoms can have significant lifetimes. The low densities and temperatures of their environment make unlikely the collisions with charge particles that can destroy them or change their state. Neutral atoms have even less influence on their existence.

Presently, astronomers seem to be approaching the excitation limits of these huge atoms. The low-frequency limit of 30 MHz for the DKR-1000 radio telescope in Pushchino allows searches for α -type RRLs up to $n \approx 600$. However, searches for higher-order transitions involving larger quantum levels are being carried out, such as the detection of the C748 β line toward Cas A (Lekht, Smirnov and Sorochenko, 1989) shown in Fig. 2.39.

To date, carbon lines with the highest quantum levels have been observed at Kharkov with the UTR-2 radio telescope. Figure 2.39 shows the average of the C764 α –C768 α lines in the 14.7-MHz range toward Cas A, obtained by

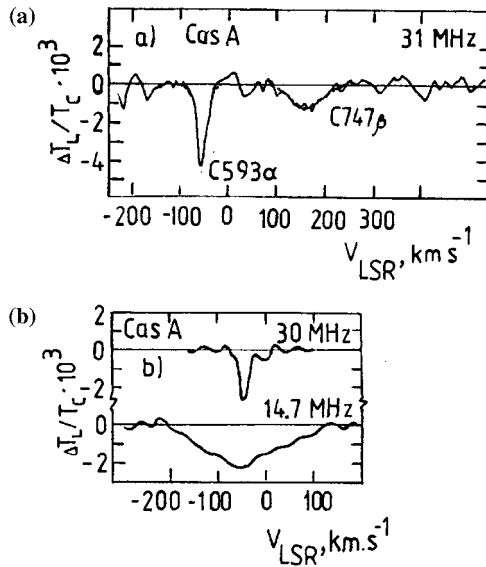


Fig. 2.39 Carbon RRLs detected toward Cas A at the highest known excitation levels of atoms. **a** The C747 β line close to the frequency of the C593 α line. The velocity scale is that of the C593 α line. From Lekht et al. (1989). **b** At the *bottom* is the averaged spectra of the lines C764 α –C768 α ; at the *top* is the spectrum of the C603 α line shown for comparison (Konovalenko, 1990)

Konovalenko (1990). The integration time for one line was 50 h. Comparison of the C747 β line with the C593 α line, and of the averaged C764 α –C768 α lines with the C593 α line, shows that the line widths increase with n due to the shortening of the lifetimes of the excited levels near the detection limit. For example, the width of the C764 α –C768 α averaged profile is already 20% of the separation between adjacent lines or $1,180 \text{ km s}^{-1}$ in velocity. At $n \approx 1,000$, the widths should be so large that the adjacent lines will blend with each other into a continuous spectrum and, therefore, should disappear.

More recently, Stepkin et al. (2007) reported the detections of additional carbon lines at 26 MHz in absorption against Cas A after integrations of up to 500 h. Some of these lines are approximately 1009 δ transitions and, as such, are more than 10^6 times larger than the ground-state atoms, corresponding to carbon atoms with classical diameters of 108 μm . As we have mentioned earlier, 0.1 mm is a dimension comparable to the thickness of this page.

2.6 Summary

Figure 2.40 illustrates the progress of our knowledge about atoms, the existence of excited levels, and the spectral lines from transitions between them. These data are of great interest for physics and were obtained from radio

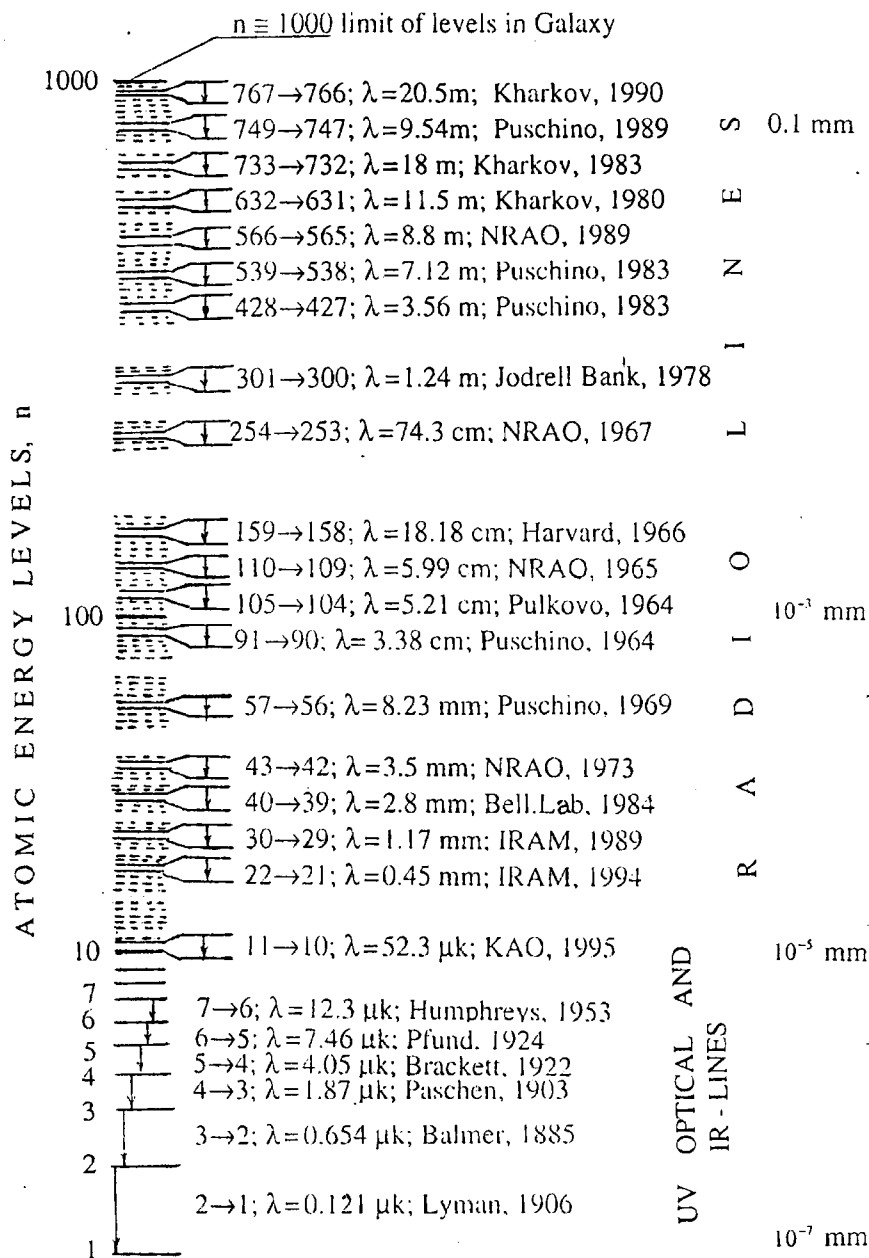


Fig. 2.40 A graphic display of the range of detected α -type recombination lines sorted by the upper principal quantum number n . The range includes the UV, IR, submillimeter, millimeter, centimeter, meter, and decameter parts of the electromagnetic spectrum. Note that interstellar atoms can exist up to excitation levels of $n \approx 1,000$. The right ordinate is the equivalent diameter of the associated atoms

astronomical methods of study. Initially performed in the centimeter range at $n \approx 100$, the observations of RRLs advanced toward the longer meter and decameter wavelength ranges, where the lines occurring near $n \approx 1,000$, limited by the conditions of our Galaxy, were detected.³³ Simultaneously, observations of RRLs advanced toward the shortest wavelength region – millimeter and submillimeter waves. After observations of IR lines as far as H10 α in the MWC349 hydrogen maser, this modern research became joined to the classical measurements of hydrogen lines in the IR, visible, and UV ranges which lie at the foundation of the Bohr quantum theory of atoms.

It is interesting to note that N. Bohr partially foresaw that the most highly excited atoms would be observed in space. In his classical article “About the spectrum of hydrogen” (Bohr, 1914), when explaining why the high-order lines of the Balmer series seen in celestial spectra were extremely difficult to observe in the laboratory, he wrote: “In order that the large orbits of electrons may not be disturbed by electrical forces from the neighboring atoms the pressure will have to be very low, so low, indeed, that it is impossible to obtain sufficient light from a Geissler tube of ordinary dimensions. In the stars, however, we may assume that we have to do with hydrogen which is exceedingly attenuated and distributed throughout an enormously large region of space.”

Naturally, Bohr could not foresee that the most highly excited atoms would be detected by radio astronomy techniques that did not exist when he created the quantum theory of atoms. Certainly, he could not have foreseen that atoms may have up to 1,000 distinct principal quantum levels.

With these discoveries involving basic physics, one can hardly doubt that our cosmos is truly a wonderful laboratory.

³³ Figure 2.40 does not contain the more recent observations of Stepkin et al. (2007), which were obtained after the original edition of this book had been published.

Chapter 3

RRLs: Tools for Astronomers

Abstract This chapter describes what astronomers have learned from observations of radio recombination lines since their detection. It discusses the characteristics of gaseous nebulae, the state of ionized hydrogen and helium in these nebulae, the characteristics of ionized carbon found in the comparatively cool interstellar gas, planetary nebulae, the Sun, an unusual early star (MWC 349A) emitting time-varying masering lines, and nearby galaxies in which radio recombination lines have been detected.

Radio recombination lines (RRLs) turned out to be a powerful tool for astrophysical research. They are unique both in the number of transitions that can be detected and in the wavelength range over which they are observed. They occupy about five orders of magnitude of the wavelength scale of electromagnetic waves,¹ which is why they can be used for the study of astronomical objects that significantly differ in their physical parameters. The physical characteristics of “radio” also play a significant role here. Unlike the electromagnetic waves of the ultraviolet, optical, or infrared ranges, huge wavelength ranges of radio waves are almost unabsorbed by the interstellar medium (ISM) and therefore can be detected from very large distances.

RRLs provide us with a great deal of information about the ISM. Although its mass is only 3% of the total mass of our Galaxy ($1.5 \times 10^{11} M_{\odot}$), the ISM is the main component in terms of occupied volume. The study of the ISM enables us to understand the evolutionary processes that take place in the Galaxy and, by extension, in other galaxies. A continuous exchange of matter takes place between stars and the ISM. According to current thinking, stars form from the interstellar matter in regions where the physical conditions

¹ The etymology of “radio” refers to radiant (electromagnetic) energy used for communication or, more primitively, signaling over distances. The term was first used in the late nineteenth century, allegedly in 1898 by the French physicist Brandly in reference to a “coherer” detector. At this writing, the radio domain is considered to range from about 10 kHz to, say, 1 THz – eight orders of magnitude – but is often extended in practice, particularly toward higher frequencies. Officially, the range can be more restricted as in the definitions used within the communications industry (Emerson, 2002).

in the ISM clouds – density and temperature – start the process by gravitational collapse that heats the gas that, eventually, triggers the thermonuclear fusion that causes the stars to “shine.” In this way, portions of the ISM are transformed into stars.

This conversion also runs the other way. The stars return part of their mass to the ISM through stellar winds, planetary nebulae, novae, and supernova explosions. Because the thermonuclear processes within these stars have enriched their original material with new elements, this process changes the chemical composition of ISM.

At the same time, the energy radiated outward by stars in the form of UV radiation, stellar winds, and expanding shells causes fundamental changes in the structure and physical conditions of the ISM as well as in its chemical composition. That part of the ISM close to stars is ionized by the stellar UV radiation, forming H II regions with temperatures of several thousands of Kelvin. In time, these H II regions expand and the star can leave the H II region because its trajectory may differ from that of the surrounding gas. Or, in the case of a nova or supernova, the star can release its ionized shell altogether. Because of these dynamic processes, other parts of the ISM will radiate their heat energy away, allowing cold dense regions to form again, which in turn will form into new generations of stars.

In this way, a circulation between stars and the ISM takes place. This duality of cosmic processes is often called “astration,” the astronomical equivalent of the biological term “symbiosis.” The stars require the ISM and the ISM requires the stars. Moreover, on a large scale, the similarity of the term *astration* to the medical word “aspiration” is also appropriate, with its implication of gas moving in and out of an organism – as in breathing. It is important to note that astration is not reciprocal everywhere. That part of the ISM that forms into small and very small stars often remains there. It is the large stars that participate most vigorously in the astration process.

Hydrogen is the principal component of the ISM. It accounts for about 70% of the mass of the ISM. The remaining parts apportion by mass to 28% for helium and 2% for all other elements. Approximately, half of the hydrogen in the ISM (by mass) is in the form of molecules inside dense, cold clouds and the other half is in the form of neutral atoms (H I) and ions (H II).

Despite its importance in the astration process, molecular hydrogen does not emit spectral lines at radio wavelengths. Astronomers study it through indirect techniques or directly through spectral lines emitted in the infrared where, unfortunately, the opacity can be quite large. Hydrogen molecules collide with other molecules that can emit in the radio range, exciting quantum mechanical energy states within them, thereby allowing these molecules to radiate through rotational transitions at millimeter and centimeter wavelengths. Studying the spectra of abundant secondary molecules like CO allows astronomers to deduce the characteristics of the radio-invisible, interstellar hydrogen molecules.

On the other hand, atomic hydrogen emits the well-known, ubiquitous $\lambda = 21$ cm hyperfine line discovered in 1951 (Ewen and Purcell, 1951; Muller and

Oort, 1951). Observations of this line provided, and are still providing, the fundamental data about the distribution of neutral hydrogen in our Galaxy and in many other galaxies.

RRLs enable astronomers to study another basic component of the ISM: the interstellar ionized gas. Initially, this term referred only to discrete H II regions, which are very widespread in the Galaxy. In fact, the existence of hydrogen RRLs is the primary criterion for classifying cosmic radio sources as either thermal (H II regions) or nonthermal. Such observations revealed that the majority of discrete continuum sources at centimeter wavelengths near the plane of Galaxy are H II regions. Lockman (1989) found 462 of 500 such sources located within $\pm 1^\circ$ of the Galactic plane to emit RRLs. Moreover, among the other 38 sources, not all are nonthermal. RRLs were not detected in some of these owing to insufficient sensitivity of the survey.

Information obtained from hydrogen RRLs enables us to determine the basic physical conditions of H II regions as well as distribution of ionized hydrogen in Galaxy. Helium, the second-most abundance element of the ISM, is also ionized in the majority of H II regions. The ratio of the intensities of hydrogen and helium RRLs enables us to determine with high accuracy the relative abundance of helium, which has great significance not only for understanding the physics of the ISM, but also for understanding how the Universe formed.

Hydrogen is generally in a neutral form in the cold ISM, in H I regions, and at the surface of molecular clouds because of its high ionization potential with respect to the ambient interstellar radiation field. Figure 3.1 illustrates the

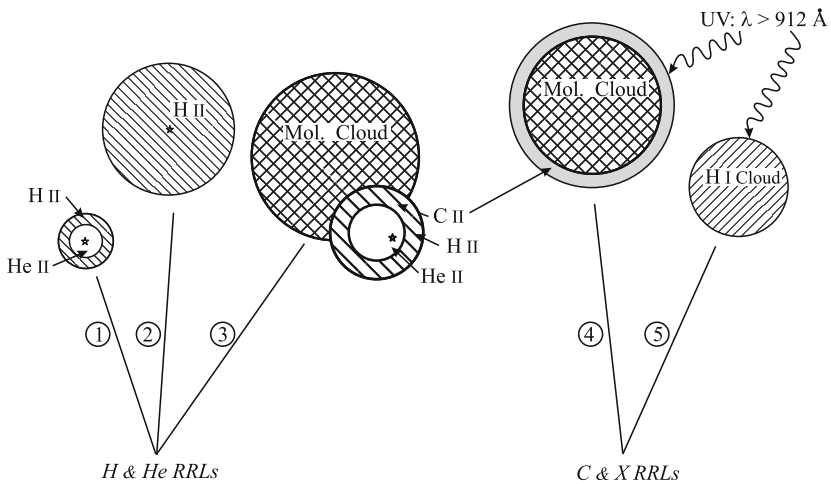


Fig. 3.1 This cartoon shows the types of ISM objects studied with RRLs. Item (1) represents a dense, bright H II region like the Great Nebula in Orion or a planetary nebula; (2) is an extended, low-density H II region; (3) represents C II regions at the interface between H II regions and molecular clouds; (4) shows the C II region boundary between molecular clouds and the diffuse ISM; and (5) illustrates C II regions within atomic H I clouds. “X” indicates RRLs from atoms other than carbon, helium, or hydrogen

types of ISM objects that are studied with RRLs. Note that elements with lower ionization potential can be ionized in these locations. Among these, carbon is the most abundant. Carbon RRLs are detected at many different frequencies (see Sect. 2.4.2). We shall show below that carbon lines contribute important information about intermediate ISM layers between H II regions and the parent molecular clouds from which they were formed.

In addition to these objects, RRLs have been detected from the Sun, two stellar systems, and a number of extragalactic sources.

Figure 3.2 shows the general form of this information obtained from RRLs. It shows a generalized RRL spectrum in units of antenna temperature (T_A) vs. arbitrary frequency units (ν) that could be obtained from any H II region in our Galaxy. The helium RRL is detected simultaneously with that of hydrogen, being separated by $4.078 \times 10^{-4}\nu$ owing to the difference in the Rydberg constants of the species. If the parent molecular cloud with the C II region on its border is included in beam of the radio telescope, the carbon line and the lines of heavier elements like S, Mn, Si, and Fe (see Sect. 3.3.1)

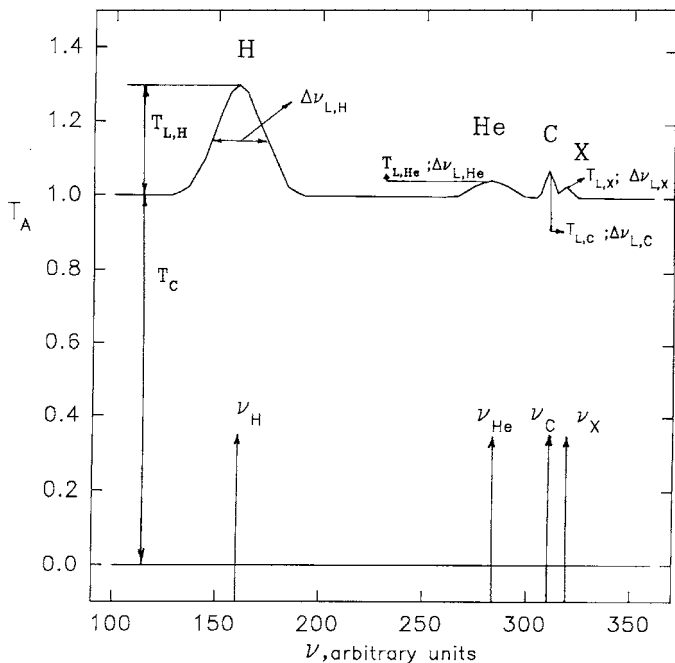


Fig. 3.2 A hypothetical spectrogram toward an H II region showing the radio recombination lines of hydrogen, helium, and carbon. The lines from heavier elements are merged into common line marked "X." The composite profile contains information about the ISM in the form of the measured antenna (brightness) temperatures of each of the constituent lines (T_L), their full widths at half-maximum ($\Delta\nu_L$) and their line shapes, the observed (Doppler-shifted) frequencies of the line centers (ν), and the underlying continuum temperature (T_C) contributed by the free-free emission from their ionization products

would also be detected. The frequencies of the lines in this group would be blue-shifted relative to the helium and carbon lines and would overlap due to the small differences in their Rydberg constants (see Table A.2). All of this RRL emission would be superimposed on the thermal continuum contributed mainly by the H II region but also by the ionization products of the heavier elements.

A large number of parameters can be measured from the spectrogram: antenna (brightness) temperatures of each of the lines and their widths, profiles and frequencies that can determine the radial velocities of the emitting medium with respect to the local standard of rest V_{LSR} . The antenna temperature (intensity) of the underlying continuum is measured simultaneously and, when used with the line intensities, determines the excitation status of the line components. These data tell us about the physical environment of the ISM.

We now turn to what has been learned, and what can be learned, from observations of RRLs from cosmic sources.

3.1 Physical Conditions in H II Regions

3.1.1 *Electron Temperature of H II Regions*

RRLs of hydrogen provide the simplest and most precise method of determining the electron temperature of H II regions. Unlike observations of optical emission lines, RRLs are unaffected by reddening by interstellar dust. Further, they can be accurately measured even in weak astronomical sources.

3.1.1.1 Line-to-Continuum Observations

The possibility of such measurements stemmed from the pioneering work of Kardashev (1959). This work showed that the paramount parameter specifying the recombination and ionization processes associated with RRLs was the electron temperature T_e . This single parameter specified not only the population of the principal quantum levels of the atom, but also the degree of ionization, such that it determined the distribution of the cosmic gas between bound and unbound quantum domains. The temperature dependence of the populations of the two domains is different. As a result, the measurement of the ratio of antenna temperatures in a line and in a part of the spectrum adjacent to it allows one to determine electron temperature of H II regions.

As is almost always the case in astronomy, there are several complications in using RRLs to determine the temperature of H II regions. One is departures of the level populations from thermodynamical equilibrium values as

described in Sect. 2.3.9. Another – recognized well after the initial detections of RRLs (Simpson, 1973a; Shaver, 1975) – was a systematic underestimate of the energy radiated in the line owing to Stark broadening that shifts emission from the line core to the line wings, where it easily blends with the underlying continuum. In this circumstance, fitting baselines to the profile results in mistaking wing emission for continuum, leading to the underestimate of the line emission relative to the continuum and, by (2.144), an overestimate of the electron temperature. Happily, however, there are frequencies where this diminishment of the measured line power offsets the enhancement of the line through the partial maser effect. In other words, the derived temperatures are correct.

Figure 3.3 illustrates a typical situation. The dotted line shows values of LTE L/C intensity ratios calculated from (2.124) and (2.26) for an electron temperature of 8,500 K and a turbulence broadening component of 10 km s^{-1} . These values would be typical for an H II region. Also plotted in the figure are actual observations of L/C from $\text{Hn}\alpha$ lines from the Orion nebula, a nearby compact H II region in the northern sky. The data cover a frequency range from 613 MHz to 135 GHz, a little over two orders of magnitude.

At low frequencies, the L/C ratios fall below the LTE values. This occurs because Stark broadening shifts radiation energy from the core of the line to the wings, where it becomes indistinguishable from the baseline of the

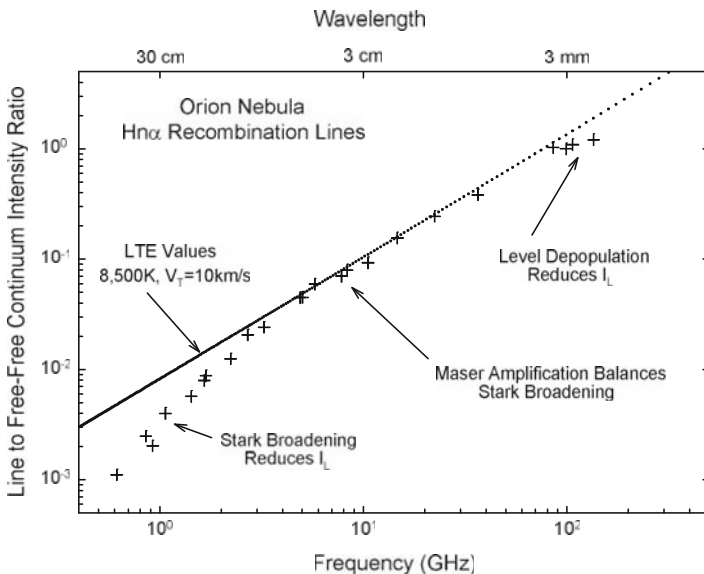


Fig. 3.3 The “line” consists of a series of points giving the LTE ratio of the line-to-continuum emission of RRLs from an H II region of 8,500 K and a turbulence broadening component of 10 km s^{-1} , calculated from (2.26) and (2.124). Each point marks the rest frequency of an RRL. Juxtaposed are observations of the L/C ratio from the Orion nebula (Lockman and Brown, 1975a; Gordon, 1989; Gordon and Walmsley, 1990)

spectra as described earlier. At high frequencies, two effects occur. First, departures from LTE lower the population of the quantum levels as described in Sect. 2.3.8, decreasing the number of atoms available to emit, and consequently decreasing the line intensity below LTE values. Second, continuum emission from dust begins to contribute significantly to the background emission as shown in Fig. 2.16, increasing the observed continuum beyond that radiated only by free-free emission. Both of these effects decrease the observed ratios of L/C below the LTE values.

There is an additional problem using observations of RRLs at low frequencies. As shown in Fig. 2.17, the opacity of the free-free emission can be significant at low frequencies. Consequently, radiation reaching the observer may not include all the matter along the line of sight. The observed values of L/C would nominally come only from regions where $\tau_C < 1$. This restriction may apply not just to a gas volume of the H II region in the foreground. Because H II regions contain density variations, the observed values of L/C could selectively reflect only the characteristics of the more tenuous gas where $\tau < 1$.

In the middle frequency range, however, it is possible to determine accurate values of electron temperature for an H II region, as discussed in detail by Shaver (1975). Stark broadening shifts energy from the line core to the line wings, thereby reducing the observed ratio L/C . Here, a fortuitous compensation occurs. The partial maser effect discussed in Sect. 2.3.8 offsets this weakening of the peak line intensity by Stark broadening such that the observed values of L/C actually fall very near the LTE values. However, owing to density inhomogeneities, the more representative observations will probably come from the upper end of this frequency range where the opacity is smaller.

Discussed in detail earlier, (2.144) provides a tool to extract an electron temperature from RRLs averaged along the line of sight in some unknown way. Equation (2.143) gives form for the enhancement of the observable quantities for the case in which $|\tau_L|$ and τ_C are much less than one. It is relatively straightforward to recast these equations into a variety of forms convenient for some particular analysis.

The problem is that, formally, one must know T_e before being able to use these equations. One must know the answer before calculating it; i.e., one must be able to determine β and b_{n_2} a priori to solve for T_e .

The solution was quantitatively given by Shaver (1980) based upon the behavior illustrated by Fig. 3.3 and an examination of the magnitude of departures from LTE. Figure 3.4 shows the calculated variation of the ratio T_e^*/T_e , i.e., the ratio of the apparent electron temperature derived from LTE considerations to the actual electron temperature, as a function of frequency and emission measure EM . The calculations assumed a filling factor for the gas of 0.1 and an excitation parameter for the central stars of $U = 100 \text{ pc cm}^{-2}$. The local electron density relates to the emission measure by $EM = 2 U N_e^{4/3} f^{2/3}$. Note the large area of the plot occupied by values near 1.0, which implies

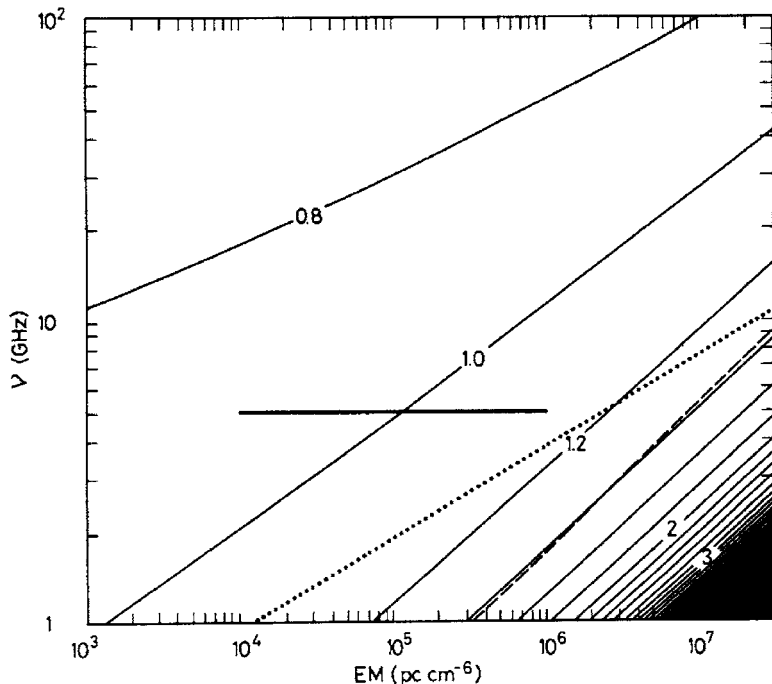


Fig. 3.4 Observing frequency vs. emission measure for various values of T_e/T_e^* . The excitation parameter $U = 100 \text{ pc cm}^{-2}$, $T_e^* = 10,000 \text{ K}$, and the filling factor $f = 0.1$ for all calculations. The horizontal line segment marks the range of H II regions observed at H109 α (5 GHz) (Reifenstein et al., 1970; Wilson et al., 1970). Figure taken from Shaver (1980)

that it is difficult to make substantial errors by calculating T_e from LTE equations. To illustrate this point specifically, Shaver plotted the range of emission measures involved in two surveys of RRLs from H II regions made at 5 GHz with the H109 α line (Reifenstein, Wilson, Burke and Altenhoff, 1970; Wilson, Mezger, Gardner and Milne, 1970). These observations involve the small range of $T_e/T_e^* \approx 1.0 \pm 0.1$ and, therefore, the values of T_e derived from the observations will be close to the correct values.

Shaver (1980) plotted these calculations in a different way. Figure 3.5 shows the loci of the ratio $T_e/T_e^* = 1$ for $T_e^* = 5,000$ and $15,000 \text{ K}$ and for two values of the filling factors for each temperature. The narrowband between the lines would include most H II regions. The centroid of this band is

$$\nu = 0.081 EM^{0.36}, \quad (3.1)$$

where ν is in units of GHz and EM is in units of pc cm^{-6} . This equation defines a frequency at which $T_e = T_e^*$, i.e., where the LTE electron temperature calculated from (2.124) is correct.

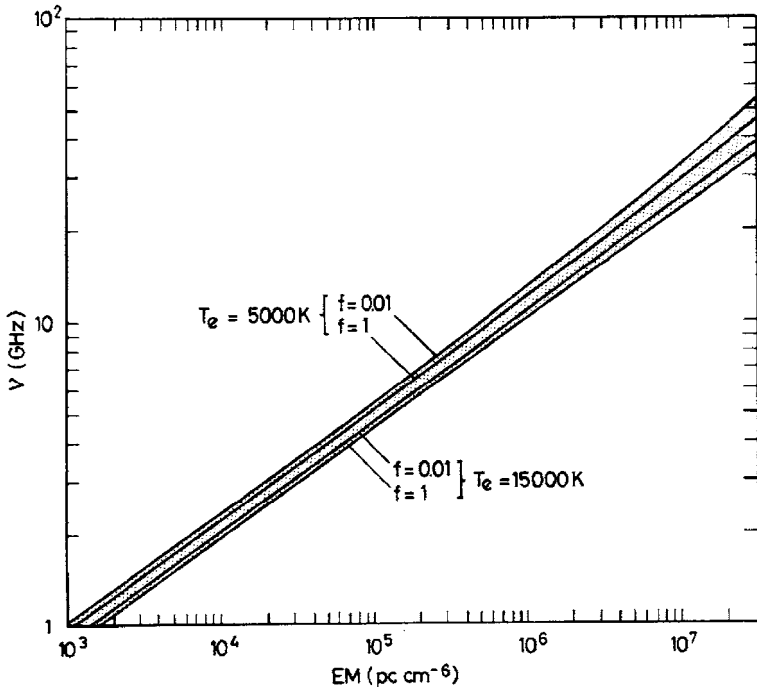


Fig. 3.5 Observing frequency vs. emission measure for $T_e/T_e^* = 1$ for $T_e^* = 5,000$ and $15,000$ K, and the filling factors $f = 1$ and 0.01 for each temperature. Figure taken from Shaver (1980)

Still, another figure is helpful to illustrate this conclusion. Figure 3.6 shows the variation of the ratio T_e/T_e^* as a function of frequency for a range of temperatures and filling factors. Again, we see that $T_e/T_e^* \approx 1.0$ at the frequency given by (3.1).

The reason for this behavior is that, in general, there is a frequency for each H II region where the collision rates populating the levels become dominant over the radiative rates, where the line amplification induced by the slope db_n/dn of the population curve is just offset by the line weakening due to underpopulation of the quantum levels themselves, thereby obviating the effects of departures from LTE. This point is a function of the emission measure, which parameterizes the densities involved in the departure coefficients.

Using this technique requires knowing the emission measure. The free-free optical depth given by (2.95) can be rewritten in terms of the optical depth τ_C :

$$\tau_C = \frac{0.08235 EM}{\nu^{2.1} T_e^{1.35}}, \quad (3.2)$$

where ν is in GHz and T_e is in K. As can be seen in Fig. 2.17, identifying the turnover frequency from the continuum spectrum of an H II region determines

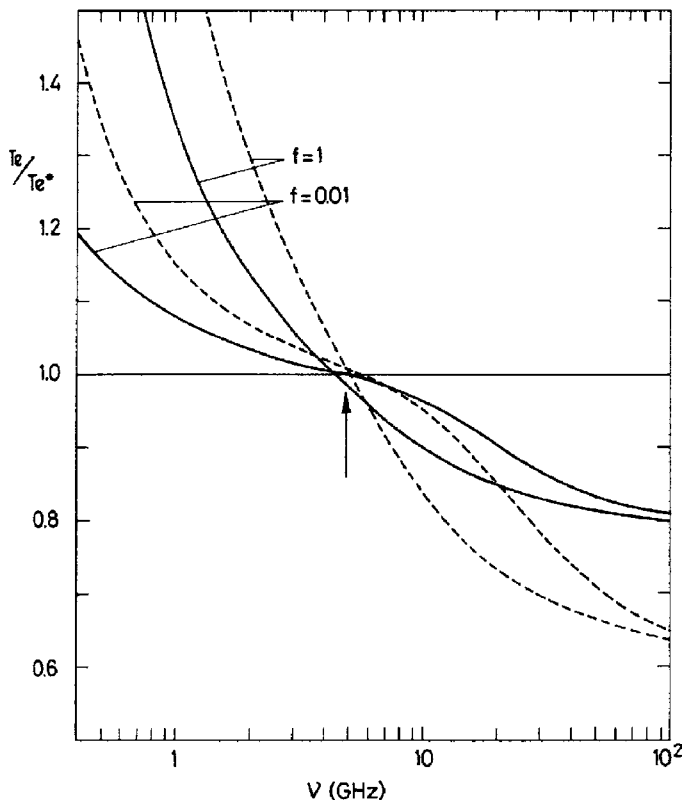


Fig. 3.6 The ratio T_e/T_e^* plotted against frequency for $EM = 10^5 \text{ pccm}^{-6}$ and $U = 100 \text{ pccm}^{-2}$. The solid and dashed lines mark $T_e = 15,000$ and $5,000 \text{ K}$, respectively, along with the indicated filling factors. The arrow marks the frequency given by (3.1). Figure taken from Shaver (1980)

the emission measure from (3.2) if the electron temperature is known. Fortunately, as Fig. 3.4 shows, the technique is tolerant to errors in the emission measure. In fact, iteration between values of T_e determined from the RRLs and subsequent re-estimates of EM from (3.2) should lead to closure, i.e., to reasonably accurate values of T_e for a given H II region.

Table 3.1 lists values of the electron temperature T_e calculated from representative millimeter and short centimeter wave RRLs from four nebulae. All observations were made with filled-aperture (single-dish) radio telescopes with beamwidths of $1'-2'$ at half-power. For reasons described earlier, $T_e \approx T_e^*$ for the short centimeter wave lines; i.e., the “true” electron temperature T_e is given by the LTE equation (2.124).

This situation does not apply to the millimeter wave RRLs, where underpopulation of the upper quantum levels weakens the line intensities, resulting in electron temperatures that are too large. Therefore, for the H40 α and H56 α lines, we corrected the temperatures (T_e^*) derived from the LTE formula by

Table 3.1 Electron temperatures derived from H II regions

| RRL | T_e (K) | | | |
|--------------------|---|---|---|---|
| | Orion nebula | NGC2024 | W3 | M17 |
| H40 α^a | 8,520 \pm 230 (11,200 \pm 300) $b_{41} = 0.73$ | 6,530 \pm 450 (8,800 \pm 600) $b_{41} = 0.71$ | 8,310 \pm 400 (10,800 \pm 500) $b_{41} = 0.74$ | 7,750 \pm 500 (10,700 \pm 700) $b_{41} = 0.69$ |
| H56 α | | 7,500 \pm 580 (8,400 \pm 650) $b_{57} = 0.88^b$ | 7,610 \pm 550 (8,260 \pm 600) $b_{57} = 0.91^b$ | 7,440 \pm 450 (8,400 \pm 500) $b_{57} = 0.87^c$ |
| H64 α^d | 8,400 \pm 400 | | | |
| H66 α^e | 8,200 \pm 300 | 7,200 \pm 500 | | 8,000 \pm 300 |
| H76 α^f | 8,600 \pm 430 | 8,200 \pm 400 | | 7,300 \pm 360 |
| H92 α^g | | | 7,940 \pm 140 | |
| Several lines | 8,100 \pm 100 H40 α , H56 α , H66 α^h | 7,400 \pm 500 H41 α , H63 α , H90 α^i | | |
| Contin. 330 MHz | 7,865 \pm 300 ^j | 8,400 \pm 1,000 ^k | | 7,600 ^{+700^l} ₋₂₁₀ |

Parentheses indicate T_e^* s, giving the T_e s above after correction by the listed b_{n_2} s

^aGordon (1989), ^bBerulis et al. (1975), ^cBerulis and Sorochenko (1983), ^dWilson and Filges (1990), ^eWilson et al. (1979), ^fShaver et al. (1983), ^gAdler et al. (1996), ^hSorochenko et al. (1988), ⁱWilson et al. (1990), ^jSubramanyan (1992a), ^kSubramanyan (1992b), and ^lSubramanyah and Goss (1996)

$$T_e \approx T_e^* (b_{n_2})^{0.87}, \quad h\nu \ll kT_e, \quad (3.3)$$

derived from (2.124) and (2.144). This approximation obtains because the small value of τ_C at high frequencies weakens the effect of β , the factor representing the gradient of population across the principal quantum levels of the upper and lower principal quantum levels. The departure coefficients are from Walmsley (1990), calculated from the FORTRAN program listed in Appendix E.1.

Note that the T_e s shown in Table 3.1 agree with each other remarkably well for each H II region. The variation is only a few percent, indicating the accuracy of the measurements and the reliability of the theory of radiation transfer developed in Chap. 2.

The last line of the table gives the results of T_e measurements of H II regions in the continuum at 330 MHz. They were carried out at the VLA with an angular resolution of 1', comparable with the resolution of the RRL observations. These data included only the brightest sources. At 330 MHz, all H II regions listed in Table 3.1 are optically thick, such that their observed brightness temperatures equal their electron temperatures, thereby providing direct measurements of T_e as described in Sect. 2.3.3.

Note the excellent correspondence between the electron temperatures derived from the RRLs and those derived from the continuum observations. Despite the higher error of the continuum measurements, they give an additional

proof of the correctness of the T_e values obtained from the RRLs from H II regions. This is especially significant because the analytical techniques are very different from one another.

Determinations of electron temperatures from RRLs should be far more accurate than from the traditional methods that use the forbidden auroral and nebular emission lines² of nitrogen and oxygen. The problem is that the optical techniques require a high temperature to excite the auroral lines, creating a temperature threshold of about 7,000 K that applies only to certain objects such as hot, bright H II regions. These techniques cannot be used for cooler material in the interstellar medium.

No such limitation obtains for electron temperatures determined from RRLs. The I_L/I_C ratio actually increases for cooler objects. Furthermore, the radio range is relatively free of absorption and extinction such that RRLs can be observed from highly obscured objects like H II regions throughout the plane of our own Galaxy.

Observations of RRLs have provided the most accurate and extensive measurements of the electron temperature of gaseous nebulae in our Galaxy. The observations include nearly 200 H II regions seen from both the northern and southern skies, including those that are cool, faint, or obscured by dark interstellar matter. The data establish that the electron temperature falls into a range of $4\text{--}12 \times 10^3$ K. Churchwell and Walmsley (1975) detected a gradient in T_e as a function of Galactocentric radius, which was confirmed by Churchwell et al. (1978), Mezger et al. (1979), Lichten et al. (1979), Churchwell (1980), and Garay and Rodríguez (1983).

Carefully, analyzing the electron temperatures 67 H II regions observed in the southern hemisphere with observational errors of about 5%, Shaver et al. (1983) obtained the regression equation of

$$T_e = (3,150 \pm 110) + (433 \pm 40)R_G \quad (3.4)$$

for the dependence of T_e in K as a function of the Galactocentric radius R_G measured in kpc. Figure 3.7 shows the actual data obtained from the H76 α and H110 α lines. The temperature gradient is easily seen.

² The terminology comes from early spectroscopy. “Auroral” transitions are named after the emission lines in the Earth’s permanent aurora. They violate the LaPorte parity rule for electric dipole transitions of $\Delta\ell = \pm 1$. Instead, auroral lines involve electric quadrupole transitions where $\Delta\ell = \pm 2$ or 0 as in the oxygen line $[\text{O I}] \ ^1D_2 \rightarrow \ ^1S_0$ at $\lambda = 5,577 \text{ \AA}$. For this reason, they are called “forbidden” lines and are designated by enclosing the atomic symbol in square brackets. “Nebular” lines are so named because they are found in gaseous nebula as well as in the Earth’s aurora. Nebular lines involve electric quadrupole and magnetic dipole contributions, resulting in changes in the electron configuration forbidden by the normal spectroscopic selection rules. An example is the jump from a triplet to a singlet configuration in neutral oxygen, $[\text{O I}] \ ^3P_0 \rightarrow \ ^1D_2$ at $\lambda = 6,300 \text{ \AA}$. In both types of transitions, the energy levels involved are called “metastable” because the transition probabilities between them are small and the lifetimes of the levels may be correspondingly long. Accordingly, the intensities of forbidden lines are sensitive to ambient temperature and density through collisions, and can be used to determine the physical conditions of their environment.

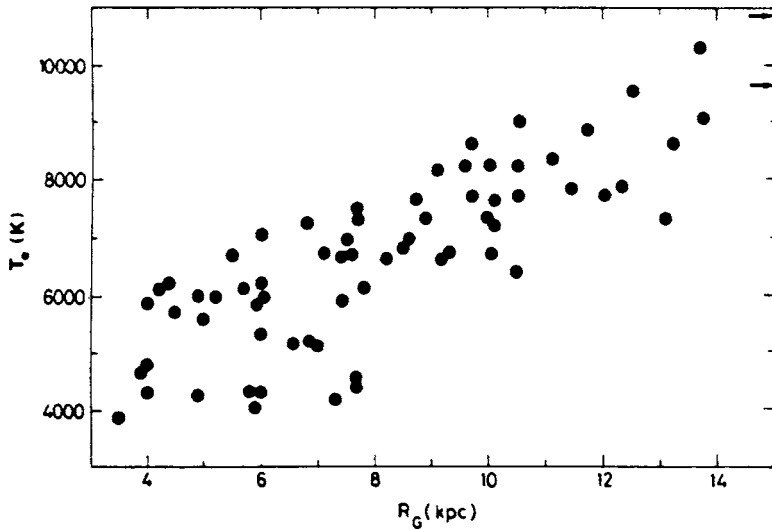


Fig. 3.7 Electron temperature of H II regions as a function of Galactocentric radius. The horizontal arrows at upper right indicate the N66 and 30 Doradus nebulae in the Magellanic Clouds. From Shaver et al. (1983). Reproduced with permission of Monthly Notices of the Royal Astronomical Society

Why should this gradient exist? Shaver et al. (1983) suggested that the temperature gradient could be explained by a corresponding Galactocentric gradient of metallic elements³ in the Galaxy. Of these, the most abundant are oxygen and nitrogen that cool H II regions through radiation in spectral lines. Collisions with free electrons easily excite metastable levels associated with these elements. This kinetic energy is later released from the H II regions largely through radiation in the [O III], [O II], and [N II] forbidden lines. Consequently, H II regions with higher metallicities can cool at a faster rate than others and should have lower electron temperatures. Because the star formation rate is higher in the inner parts of the Galaxy, the metallicity should decrease – and electron temperature should increase – with increasing Galactocentric radius consistent with the trend shown in Fig. 3.7. For example, a twofold increase in the metal abundance typical of the solar vicinity will cause a decrease in the temperature of the nearby H II regions by almost 2,000 K (Afflerbach, Churchwell, Accord, Hofner, Kurtz and DePree, 1996).

Observations of optical spectral lines also indicate a decrease of metallicity with Galactocentric distance. Figure 3.8 shows the variation of the normalized abundances O/H and N/H with the Galactocentric radius of our Galaxy. The radial decrease of both constituents is obvious.

³ In astronomy, “metals” refer to elements heavier than hydrogen and helium. Unlike hydrogen and helium, metals are principally produced by nucleosynthesis within the stars. The term “metallicity” refers to the relative abundance of these elements.

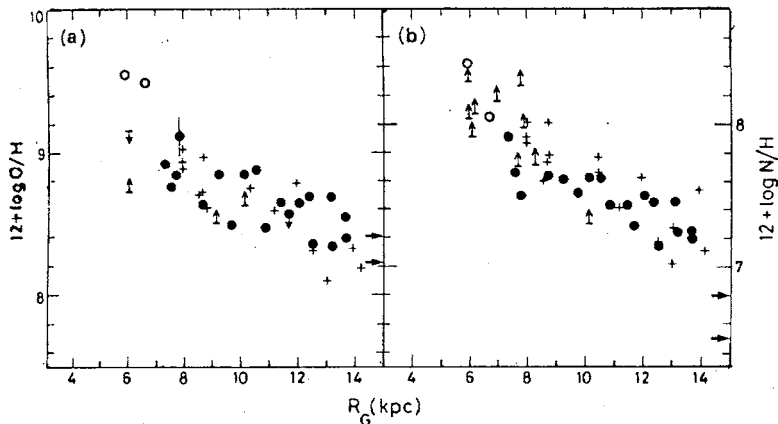


Fig. 3.8 Abundance of O and N vs. Galactocentric distance. The *open circles* denote S38 and S48 and the *horizontal arrows* represent the two Magellanic Cloud sources N66 and 30 Doradus. The *crosses* represent early data from Peimbert (1979). From Shaver et al. (1983). Reproduced with permission of Monthly Notices of the Royal Astronomical Society

At fixed distances (Galactocentric radii) from the Galactic center, Fig. 3.7 shows the electron temperatures determined from RRLs to have a large scatter. This scatter results from real differences in the electron temperatures of H II regions, in addition to measurement errors, and should be expected. Even in the same region of the Galaxy, H II regions will differ from each other in density and in the radiation field from the exciting stars. Higher densities mean more effective depopulation of metastable energy levels by collisions, reduced intensities of the associated forbidden lines, and, in turn, less effective radiative cooling and higher gas temperatures. Also, hotter exciting stars – or more of them – will increase the ambient radiation in the nebular gas and, correspondingly, increase the electron temperature of the gas.

The characteristics of ultracompact H II regions illustrate the effect of density on temperature. These are small nebulae with higher electron densities than the usual H II regions. Afflerbach et al. (1996) used H42 α , H66 α , H76 α , and H93 α lines to measure the electron temperatures of ultracompact nebulae. Despite the scatter of the observations, Fig. 3.9 shows the values of T_e for the ultracompact nebulae to exceed those of the ordinary H II regions, especially near the Galactic center. This is probably a consequence of radiation cooling being impaired by the higher nebular densities.

Together, the variation of T_e with Galactocentric distance determined from normal and ultracompact H II regions shows that the metallicity on the interstellar gas of our Galaxy decreases outward from the center. Indirectly, this gradient indicates increased star formation in the Galactic center where the nucleosynthesis enriches the heavy elements.

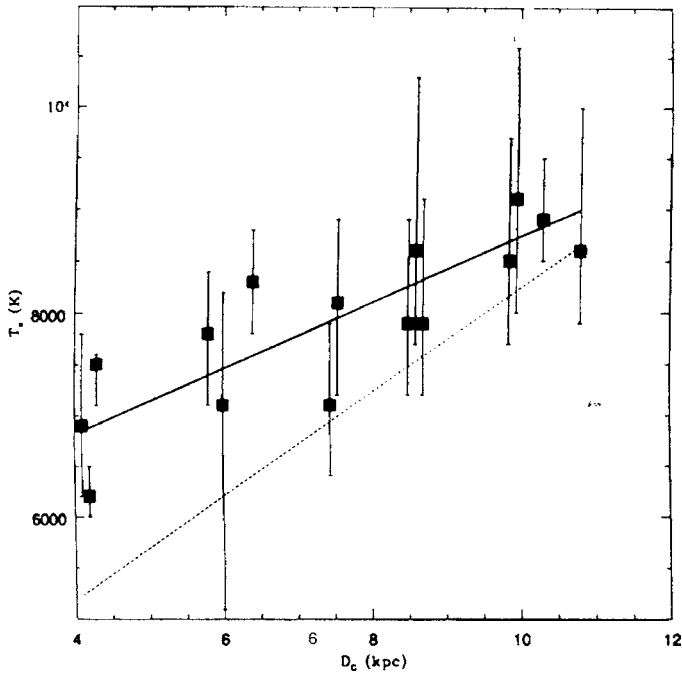


Fig. 3.9 The electron temperature of ultracompact H II regions obtained from radio recombination lines plotted against Galactocentric distance. The *solid line* is a least-squares fit to the data. The *broken line* is the gradient found by Shaver et al. (1983) for normal H II regions. From Afflerbach et al. (1996)

By providing an opportunity to measure electron temperatures through the interstellar gas of our Galaxy without concern for interstellar extinction, RRLs provide information of great value regarding evolutionary processes in spiral galaxies in general.

3.1.2 Electron Density of H II Regions

Probably, the most direct method of determining N_e for an H II regions is from the free-free continuum emission after first determining T_e , as discussed in Sect. 2.3.4. This procedure determines the optical depth of the free-free emission and, in turn, gives the emission measure of the nebula that provides the electron density once the size of the nebula has been determined.

However, it is also possible to measure electron densities from the RRLs themselves from Stark broadening. Equation (2.74) can be rewritten as

$$N_e = \frac{\Delta\nu_L}{8.2 \left(\frac{n}{100}\right)^{4.4} \left(1 + 2.2\frac{\Delta n}{n}\right)} \text{ cm}^{-3}, \quad (3.5)$$

where $\Delta\nu_L$ is the Lorentzian component of the line width in Hz caused by Stark broadening. Determining $\Delta\nu_L$ requires observations of RRLs over a wide frequency range made with the same angular resolution or beamwidth, i.e., observations of the same volume of gas. Specifically, observations of a high-frequency (low- n) RRL would provide a purely Doppler line profile. Observations of a low-frequency (high- n) line would provide the Lorentzian profile that, with the Doppler profile, would then provide the Stark broadening component $\Delta\nu_L$ needed for evaluation of (3.5).

Presuming that the Stark broadening is well measured if it equals or exceeds the Doppler width, we can combine (2.26) and (2.74) to derive

$$n \geq \left(\frac{3.6 \times 10^{19} \Delta n}{N_e} \right)^{0.135}, \quad (3.6)$$

if $V_t = 0$ and $\Delta n/n \ll 1$. This equation determines the minimum principal quantum number that should be used to determine the Stark broadening component for any given electron density for an RRL of type Δn . For example, for α lines at $N_e = 10^4 \text{ cm}^{-3}$, $n \geq 125$ whereas at $N_e = 10^3 \text{ cm}^{-3}$, $n \geq 171$. Because $n \propto N_e^{-0.135}$, suitable observations of high-density H II regions like planetary nebulae could be made at modestly low values of n . Here, the line intensities would be more detectable relative to the underlying continuum because I_L/I_C varies inversely with n as can be seen in theory (2.124) and in practice (Fig. 3.3).

In practice, one implements this analytic technique by fitting a Voigt function to the higher- n line profile, using the lower- n profile to establish the purely Doppler profile. Equation (2.72) will be useful here. Rearranging this approximation, we obtain the full width of the Lorentz component to be

$$\Delta\nu_L = 7.79 \Delta\nu - \sqrt{14.6 \Delta\nu_G^2 + 46.1 \Delta\nu^2}, \quad (3.7)$$

where $\Delta\nu$ and $\Delta\nu_G$ are the full widths at half-intensity of the Voigt and Doppler (Gaussian) profiles, respectively.

Excellent determinations of the Lorentz component can be made by using a series of $n\alpha$ and higher-order lines observed near a single frequency, like the series reported by Pedlar and Davies (1972) and others described in Sect. 2.2.6. Measuring the progressive changes in the pressure broadening from a series de-emphasizes the measurement errors of any single line profile and, therefore, helps define the pressure broadening accurately. Making the observations near a single frequency also ensures that the lines will originate from the same volume of ionized gas.

Values of electron density calculated from Lorentz broadening depend weakly on the temperature of the H II regions. For $5,000 \leq T_e \leq 10,000 \text{ K}$, the Lorentz broadening will vary by 10% for $100 \leq n \leq 200$ as can be seen in Sect. 2.2.3.

Table 3.2 Electron densities in H II regions and planetary nebulae

| Sources | From RRLs N_e (10^4 cm^{-3}) | From continuum $\langle N_e \rangle$ (10^3 cm^{-3}) | Ratio $N_e/\langle N_e \rangle$ |
|---------------|---|--|------------------------------------|
| M42 (Orion A) | 1.0 ± 0.3^a | 2.3^b | 4.3 |
| W3 | 1.5 ± 0.3^c | 3.7^b | 4.3 |
| DR21 | 4.3 ± 0.4^c | 1.3^b | 3.3 |
| NGC7027 | 6.7 ± 0.5^d | 57 ± 3^d | 1.2 |
| IC418 | 1.8 ± 0.4^e | 20^e | ≈ 1 |

^aSmirnov et al. (1984), ^bBerulis and Sorochenko (1983), ^cSmirnov (1985), ^dErshov and Berulis (1989), and ^eGaray et al. (1989)

Whatever method is used to determine N_e from H II regions, it is important to note that the results may be technique dependent. For example, densities determined from the free-free continuum spectra involve the emission measure; i.e., they derive from the square root of the path-averaged emission measure, the mean electron density $\langle N_e \rangle = (EM/L)^{1/2}$. In comparison, the electron densities determined from pressure broadening result from collisions with excited atoms and give the localized actual values – a very different kind of density weighting.

We close this discussion by listing electron densities in Table 3.2 for a few H II regions and planetary nebulae determined both from RRL pressure broadening and from the free-free emission spectrum.

The rightmost column of Table 3.2 gives the ratio of the local to the mean values of N_e determined by the two methods. The RRL values – determined by the ambient conditions of the emitting atoms – exceed the continuum values by factors of 3–4 for Orion, W3, and DR21. This ratio indicates considerable fluctuations in electron density along the lines of sight through these H II regions. As one might guess, this effect is absent in the two much smaller and denser planetary nebulae NGC7027 and IC418. Not only do we learn the electron densities from the RRL line widths, but also we learn about the density structure of the sources by comparison with the continuum observations.

Odegard (1985) developed a similar method of determining the electron density of H II regions from the line profiles. He selected a high-frequency RRL to determine the gas temperature and used this information to predict the Gaussian shape and I_L/I_C ratio at line center of a lower-frequency RRL that should have Stark broadening. To correct the line intensities for departures from LTE, he used published departure coefficients b_n s and population gradients β s (Salem and Brocklehurst, 1979). Comparison of the measured line intensity with the predicted one is then a measure of the intensity of a Voigt profile and, hence, of the electron density. An important requirement for this method is that both line observations involve the same volume of gas.

Odegard's measurements show good agreement with electron densities determined from the optical forbidden lines. However, the electron densities

determined directly from the line widths (see Table 3.2) are even more accurate than those he determined from the line intensities of the low-frequency lines.

3.1.3 Velocities of Turbulent Motion

In the millimeter and short centimeter wavelength ranges, the principal quantum numbers n of detectable RRLs are small. In this regime, Stark broadening is undetectable because of its steep dependence on n as seen in (2.60); i.e., the broadening $\propto n^{4.5}$. Here, however, RRL measurements can determine T_e with great accuracy and, accordingly, measure the line broadening due to turbulence within the telescope beam (microturbulence). Rewriting (2.26), we obtain an expression for the turbulence velocity V_t :

$$V_t = 0.6 \sqrt{(\Delta V_G^2) - 4.55 \cdot 10^{-2} T_e} \quad \text{km s}^{-1}, \quad (3.8)$$

from the velocity width ΔV_G of the Gaussian profile of a hydrogen RRL at half-intensity measured in km s^{-1} .

RRL observations (Berulis, Smirnov and Sorochenko, 1975; McGee and Newton, 1981; Wink, Wilson and Bieging, 1983) show that the microturbulence within H II regions lies in the range of $5 \rightarrow 25 \text{ km s}^{-1}$. Such values depend upon the angular resolution of the telescope with respect to the angular extent of these nebulae as illustrated by Fig. 2.4. The reason for this dependence of V_t upon the beamwidth is there are significant gradients of line-of-sight velocities normal to the beam as well as along the sight through the H II region as shown by Fig. 3.10.

The velocity broadening of the centimeter wave RRLs agrees well with the velocity dispersion of optical emission lines – for the Orion nebula, at least. Weedman (1966) integrated the radial velocities of H γ measured at several thousand points at $1''/3$ intervals within a $4' \times 4'$ region of the Orion nebula centered on the Trapezium (Wilson, Münch, Flather and Coffeen, 1959). Summing the radial velocities of the optical lines and convolving the result with the average broadening of each of the components produced a composite line profile that agreed well with the profile observed for the radio H109 α line made with a beam of $6'.5$ (Höglund and Mezger, 1965). The implication is that the RRLs give a faithful representation of gas dynamics internal to the nebula; i.e., of the microturbulence within the radio beam along that particular sight line.

From a survey of 82 H II regions observed in the H109 α line (Reifenstein et al., 1970), the RMS turbulence⁴ $\langle V_t^2 \rangle^{1/2} = 13.4 \text{ km s}^{-1}$, so that $V_t =$

⁴ Frequently used in literature, the root-mean-square (RMS) velocity of turbulent motion $\langle V_t^2 \rangle^{1/2}$ is related to V_t by the expression $\langle V_t^2 \rangle^{1/2} = 1.22 V_t$.

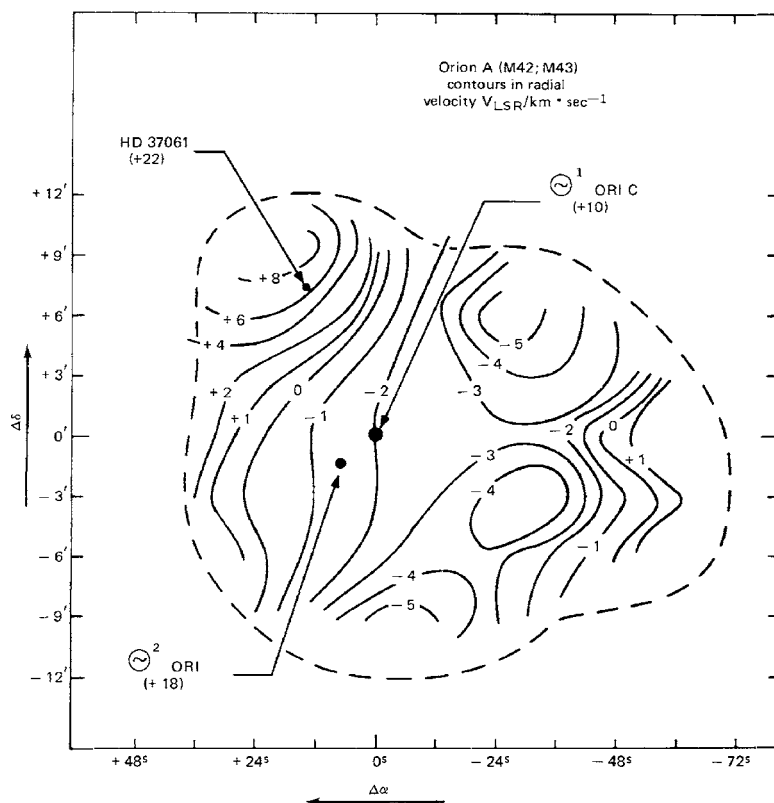


Fig. 3.10 The gradients of the radial velocity of the H109 α RRL in km s^{-1} across Orion A (M42 and M43) as a function of offsets in RA and declination from the exciting star Θ^1 Ori. The telescope beam is $6.5'$ at half-intensity. From Mezger and Ellis (1968)

11 km s^{-1} for the average H II region observed with the $6.5'$ beam of the NRAO 140-ft. radio telescope. These values were calculated by assuming LTE values of the electron temperature T_e .

3.2 Ionized Hydrogen and Helium in the Galaxy

3.2.1 Distribution of H II Regions

3.2.1.1 Astronomical Doppler Shifts

The observed frequencies of RRL (see Fig. 3.1) usually differ somewhat from the calculated values because of Doppler shifts due to the motion of the telescope with respect to the source. The Earth rotates on its axis, the Earth

Table 3.3 Velocity components affecting observed frequencies

| Component | Approximate V_{max} (km s^{-1}) |
|---|---|
| Source \rightarrow LSR | (Source dependent) |
| LSR \rightarrow solar system barycenter | 20 |
| Solar system barycenter \rightarrow Earth–moon barycenter | 30 |
| Earth–moon barycenter \rightarrow Earth center | 0.1 |
| Earth center \rightarrow telescope | 0.5 |
| Planetary perturbations upon Earth orbit | 0.013 |

revolves around the Sun, the Sun moves within the Galaxy, and the Galaxy moves with respect to the Local Group of galaxies of which it is a member.

For convenience, astronomers adopt a kinematic reference frame in the Galaxy toward which the Sun is headed. This frame is based upon the average velocity of stellar spectral types *A–G* in the vicinity of the Sun without regard for luminosity class. It assumes the Sun to be moving at 20.0 km s^{-1} toward 18^m RA and $30^\circ \delta$ 1900.0 and is known as the “local standard of rest” (LSR). Table 3.3 lists the magnitudes of the velocity components between the telescope and the LSR. Ball (Meeks, 1976) gives a FORTRAN subroutine for calculating the velocity of a telescope with respect to the LSR.

The LSR itself moves with respect to the Galactic center. Observations (Kinman, 1959) of globular clusters show the Sun to move at $167 \pm 30 \text{ km s}^{-1}$ toward a Galactic latitude and longitude of $(\ell, b) = (90^\circ, 0^\circ)$. Similar observations (Humason and Wahlquist, 1955) of galaxies in the Local Group show the Sun to move at $291 \pm 32 \text{ km s}^{-1}$ toward $(106^\circ, -6^\circ)$. The difference between these velocity vectors is not understood. Therefore, the assumed value is taken to be 250 km s^{-1} toward $(90^\circ, 0^\circ)$ about the “Galactic standard of rest.”

Most importantly, the rotational velocity of the Galaxy – and of most other spiral galaxies – changes as a function of Galactocentric radius as shown by Fig. 3.11. This “differential rotation” creates a variation of radial velocity with distance along a line of sight through the Galactic plane. In principal, this velocity gradient should make it possible to locate an H II region along the sight line based upon the radial velocity observed for its RRLs.

There may be a distance ambiguity, however. With the differential rotation, it is possible to relate the radial velocities V_{LSR} observed for RRLs to a distance from the center of the Galaxy. In contrast, it is not generally possible to locate the H II regions *uniquely* along sight lines passing through Galactic longitudes of $|\ell| < 90^\circ$ on the basis of the radial velocities alone. There is no such distance ambiguity for sources located at $|\ell| \geq 90^\circ$.

Consider the geometry shown in Fig. 3.12 for a sight line at $\ell < 90^\circ$. Except for the locus of the subcentral points, along such sight lines are two possible locations that would have the same radial velocity with respect to

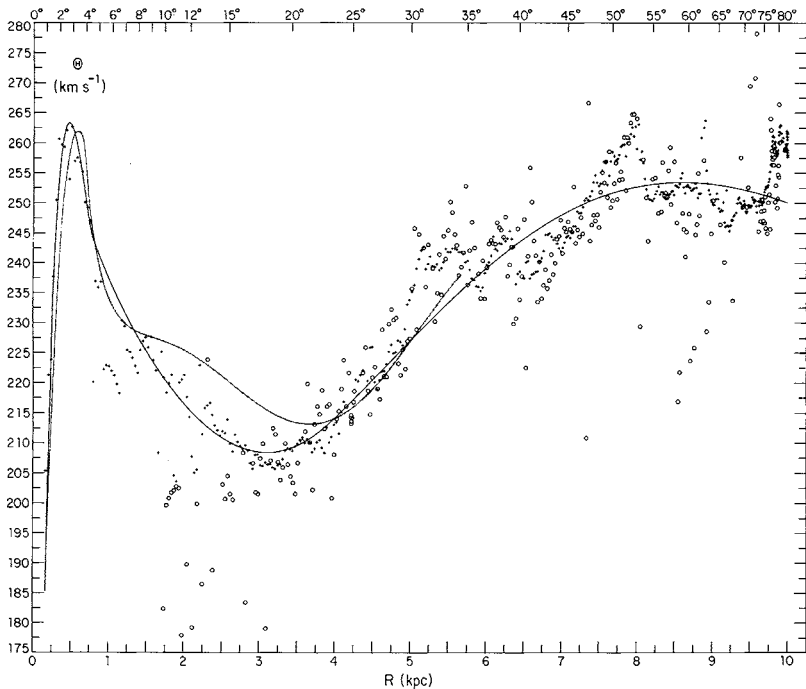


Fig. 3.11 The variation of the linear velocity of differential Galactic rotation as a function of Galactic radius. In this plot, the Sun lies at 10 kpc. *Small plus symbols* are from $\lambda = 21$ cm HI emission and *open circles* are from CO emission. The *black line* is the fit to all observations. The *shorter, lighter line* is an earlier rotation curve proposed by Simonson and Mader (1973). From Burton and Gordon (1978)

the observer: a near and a far point. The Galactocentric radius would be the same for each location but the distance from the observer could be either of the two. Such position ambiguities can be distinguished only by additional information such as optical extinction, implied physical size, elevation above the Galactic disk, etc.

3.2.1.2 Physical Location of H II Regions in the Galaxy

Nonetheless, surveys of H II regions in hydrogen RRLs were carried out in both northern and southern hemispheres to determine the distribution of H II regions. These first surveys were initiated almost simultaneously in the northern sky in the range of Galactic longitude $\ell = 348^\circ \rightarrow 360^\circ \rightarrow 209^\circ$ (Reifenstein et al., 1970) and in the southern sky for $\ell = 189^\circ \rightarrow 36^\circ \rightarrow 49.5^\circ$ (Wilson et al., 1970). Both surveys searched for the H109 α line toward known sources of $\lambda = 11$ cm continuum radiation located close to the Galactic equator, i.e., at $|b| \leq 1^\circ$. Subsequently, the line was detected in 82 and 130

Galaxy, where $R < R_o$ and $270^\circ \leq \ell \leq 90^\circ$, so that their distances from the Sun could not be determined uniquely.

Additional surveys overcame the distance problem to some extent. Downes et al. (1980) observed the H110 α RRL (4,874 MHz) in 171 H II regions in the range $\ell = 357^\circ \rightarrow 360^\circ \rightarrow 60^\circ$ simultaneously with observations of the formaldehyde (H₂CO) absorption line at 4,830 MHz. Of these, the H₂CO absorption resolved the distance ambiguity of 56 H II regions in the inner Galaxy on the basis of the amount of extinction along the lines of sight.

The most detailed survey of southern sky was carried out by Caswell and Haynes (1987) in the H109 α and H110 α lines. They detected these lines in 316 sources located close to Galactic plane within Galactic latitudes $\ell = 233^\circ \rightarrow 360^\circ \rightarrow 13^\circ$. Here also, the 4,830-MHz formaldehyde line was recorded simultaneously and resolved the twofold distance ambiguity problem for 146 sources.

By enabling measurements of the distances of H II regions in the Galaxy including those located beyond the Galactic center, RRLs have provided a tool to model the large-scale structure and to locate the spiral arms. Figure 3.13 shows a composite map of H II regions plotted onto the Galactic plane. While the map was originally proposed by Georgelin and Georgelin (1976), Taylor and Cordes (1993) added observations from subsequent observations to improve the model. They also adopted the modern value of 8.5 kpc for the Galactocentric distance of the Sun.

In the figure, the spiral structure of our Galaxy is distinctly seen even though details of the arm continuity are missing. Data on the far side of the Galactic center are sparse but still adequate to suggest an asymmetry in the structure of the arms as indicated from the H II regions. Being heated by newly formed stars, the H II regions mark the location of young Galactic material, sometimes called “Population 0” material⁵ as an extension to an older stellar classification scheme introduced by Walter Baade in the 1940s. Additional observations will fill in the gaps but probably not change the conclusion that our Galaxy has asymmetrical spiral arms.

The most detailed survey of H II regions in the northern sky was made by Lockman (1989). This study detected 462 sources in the H85 α , H87 α , and H88 α lines and some in the H100 α , H101 α , H125 α , and H127 α lines as well. The study did not resolve the distance ambiguity from the Sun to those H II regions in the inner Galaxy; the observations were considered in terms of the Galactocentric distance instead.

Combining this survey with the southern survey (Caswell and Haynes, 1987) shows the distribution of 750 H II regions as a function of Galactocentric

⁵ This newer term refers to the extremely young component (O stars and H II regions) of the interstellar gas in distinction to the older term, Population I, that includes disk stars of many ages which delineate the spiral arms of a galaxy. The term “Population II” refers to Galactic stars not associated with spiral structure, in short, to everything else. Today’s revised terminology also includes subclassifications like “extreme population I,” “moderate Population I,” etc.

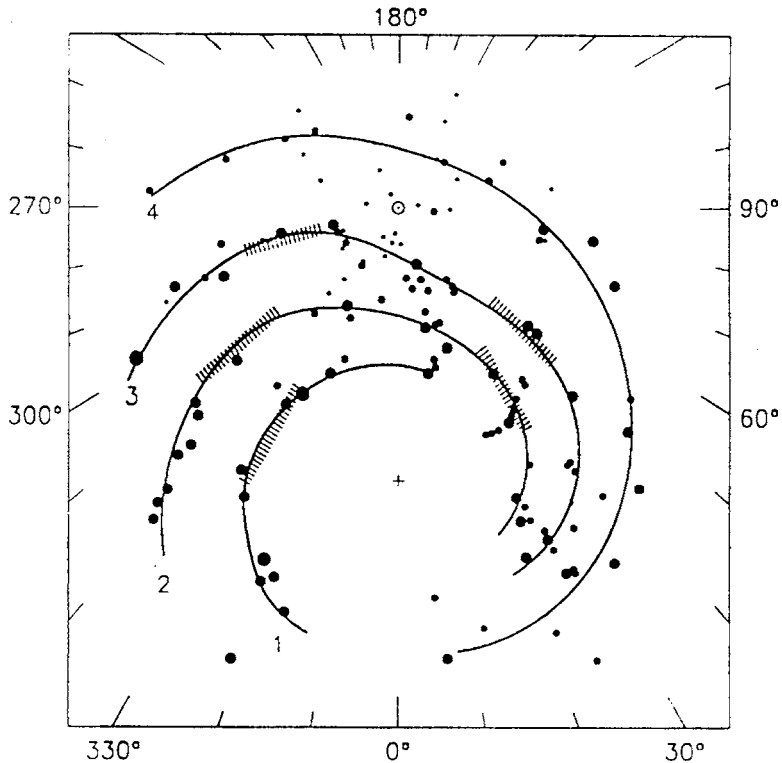


Fig. 3.13 A spiral model of the Galaxy originally proposed by Georgelin and Georgelin (1976) but now including the results of additional RRL surveys of HII regions. *Filled circles* represent the HII regions and *shaded areas* are directions of intensity maxima in the radio continuum and in neutral hydrogen. In each arm, the *lines* are cubic splines fitted to the HII region positions. A *number* marks each arm, the *cross* indicates the Galactic center, and the *open circle with dot* indicates the position of the Sun. From Taylor and Cordes (1993)

distance for our Galaxy. These distances result from the differential rotation curve proposed by Burton and Gordon (1978). The Sun is assumed to lie 8.5 kpc from the center of the Galaxy. The results shown in Fig. 3.14 confirm solidly the double-peaked, apparently toroidal distribution in the range $4 < R < 6$ kpc discovered 20 years earlier from many times fewer observations (Mezger, 1970). According to Hodge and Kennicutt (1983), with respect to surface density, the radial distributions of HII regions in spiral galaxies can be classified as (1) continuously decreasing from the center outward toward the outer boundary of the galaxy, (2) oscillating with increasing Galactocentric radius and finally decreasing toward the outer boundary, and (3) increasing from the center to a maximum and then decreasing toward the outer boundary. In this scheme, the Galaxy seems to fall into the third class.

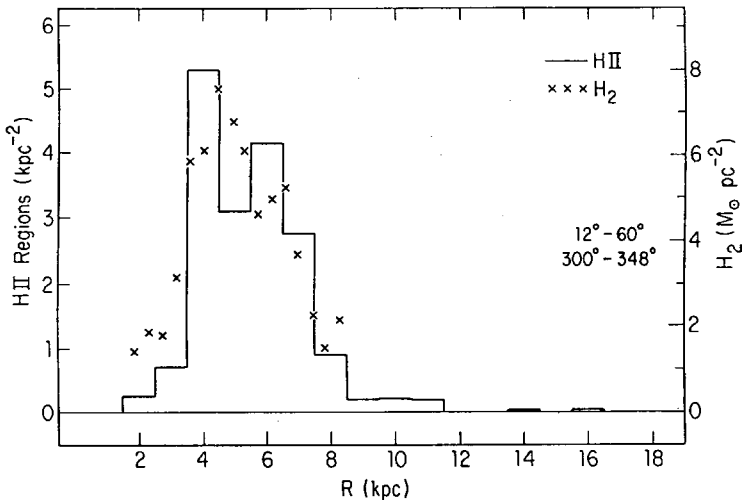


Fig. 3.14 Surface density of H II regions in the Galaxy plotted against Galactocentric distance for $R > 1.5$ kpc and $\ell = 12^\circ \rightarrow 60^\circ$ and $300^\circ \rightarrow 348^\circ$. The Sun is assumed to lie 8.5 kpc from the Galactic center and the bins are 1-kpc wide. The crosses mark surface densities of H_2 deduced from observations of CO emission. From Lockman (1990)

Figure 3.14 shows the radial distribution of the surface density of H II regions to follow closely that of the molecular hydrogen deduced from observations of carbon monoxide. This is to be expected because both constituents are Population 0 or I material.

For comparison, Fig. 3.15 shows the distribution of nucleons in all forms as a function of Galactocentric radius (Gordon and Burton, 1976). Subject to differences in the Sun–Center distance, the agreement of the H II distribution shown in Fig. 3.14 with the distribution of the gaseous constituents in the Galaxy is excellent. Note that the ratio of $\sigma(H_2)/\sigma(HI)$ decreases with increasing Galactocentric radius, showing that the density of gas capable of star formation decreases outward from the Galactic center. Subsequent observations (Lockman, Pisano and Howard, 1996) confirmed that the H II regions – both diffuse and compact – follow the Galactocentric radial distribution of molecular gas rather than of the H I gas. Unlike the H_2 gas, H II regions *require* newly formed stars and, therefore, indicate the presence of star-forming activity. Thus, knowing the distribution of H II regions, determined by RRLs, is essential to understanding large-scale star formation in the Galaxy.

3.2.2 Low-Density Ionized Hydrogen

RRLs have been detected not only from well-defined H II regions, but also from directions in the Galactic plane free of discrete sources. Gottesman and

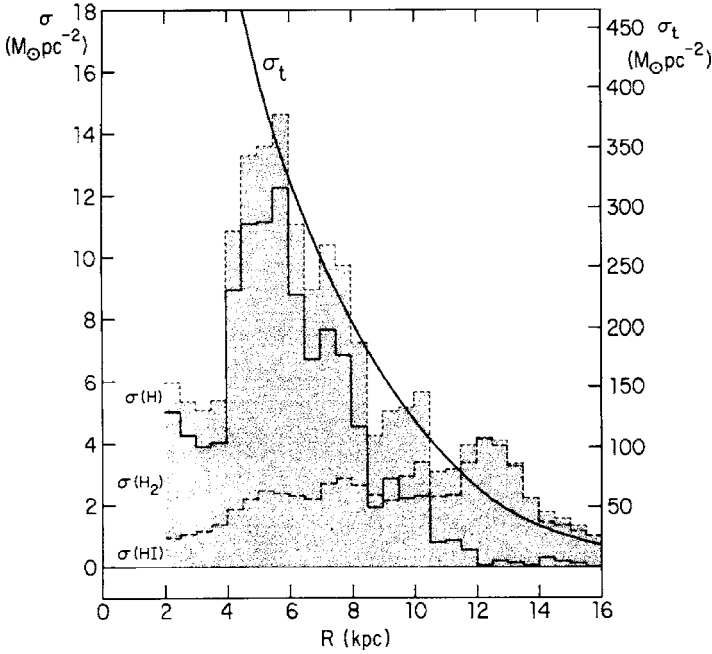


Fig. 3.15 Surface density of hydrogen plotted against Galactocentric radius for $R > 1.5$ kpc for the Galaxy. *Lower broken line*: HI. *Solid histogram*: H_2 estimated from CO observations. *Dashed histogram*: hydrogen nucleons in all forms. *Curve marked σ_t* : total surface density predicted by dynamic models of star density (Innanen, 1973). *Note*: Here, the Sun is placed at 10 kpc from the Galactic center. From Gordon and Burton (1976)

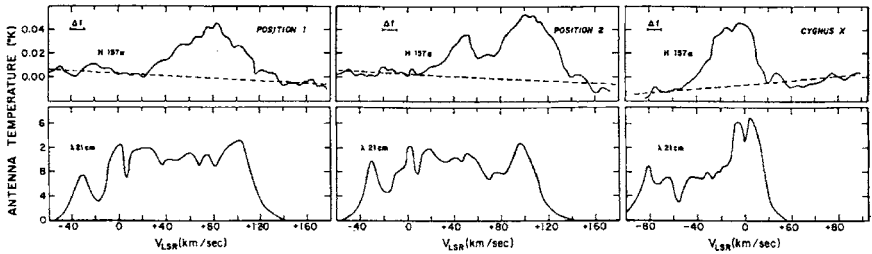


Fig. 3.16 *Top*: H157 α emission observed from three directions in the Galactic plane free of discrete sources. *Bottom*: corresponding $\lambda = 21$ cm of HI in the same directions. From Gottesman and Gordon (1970)

Gordon (1970) detected rather weak ($T_L < 0.05$ K) but distinctly obvious H157 α emission from the directions $\ell = 23^\circ 92'$, $\ell = 25^\circ 07'$, and $\ell = 80^\circ 09'$ and $b < 0.9^\circ$ free from radio sources with high surface brightness. Jackson and Kerr (1971) made similar detections almost simultaneously. Figure 3.16 shows the line spectra juxtaposed with profiles of $\lambda = 21$ cm emission from HI in nearby directions.

The qualitative agreement of the H157 α line radial velocities with those of $\lambda = 21$ cm line, the much greater line widths compared with RRLs from H II regions, and the weak intensities suggested that detected lines originated in diffuse, ionized interstellar gas. Such a possibility corresponded to the steady-state two-component model of a partially ionized ISM generally accepted at that time. According to this model, interstellar gas (excluding discrete H II regions) consisted of dense, cold, $T_e = 40 \rightarrow 60$ K clouds in pressure equilibrium with an ambient hot, $T_e = 1,000 \rightarrow 10,000$ K, rarefied, intercloud medium. It was believed that low energy cosmic rays were responsible for the ionization and heating of both components (Pikelner, 1967; Field, Goldsmith and Habing, 1969; Hjellming, Gordon and Gordon, 1969).

It was impossible to calculate the electron temperature of the diffuse component from the RRL observations. The associated free-free continuum was not known well enough to use (2.124). Furthermore, the extent of departures from LTE – if any – was also unknown. All that could be safely determined from the initial observations was the quantity $\int_{\text{path}} N_e N_i T_e^{-0.35} dl$. Accordingly, Gottesman and Gordon (1970) calculated path-averaged electron densities for three arbitrary fractions for the free-free component of the observed continuum emission: 100, 10, and 1%. If the free-free continuum accounted for 10% of the observed continuum along the sight lines, they suggested that the value of approximately 1,000 K might be an upper limit for the electron temperature of the diffuse gas, which would imply a limit of $\langle N_e \rangle \approx 0.3 \text{ cm}^{-3}$.

Not only subsequent observations (Gordon and Gottesman, 1971) of the H197 β lines from the same positions verified the reality of the H157 α line detected earlier from the diffuse gas, but also the ratios of the radiated power in the line favored a path-averaged $1,000 < \langle T_e \rangle < 10,000$ K. Unfortunately, the signal-to-noise ratio of the β line observations was insufficient to restrict the temperature further. This temperature range implied a corresponding range of the emission measure of $280 \rightarrow 5,000 \text{ cm}^{-6} \text{ pc}$ over a probable path length of 14 kpc. Although considerable fluctuations in density along the paths could be expected, these values implied $0.15 < \langle N_e \rangle < 0.6 \text{ cm}^{-3}$, respectively, for RMS densities.

Cesarsky and Cesarsky (1971) examined other constraints on the continuum emission, noted that the unknown filling factor of the cold interstellar clouds prevented direct application of (2.124) as used to interpret RRLs from H II regions, and concluded that two models were possible with respect to the theoretical models of the ISM extant at that time. The diffuse RRL emission could come either from intercloud gas of approximately 800 K or from embedded clouds of $40 < \langle T_e \rangle < 60$ K.

To pursue their suggestion observationally, Cesarsky and Cesarsky (1973) noted that T_e has a different exponent for line and for continuum emission (see (2.95) and (2.116)). They observed RRLs in the direction of the supernova remnant 3C391 and compared the line intensity with the turnover frequency observed for the source. Their results suggested that the average electron temperature for the diffuse gas must be less than 400 K. However,

the weakness of the line intensity made this a difficult experiment to perform with high accuracy. Furthermore, if the clouds along the sight line are in pressure equilibrium with hot intercloud gas, line emission from the cold clouds would dominate. Without knowing the distribution of conditions along this line of sight, one found it difficult to know what component the temperature referred to.

These models of a diffuse, partly ionized interstellar gas were consistent with observations of pulsars, discovered (Hewish, Bell, Pilkington, Scott and Collins, 1968) a short time before the detection of RRL emission from the diffuse ISM. All the pulsars exhibited a delay in the time of arrival of the pulses that varied inversely with frequency. This effect had a natural explanation of propagation dispersion caused by the ionized component (electrons) of the ISM. Fitting the frequency dependence of the delay determines the integrated electron density or “dispersion measure” along the path to the pulsar, $DM \equiv \int_{\text{path}} N_e dl$. The observed values of DM could not be explained by the ionization of ISM constituents with ionization potentials lower than H I; the resulting DM would be too small. The only possible source for the DMs was the ionization of the principal constituent of the ISM along the lines of sight – hydrogen.

The likely source of this ionization was soft cosmic rays with an energy ≈ 2 MeV. The UV radiation from embedded hot stars could not permeate the ISM widely enough; in fact, most H II regions are considered to be ionization bounded rather than density bounded for this very reason.⁶ The rate of ionization necessary to account for observed pulsar dispersion measure was $\zeta_H = 2.5 \pm 0.5 \times 10^{-15} \text{ s}^{-1}$ (Hjellming et al., 1969). The intensity of cosmic rays measured on Earth could only produce $\zeta_H = 6.8 \times 10^{-18} \text{ s}^{-1}$ but these cosmic rays are strongly attenuated by the magnetic field of the Sun and solar wind, and could be significantly weaker than the ambient flux of cosmic rays in the ISM. Calculations of the possible intensity of subcosmic rays in the ISM, based on the energy and frequency of supernova explosions in the Galaxy, gave the value $\zeta_H = 1.2 \times 10^{-15} \text{ s}^{-1}$ (Spitzer and Tomasko, 1968). This value was close to what was required to explain the observed dispersion measures.

The first radio astronomical measurement of ζ_H was even more encouraging. It agreed well with the theoretical estimate based upon supernovae and with the empirical value needed to explain the pulsar dispersion measures. Comparison of absorption in the $\lambda = 21$ cm line of H I with free-free absorption toward three sources of nonthermal emission – 3C10, 3C123, and 3C340 – gave the value of $\zeta_H = 2.0 \times 10^{-15} \text{ s}^{-1}$ (Hughes, Thompson and

⁶ The term “Strömgren sphere” (Strömgren, 1939) describes a density-bounded ionization zone around a hot star in a neutral medium. The sphere results from the balance between the UV flux emitted by the star and the number of ionizations possible in the ambient medium. This is the physical principle underlying the luminous, discrete H II regions called *gaseous nebulae*. The concept stems from the recognition that ionizing UV photons cannot travel very far in the ISM.

Colvin, 1971). This result was exactly what was needed, and the existence of a diffuse component to the ISM with ionization sufficient to account for the diffuse RRLs and the pulsar dispersion appeared to be justified.

Unfortunately, subsequent observations increasingly cast doubts on the cold gas model. New, more accurate observations gave $\zeta_H \leq 2 \times 10^{-16} \text{ s}^{-1}$, one order of magnitude below the value needed to explain the DM measured by pulsars. Additional observations confirmed this upper limit (Shaver, 1976a; Sorochenko and Smirnov, 1987) (see also Sect. 3.3). As more and more data accumulated, observations of hydrogen RRLs toward directions free from discrete H II regions indicated conditions similar to the RRLs observed from normal H II regions.

An attempt was made to determine the temperature of the diffuse gas by measuring its scale height above the Galactic plane (Gordon, Brown and Gottesman, 1972). These observations implied a scale height less than 70 pc, a crudely determined value consistent with a kinetic temperature $\approx 4,000 \text{ K}$ or less.

Contrary evidence to the cold gas model lies in the measurements of the physical conditions of the medium in which the diffuse RRLs were formed. Jackson and Kerr (1975) determined electron temperatures from observations of the H110 α line in nine directions in the Galactic plane from $\ell = 359^\circ \rightarrow 0^\circ \rightarrow 80^\circ$ that avoided discrete H II regions. The result was $\langle T_e \rangle = 4,400 \pm 600 \text{ K}$.

The second contrary evidence came from measurements of the electron density along similar sight lines. Analysis of H110 α line intensities gave $\langle N_e \rangle = 5 \rightarrow 10 \text{ cm}^{-3}$ (Shaver, 1976b). Similar values of temperature and density came from the H166 α lines. Observations in 13 directions along the Galactic plane from $\ell = -1^\circ \rightarrow 47^\circ.5$ gave $\langle T_e \rangle = 6,000 \pm 1,000 \text{ K}$ and $\langle N_e \rangle = 2 \rightarrow 10 \text{ cm}^{-3}$ (Matthews, Pedlar and Davies, 1973). The beamwidth for this survey was $\approx 30'$ where mainly extended sources contribute to the observed emission. Taken together, these results pointed to physical conditions expected in extended, low-density H II regions rather than the cold, partially ionized interstellar gas originally suggested by Cesarsky and Cesarsky (1971) for the source of the diffuse RRL emission.

The observing geometry pointed to extended H II regions. Jackson and Kerr (1975) noted that the line profile changed greatly between two directions separated by only $6'$. Significant changes in the line profiles over small changes in direction were also noticed by Lockman (1980) in a survey of H166 α RRLs. Figure 3.17 illustrates these changes with spectra from that survey taken along the Galactic plane over the range $\ell = 33^\circ.0 \rightarrow 37^\circ.0$.

The location of the diffuse RRL emission in the Galaxy was also a clue to its origin. Gordon and Cato (1972) observed the H157 α line in nine directions along the Galactic plane from $\ell = 9^\circ.4 \rightarrow 80^\circ.6$ in directions free from discrete H II regions. Most of the line emission arose from the range of Galactocentric distance of $R = 3 \rightarrow 9 \text{ kpc}$ where the H II regions are located, as shown in Fig. 3.14. Hart and Pedlar (1976) also noted this spatial correlation from their

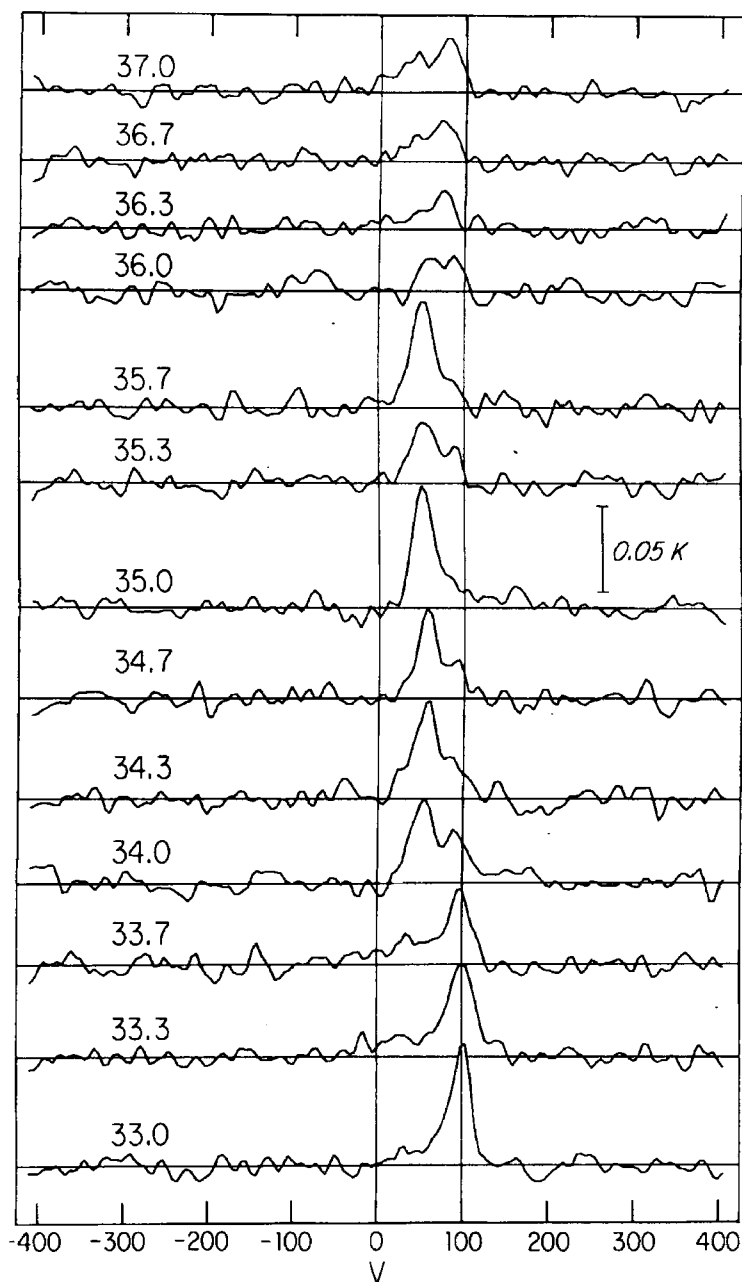


Fig. 3.17 A sequence of H166 α line profiles taken along the Galactic plane at longitudes marked in each spectrum. The *vertical scale* is antenna temperature in K at the NRAO 140-ft. telescope and the *abscissae* are velocity in km s^{-1} with respect to the LSR. From Lockman (1980)

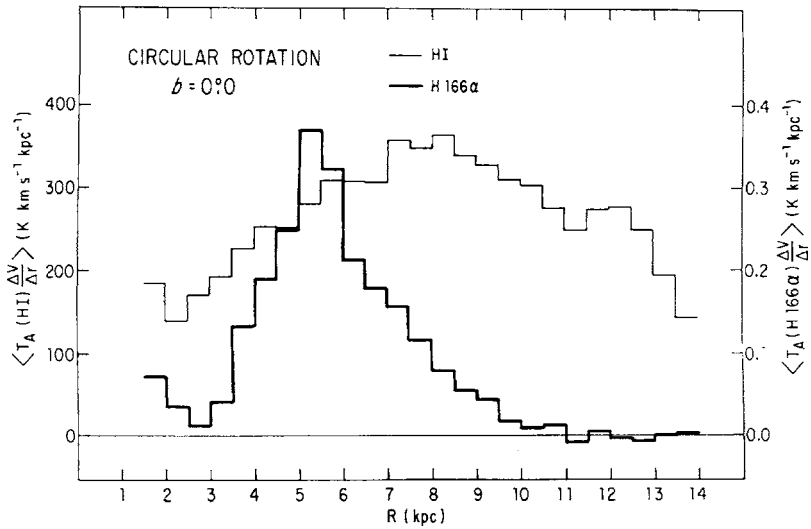


Fig. 3.18 Comparison of H166 α RRLs from Galactic diffuse gas with HI emission at $\lambda = 21$ cm. *Abcissa* is Galactocentric distance with $R_0 = 10$ kpc. From Lockman (1976)

H166 α observations. Note that only these observations do indicate spatial correlation between the diffuse gas and the discrete H II regions but these spatial distributions differ greatly from that of HI emission as can be seen by comparing Figs. 3.15 and 3.18.

The difference between the Galactocentric distribution of the diffuse RRL emission and the HI gas is especially clear in Fig. 3.18. The data result from samplings of both lines at intervals of $\Delta\ell \approx 1^\circ$ over a range of $358^\circ \leq \ell \leq 50^\circ$. Additional observations of the H166 α emission were made at $\ell \approx 44^\circ$ and $\ell = 100^\circ \rightarrow 125^\circ$. While the intensity of the $\lambda = 21$ cm line is nearly constant over the range $R = 5 \rightarrow 12$ kpc, the intensity of the H166 α line increases sharply in the range $R = 3 \rightarrow 5$ kpc and rapidly decreases in the range $R > 5$ kpc. Thus, it appears certain that the RRLs from the diffuse gas are not connected with the HI component of the ISM (Lockman, 1980).

All of these observational results – electron temperature, electron density, location – suggested that the diffuse RRL emission arises in the vicinity of H II regions rather than in a cold, partially ionized component of the general ISM. The remaining question involved the origin of the low-density, ionized hydrogen with a temperature of a few thousands of Kelvins.

Examination of the location of O stars with respect to H II regions provided a surprising answer. Mezger and Smith (1975) found that only 20% of O stars were located within H II regions.⁷ Mezger (1978) further suggested that the remaining 80% of the O stars formed extended regions of fully ionized,

⁷ For this study, H II regions or radio H II regions were defined to be those sources of thermal radio emission that provided antenna temperatures at 5 GHz of $T_A \geq 1$ K at the

low-density gas that he called the “extended, low-density” (ELD) component of the ISM. Because these enormous Strömgren spheres overlapped each other, the ELD could occupy a large volume of the ISM. He estimated that the ELD would have typical values of 7,000 K and 3 cm^{-3} for the electron temperature and density, respectively.

New low-frequency observations provided confirmation. Being more sensitive to extended emission of low surface brightness, a survey (Anantharamaiah, 1986) of the Galaxy in the H272 α (325 MHz) line showed that the main part of the diffuse emission indeed comes from the low-density, outer envelopes of normal H II regions. The observations indicated the electron densities to be $1 \rightarrow 10\text{ cm}^{-3}$, the electron temperatures to be 3,000 \rightarrow 8,000 K, and the emission measures to be $500 \rightarrow 3,000\text{ cm}^{-6}\text{ pc}$. The sizes of these envelopes were impressive: 30 \rightarrow 300 pc. In the Galactic plane, a line of sight at $\ell \leq 40^\circ$ will intersect at least one of these envelopes.

Figure 3.19 illustrates the observational results. A longitude–velocity diagram shows contours of the H166 α emission, an RRL with a frequency near

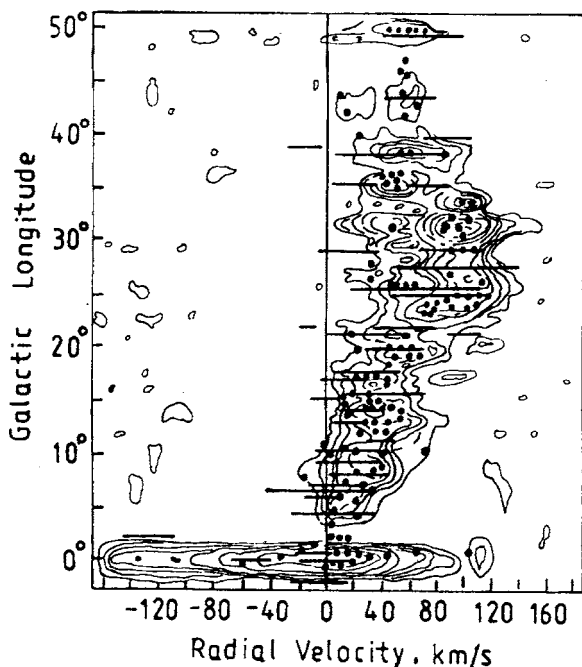


Fig. 3.19 A longitude–velocity diagram of the H272 α (*horizontal lines*), H166 α (*contours*), and H110 α (*points*) RRLs observed for our Galaxy. The length of the *horizontal lines* marks the full width at half-intensity of the H272 α line profile. From Anantharamaiah (1986)

NRAO 43-m and the CSIRO 65-m telescopes. This threshold corresponds to a minimum emission measure of $\text{EM}_{\text{min}} \approx 10^4\text{ cm}^{-6}\text{ pc}$.

the H157 α lines in which the diffuse, ionized gas was originally detected (Gottesman and Gordon, 1970). The diagram also shows the full widths at half-intensity of the low-frequency H272 α lines that are more sensitive to low emission measure gas. Superimposed upon these are points marking detections of the H110 α lines from discrete H II regions. Note that everything agrees rather well. The locations of the discrete H II regions are generally those of the ionized, diffuse gas. It is obvious that the halos of those H II regions *are* the diffuse gas, just as Mezger (1978) had suggested with his ELD.

To investigate the phenomenon even more thoroughly, Heiles et al. (1996) observed the H165 α , H167 α and, to a lesser extent, the H157 α and H158 α lines at 583 positions along the Galactic plane within the range $\ell = 0^\circ \rightarrow 60^\circ$. The angular resolution was 36'. RRLs were detected in 418 positions and were easily explained as emission from the extended low-density warm ionized medium (ELDWIM), a concept introduced by Petuchowski and Bennet (1993) that extended the older ELD.

Physical conditions derived from the new observations agreed well with those derived from the older ones. The average electron temperature was about 7,000 K when maser amplification was included in the analysis of the T_L/T_C ratios. The ELDWIM regions appeared to be located in the Galactic arms, occupying approximately 1% of the volume and containing an average electron density of approximately 5 cm^{-3} .

Taken together, the observations leave little doubt that the RRL emission observed along the Galactic plane outside of distinct H II regions is also emitted by more spatially extended, low-density H II gas. The situation is evidently somewhat different from the simpler situation originally modeled by Strömgren (1939) for ionization regions that “should be limited to sharply distinct bounded regions in space surrounding O-type stars.” The clumpy, irregular nature of the ISM evidently allows leaks of UV photons from the immediate vicinity of the geometrically distinct H II regions to form extended, low brightness, ionized regions with indistinct borders. These regions can have diameters of tens if not hundreds of pc, average electron densities of $1 \rightarrow 10 \text{ cm}^{-3}$ and emission measures of $<10^4 \text{ cm}^{-6} \text{ pc}$.

Figure 3.20 describes the probable situation. The figure shows the locations of bright H II regions and RRL emission from the ionized diffuse gas in terms of the sizes of the observing beams. The spatial relationship between the two kinds of objects seems obvious. We expect that the increasing sensitivity of radio telescopes will facilitate the detections of additional RRL emission in the inner Galaxy that will close the spatial gaps between the discrete and diffuse sources.

What is the origin of these H II regions with low densities and surface brightness? There may not be a single mechanism. Some may have evolved by the expansion of compact H II regions (Anantharamaiah, 1986; Lockman et al., 1996), leading to tenuous H II regions with lowered electron densities and emission measures. Some of them may be formed in the evolution of

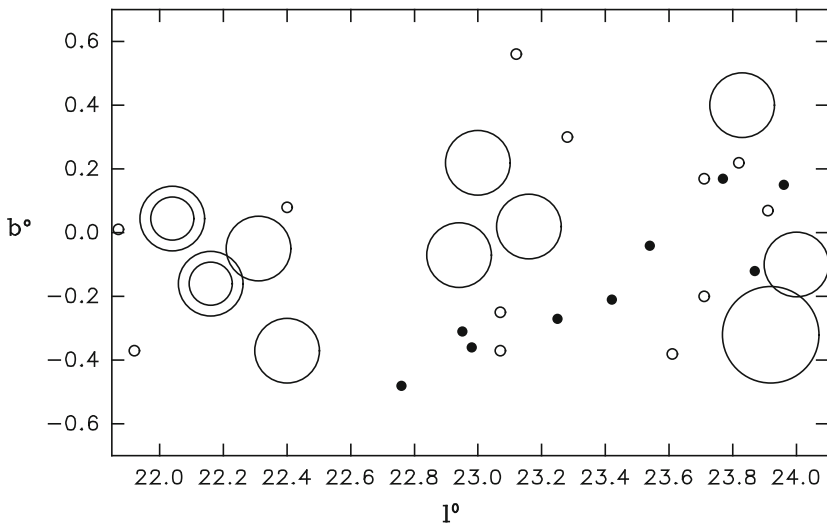


Fig. 3.20 Ionized gas observed in the inner Galaxy in a region bounded by the indicated range of b and l . The *filled and open small circles* mark discrete H II regions observed in a range of RRLs from H85 α to H127 α by Downes et al. (1980) and Lockman (1989), respectively. The sizes of the *circles* correspond to the beams used in the observations. The *open middle-sized circles* indicate extended “diffuse” H II regions with diameters $\approx 12'$ observed by Lockman et al. (1996). The *double circles* mark the location of two H II regions observed in the same directions but with different radial velocities. The *large circle* marks the direction and beam size in which RRLs from the diffuse medium were observed for the first time (Gottesman and Gordon, 1970)

H II regions from a giant molecular cloud (GMC) in a manner illustrated by Fig. 3.21. Massive stars spend only part of their lives in the parent clouds, about 20% or 10^6 years. There, these hot stars that form as young stellar objects disperse their cocoons with stellar winds as the star formation progresses through the giant cloud, increasingly leaving their UV emission available to ionize large regions of the ISM. In addition, isolated compact H II regions could disperse their H I mantles, so that they become density bounded rather than ionization bounded locally, also allowing their UV photons access to a much larger volume of the ISM (Churchwell, 1975). Finally, soft cosmic rays from supernovae surely ionize regions of the ISM to some extent as suggested earlier by many astronomers (Pikelner, 1967; Spitzer and Tomasko, 1968; Field et al., 1969; Hjellming et al., 1969).

Following Strömgren (1939), we examine the balance of the UV ionizing flux density to the recombinations in the ambient, lower density interstellar gas. The radius of a Strömgren sphere can be enormous in a rarified medium. For ionization-bounded media involving only hydrogen, this radius in meters is

$$R = \left(\frac{3N_L}{4\pi\alpha N_e N_{H^+}} \right)^{1/3}, \quad (3.9)$$

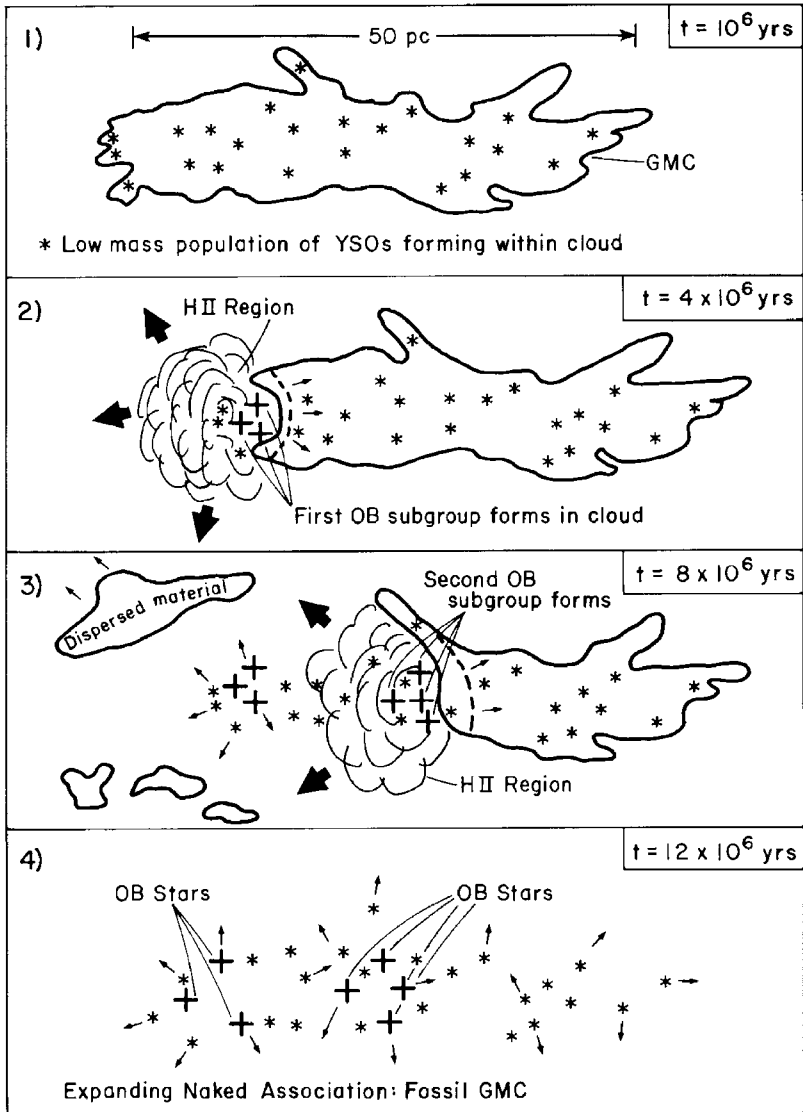


Fig. 3.21 The panels from *top to bottom* depict the formation and unveiling of an OB association from a giant molecular cloud. As the region refills with cold ISM, this process can repeat, as has happened in the Orion cloud. From Lada (1987)

where the total number of Lyman continuum photons from O stars per second $N_L \approx 10^{49} \text{ s}^{-1}$ (Spitzer, 1978) and the hydrogen recombination coefficient to all but the first level $\alpha = 2.06 \times 10^{-11} T_e^{-1/2} \phi_2$. The factor ϕ_2 is weakly dependent on temperature and is tabulated by Spitzer (1978). The electron (N_e) and ion (N_{H^+}) densities are in units of cm^{-3} . Equating these densities

allows the transformation of (3.9) into a form that gives the H II mass within the sphere:

$$\frac{M_{HII}}{[M_{\odot}]} = \frac{8.4 \times 10^{-58} N_L}{\alpha N_e}. \quad (3.10)$$

This equation demonstrates that an Ostar will ionize a larger mass in a tenuous medium than that in a dense one. Physically, the recombination rate of the ions $\propto N_e^2$. Accordingly, the lifetime of the H^+ ions is higher in lower density H II regions. For the same reason, a Lyman photon flux density ionizes more hydrogen atoms in a lower density gas. Consequently, the amount of ionized mass in a Strömgren sphere varies inversely with N_e .

Numerical examples illustrate the situation. If $T_e = 7,000$ K and $\phi_2 = 1.41$, the Strömgren sphere of an ionization-bounded H II region with $N_e = 10^4 \text{ cm}^{-3}$ will have a diameter of 0.27 pc and a mass of $2.4 M_{\odot}$ according to (3.9) and (3.10). Yet, with a lower density of $N_e = 1 \text{ cm}^{-3}$, the sphere will have a diameter of 123 pc and a mass of $2.4 \times 10^4 M_{\odot}$.

Let us apply these concepts to the diffuse, ionized component of the Galaxy. For the ELD with $\langle N_e \rangle = 3 \text{ cm}^{-3}$, the ionized mass $\approx 8 \times 10^3 M_{\odot}$. If the total number of O stars in the Galaxy is 2.5×10^4 (Petuchowski and Bennet, 1993) and 80% (Mezger and Smith, 1975) of them lie outside of the discrete H II regions in which they were formed, then the mass of low-density hydrogen ionized by these stars is the product of the mass per O star, the number of O stars, and the percentage available – or $\approx 1.6 \times 10^8 M_{\odot}$, exactly the mass estimated by Mezger (1978) from considering the total flux of Lyman continuum from O stars in the Galaxy. Evidently, the principal mass of H II in the Galaxy is contained in these extended regions ionized by unobscured O stars, supplemented by the discrete H II regions located in the spiral arms.

This ionization also accounts for the dispersion of emission from pulsars that lie in the Galactic plane, especially when the geometry of the spiral arms is considered (see Fig. 3.13). For longitudes $\ell = 280^\circ \rightarrow 310^\circ$, where lines of sight go along an arm, a large dispersion measure is observed. Twenty-two pulsars have DMs in the range $260 \rightarrow 715 \text{ cm}^{-3} \text{ pc}$. For the longitude range $50^\circ < \ell < 80^\circ$ with a wider gap between arms, only one pulsar has a $\text{DM} > 260 \text{ cm}^{-3} \text{ pc}$. In fact, Taylor and Cordes (1993) believe that they have determined the distance to pulsars from a similar Galactic model, augmented with a few additions and refinements, with an accuracy of about 25%.

3.2.3 Thickness of the Ionized Hydrogen Layer

In general, astronomers observed RRLs from the diffuse, ionized component of the Galaxy to lie at low Galactic latitudes, i.e., in a very thin layer within $|b| < 1^\circ$. In terms of scale height above the plane, these limits

ranged from $h \approx 36$ pc (Gordon et al., 1972) to $<80 \rightarrow 100$ pc (Hart and Pedlar, 1976; Mezger, 1978; Anantharamaiah, 1986).

However, the optical recombination line $H\alpha$ was observed up to much higher Galactic latitudes. Its greater latitude extent is probably a consequence of its much greater intensity compared with RRLs *and* the much greater sensitivity of the optical instruments – by orders of magnitude. For example, with a high resolution of 0.26 \AA (12 km s^{-1}), Reynolds (1990) used a Fabry–Perot spectrometer to detect $H\alpha$ emission to a limiting intensity⁸ of approximately 0.25 R , i.e., corresponding to a threshold emission measure of approximately $0.5 \text{ cm}^{-6} \text{ pc}$ at a temperature of $8,000 \text{ K}$. For comparison, the most sensitive survey (Heiles, Reach and Koo, 1996) of RRLs from the diffuse gas could only detect emission measures $\geq 120 \text{ cm}^{-6} \text{ pc}$. Therefore, the detection of diffuse $H\alpha$ emission at high Galactic latitudes does not conflict with the observations of the narrower confinement of the RRLs to the Galactic plane.

While strong interstellar absorption makes it difficult to observe in the Galactic plane, $H\alpha$ emission is an effective tool to observe ionized gas at high Galactic latitudes. Observations show that $H\alpha$ is emitted from every direction with an intensity ranging from $0.25 \rightarrow 0.8 \text{ R}$ at the Galactic pole to $3 \rightarrow 12 \text{ R}$ in directions approaching the Galactic plane. The implication is that the diffuse, ionized gas extends to heights above the Galactic plane of $|z| \approx 1,000 \text{ pc}$ – considerably higher than the ionized gas observed with RRLs. Reynolds (1990) estimates the temperature of this gas to be approximately $8,000 \text{ K}$ from the width of the $H\alpha$ profile.

Pulsar observations indicated the presence of ionized gas at high Galactic latitudes. Taylor and Manchester (1977) used the dispersion measures to determine that $|z| \approx 1,000 \text{ pc}$, in good agreement with the value measured later with $H\alpha$ emission. Furthermore, observations of pulsars in high-latitude globular clusters at $|z| > 3 \text{ kpc}$ determined the structure and electron density of the ionized halo (Reynolds, 1991) from their dispersion measures. Observations of $H\alpha$ emission in the same directions and, hence, from the same gas contributing to the signal dispersion determined an average electron density. Because the emission measure $EM \equiv \int N_e^2 ds$ and the dispersion measure $DM \equiv \int N_e ds$, their ratio gives $\langle N_e \rangle$ while the ratio DM^2/EM gives the characteristic sizes of the ionized regions $\langle D \rangle$. These ratios indicated that the ionized halo is inhomogeneous. The average electron density is approximately 0.08 cm^{-3} in “clouds” that occupy $\geq 20\%$ of the columns along the lines of sight (Reynolds, 1991).

⁸ The Rayleigh (R) is a unit of photon emission rate named in honor of the fourth Lord Rayleigh (R.J. Strutt) who made the first measurement of the night airglow in 1930. One R is defined as $10^6/4\pi \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ (Hunten, Roach and Chamberlain, 1956). Consequently, at the wavelength of $H\alpha$ emission, $1 \text{ R} = 2.4 \times 10^{-7} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$. The emission measure of $H\alpha = 2.75 T_4^{0.9} I_\alpha \text{ cm}^{-6} \text{ pc}$, where T_4 is the temperature in units of 10^4 K and I_α is the intensity of $H\alpha$ line in Rayleigh (Reynolds, 1990). Therefore, at $T = 8,000 \text{ K}$ and $I_\alpha = 0.25 \text{ R}$, the threshold emission measure of the diffuse $H\alpha$ emission $\approx 0.5 \text{ cm}^{-6} \text{ pc}$.

We conclude that observations of RRLs, pulsar dispersion measures, and $H\alpha$ emission indicate a layer of warm, tenuous ionized gas above and below the Galactic plane extending to a height of approximately $|z| \approx 1,000$ pc. In contrast, the discrete $H\text{II}$ regions are generally confined to $|z| \approx 100$ pc. Both components have approximately the same temperature. The sources of this ionization are not yet known in detail but O stars are certainly capable on the basis of their collective, radiated energy. However, it is not clear how UV photons from these stars can travel hundreds of pc through the $H\text{I}$ gas of the Galactic plane (Reynolds, 1984; Reynolds, 1993) to form the ionized layer.

Heiles et al. (1996) described a possible mechanism for the transport of the UV photons and of ionized gas from the Galactic disk to the halo domain. Based upon their observations of RRLs near 1.4 GHz, they believed that the Galaxy contains “chimney-like” structures connecting the disk constituents with the halo like the one sketched in Fig. 3.22. Similar to the closed-end “worms” seen in earlier observations of the $\lambda = 21$ cm emission of $H\text{I}$, the 408-MHz continuum emission, and most recently the IR (Koo, Heiles and Reach, 1992), these open-end chimneys allow UV photons and hot, ionized gas to flow freely from the disk to the halo. The chimneys themselves result from explosions of supernovae in the disk, blowing conduits to the halo. Typical dimensions of these structures could be 1,000 pc in diameter and hundreds of pc in length.

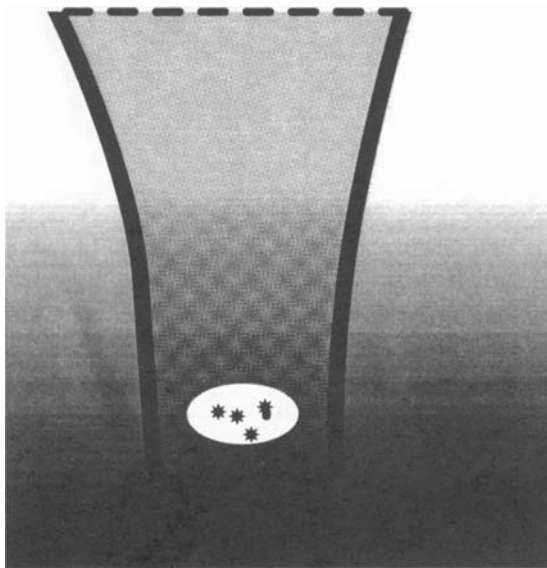


Fig. 3.22 A sketch of a “chimney” connecting the disk ISM with the gaseous Galactic halo. It would result from an older generation of massive stars in a disk star cluster that exploded as supernovae, creating the large cavity that contains the hot rarefied gas. The new O stars in the same cluster would produce ionizing photons that would travel freely through the chimney. The *white ellipse* represents the molecular cloud from which a new generation of stars will form. From Heiles et al. (1996)

Additional observations – including RRLs – will resolve the origin of the observed warm, tenuous, ionized component of the Galaxy. The worms themselves might explain all of the ionization or additional mechanisms may be found to account for the propagation of the UV flux and diffuse, ionized gas through the Galaxy.

3.2.4 Helium in the Galaxy

Helium recombination lines, i.e., from the common isotope ^4He , were detected soon after those of hydrogen. In many cases, both lines fell within the same spectral window of the spectrometer because the 0.04% change in the Rydberg constant (Table A.2) shifts the helium RRLs only to slightly higher frequencies (1.17) than those of hydrogen. In almost all cases, the line widths are such that the H and He lines of the same order fall side by side; i.e., the cores of the two RRLs are well separated in a spectrum and the lines do not overlap significantly. Figure 3.23 shows the original detections of the $\text{He158}\alpha$ (1.72 GHz) and $\text{He159}\alpha$ (1.62 GHz) lines from the H II region M17 with the 60-ft. telescope of Harvard College Observatory (Lilley, Palmer, Penfield and Zuckerman, 1966). That the helium recombination lines fell at the “correct”

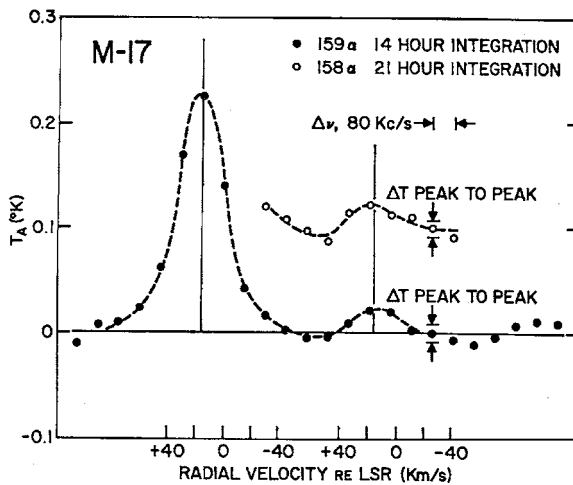


Fig. 3.23 The first detection (Lilley, Palmer, Penfield and Zuckerman, 1966) of helium recombination lines: plots of the H158 α and H159 α spectra (*left*) from M17 with extensions covering the corresponding helium lines (*right*), in units of antenna temperature and radial velocity with respect to the LSR. The *vertical lines* mark the theoretical separation between the two lines. Figure taken from Palmer (1968). Reproduced with permission of Nature

velocity offset⁹ from the hydrogen RRLs made it certain that the detections were actually helium RRLs rather than some other emission line.

The initial detections brought no surprises. The velocity separation of the H and He RRLs was just as predicted by theory. At that time, the cosmological values of the density¹⁰ ratio $N_{\text{He}}/N_{\text{H}}$ were predicted to range from $0.08 \rightarrow 0.10$, and optical observations of extragalactic objects gave the ratio to be 0.10 (Peimbert and Spinrad, 1970). The integrated intensity ratio $I_{\text{He}}/I_{\text{H}}$ of the new RRL detections was 0.10 ± 0.05 , roughly consistent with the optically determined values even though the relative sizes and temperatures of the H and He ionization zones in M17 were not known in detail.

The importance of the helium RRL detection was that it offered a new tool to investigate the ionization structure within H II regions and, possibly, through better determinations of the elemental abundance ratio $N_{\text{He}}/N_{\text{H}}$ an opportunity to investigate the evolution of the Galaxy and the origin of the universe.

3.2.4.1 Helium in H II Regions

The physical characteristics of the helium atom allow it to behave very differently than hydrogen within an astronomical H II region. First, its ionization energy is 24.6 eV ($\lambda = 504 \text{ \AA}$) compared with the 13.6 eV ($\lambda = 912 \text{ \AA}$) of hydrogen. This means that the size of a Strömgren sphere for He II may be smaller or larger than that of H II in the same nebula, depending primarily upon the spectrum of the UV radiation emitted by the exciting stars. Accordingly, astronomers often refer to these two spheres as being either spatially coincident or noncoincident.

Second, helium has two electrons, allowing it to exist in the He III form in addition to the more common He II. The ionization energy to produce this form is 54.4 eV ($\lambda = 228 \text{ \AA}$), requiring considerably more energy than that required to create the He II form.

For these reasons, the abundance of helium within an H II region is the sum of the relative number densities of the neutral, singly ionized, and doubly ionized forms of helium:

$$y = y^0 + y^+ + y^{++}. \quad (3.11)$$

⁹ Because the rest frequencies of RRLs scale linearly with the Rydberg constant, the *radial velocity* offsets of lines of increasing mass (He, C, etc.) from the same gas will be the same relative to the H RRLs for the same values of n and Δn . Figure 3.23 is an example. Also see footnote 2.

¹⁰ Cosmological values of hydrogen, helium, and heavier elements are often given in terms of their fractional mass X , Y , and Z , respectively. $X + Y + Z = 1$. Here, we refer to units of fractional elemental number density, expressed by the lower case letters x , y , and z , which are more closely related to the intensity of the RRL emission. *Note:* For convenience, authors sometimes use $y = N_{\text{He}}/N_{\text{H}}$ which, of course, is an approximation to the formal definition of y made possible by the cosmological dominance of hydrogen gas.

By definition, $x + y + z = 1$, where x and z refer to the number densities of hydrogen and heavier atoms, respectively. Each of these values refers to the integral of the constituent over the entire H II volume of the nebula. Only the components y^+ and y^{++} can be observed through RRLs. The neutral component of helium within the H II region, y^0 , must be determined by other means.

Because of the enormous energy required to produce significant amounts of y^{++} with respect to y^+ , this component can be ignored in (3.11). Calculations of blanketed stellar atmospheres show that the flux at $\lambda = 228 \text{ \AA}$ is about two orders of magnitude less than that at $\lambda = 504 \text{ \AA}$ even in the hottest stars (Kurucz, 1979). More importantly, observations of helium RRLs in H II regions gave limits for the y^{++} lines of approximately one order of magnitude below detections of y^+ RRLs (Churchwell, Mezger and Huchtmeier, 1974). Optical searches for the $\lambda = 4,686 \text{ \AA}$ line of He II lower these limits by another order of magnitude (Peimbert and Goldsmith, 1972; Shaver, McGee, Newton, Danks and Pottasch, 1983). However, y^{++} lines have been detected in the much hotter environment of planetary nebulae.

In contrast, the neutral component y^0 cannot be ignored. As mentioned above, the He II zone may be smaller than the H II zone; i.e., all of the helium may not be ionized depending upon the spectral types of the exciting stars. Additional factors involve properties of the nebulae itself, manifesting themselves through radiation transfer factors like line profiles, departures from LTE, and continuum opacity including the effects of dust.

The stellar type is the most significant factor. Figure 3.24 shows the ratio γ of the helium ionizing photons to the hydrogen ionizing ones plotted

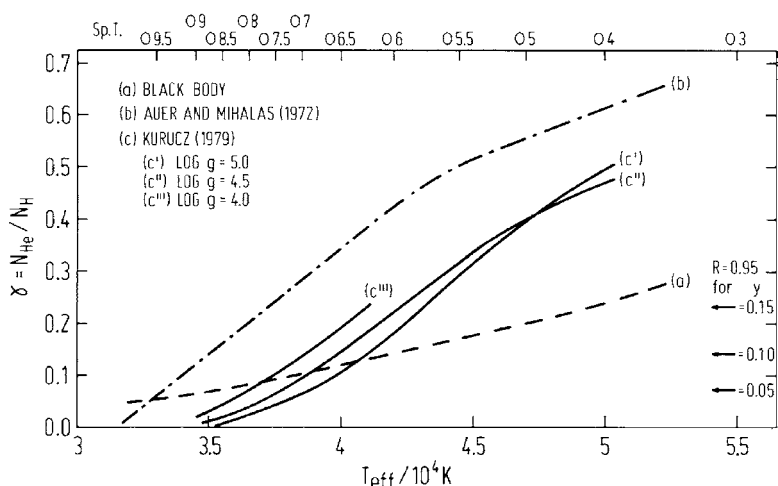


Fig. 3.24 The ratio γ of He ionizing to H ionizing photons as a function of stellar type and effective temperature. Curves *a*, *b*, and *c* refer to a black body, a non-LTE atmosphere, and three model atmospheres that include line blanketing, respectively. From Mezger (1980)

as a function of stellar type for three kinds of atmospheric models. In general, when $y = 0.10$, the diameters of the He II and H II Strömgren spheres are approximately equal (spatially coincident) for $\gamma > 0.20$ (Mathis, 1971). According to the figure, this would occur for stellar types earlier than approximately O6 based upon calculations of model atmospheres that include line blanketing (Kurucz, 1979). More recent calculations give similar results.¹¹ Arrows on the right side of the figure indicate where the diameter of the He II zone reaches 95% of the diameter of the H II zone for other values of y .

The compact H II region NGC2024 (Orion B) is an example of non-coincidence, where the sizes of the H II and He II ionization spheres are very different within the same nebula. Early RRL observations indicated $y^+ < 0.02$ (Gordon, 1969; Churchwell et al., 1974) compared with the value of $y^+ \approx 0.08$ observed for its nearby compact nebula, NGC1976 (Orion A). Later observations did detect weak helium RRLs in NGC2024, giving $y^+ = 0.02 \rightarrow 0.06$ depending upon where one looks (McGee and Newton, 1981; Krügel, Thum, Martín-Pintado and Pankonin, 1982).

While the exciting star of this nebula is highly obscured, some optical observations suggested that it could be a O9.5Ib star (Becker and Fenkart, 1963). If so, the calculations shown in Fig. 3.24 indicate that the He II Strömgren sphere would be much, much smaller than that of H II even for $y \approx 0.1$, thereby accounting for the weak helium RRLs in this nebula. More recent observations in the near IR show the presence of two candidate stars but their spectral types are unknown, so that the exciting star(s) of NGC2024 remains a mystery (see Frey et al. (1979)).

Other discrete H II regions also exhibit anomalously small values of y^+ . A prominent example is the giant H II region near the Galactic center, Sgr B2, for which RRLs give $y^+ \leq 0.024$ (Churchwell et al., 1974). Later, Lockman and Brown (1975b) detected $y^+ = 0.08$ from Sgr B2 with a smaller beam and higher frequency. They claimed the earlier nondetections resulted from instrumental effects. Subsequent observations by the VLA showed that the differences in observed values of y^+ were due to a combination of the beam size and a considerable variation in y^+ over this huge, obscured H II region (Roelfsema, Goss, Whiteoak, Gardner and Pankonin, 1987). In other words, $y^+ = 0.8 \rightarrow 0.10$ for Sgr B2 if one looks in the right places with the right angular resolution. This confirms the “geometric effect” originally suggested by Mezger (1980).

A few H II regions are close enough to have sufficiently large angular extents to allow imaging even by single dishes.¹² These observations indicate

¹¹ In his Chap. 2, Osterbrock (1989) calculates the ratios of the radii of the He II and H II ionization-bounded spheres as a function of effective temperature (related to stellar type) using an “on-the-spot” approximation.

¹² Single dishes capture all of the flux within their beams but cannot respond to detailed angular structure within these beams. In contrast, synthesis telescopes “see” the detailed angular structure within the primary beams of their individual antennas through their interferometric nature. However, they can miss the angularly “extended” component unless

spatial variations of He II within these nebulae. For example, Pankonin et al. (1980) used the He101 α line to find a radial decrease of y^+ in the Orion nebula from 10% near the center to 6% toward the edge. Using different frequencies, Tsivilev et al. (1986) found an *increase* of y^+ from approximately 8% at the center to approximately 11% at the edge, for which they proposed a blister model for the He II component of the nebula. Similar observations of helium RRLs in the H II region W3 show variations of the observed values of $y^+ \approx 6 \rightarrow 8\%$ as the angular size of the telescope beam changes.

Higher-resolution observations reveal even more structure in He II within nebulae. Observations of y^+ for W3 made with a synthesis telescope at an angular resolution of $4''$ shows values up to 9% (Roelfsema, Goss and Geballe, 1989). More recent radio interferometric observations in W3A give even greater values, with an extreme of $y^+ = 34 \pm 6\%$. Figure 3.25 shows these observations (Roelfsema, Goss and Mallik, 1992).

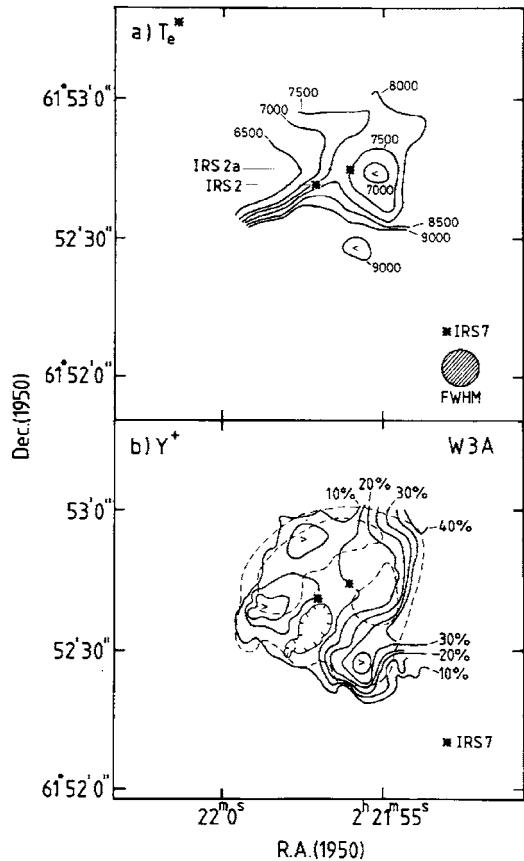


Fig. 3.25 *Top:* distribution of T_e over the H II region W3A derived from H76 α RRLs (14.7 GHz) observed with the VLA at a resolution of $4''$. *Bottom:* the corresponding variation of y^+ derived from the ratios of the He76 α to H76 α line intensities, superimposed upon the free-free emission. From Roelfsema et al. (1992)

special provision is made to obtain it. Therefore, the two kinds of radio telescopes often see different aspects of the same source.

What could be happening within these nebulae? The situation is clearly more complicated than a simple noncoincidence of the He and H Strömgren spheres. The locally observed values of y^+ can be much greater than the cosmologically predicted values of approximately 10% even though some helium is known to exist in the neutral y^0 form.

Brown and coworkers (Brown and Gómez-González, 1975; Brown and Lockman, 1975) suggested that the explanation lies in non-LTE radiation transfer effects. The basis for this idea was that measured values of y^+ seemed to be a function of the observing frequency ($n\alpha$ lines) used for the observations. However, subsequent ad hoc observations did not confirm this.

Another suggestion was that a peculiar stellar radiation field could preferentially ionize helium with respect to hydrogen such that the intensity ratio of He/H $n\alpha$ RRLs is enhanced because of a local underabundance of H II (Roelfsema et al., 1992). Unfortunately, their calculations indicated that this is rather difficult to achieve to the extent required to explain the observations of large values of y^+ observed in W3A.

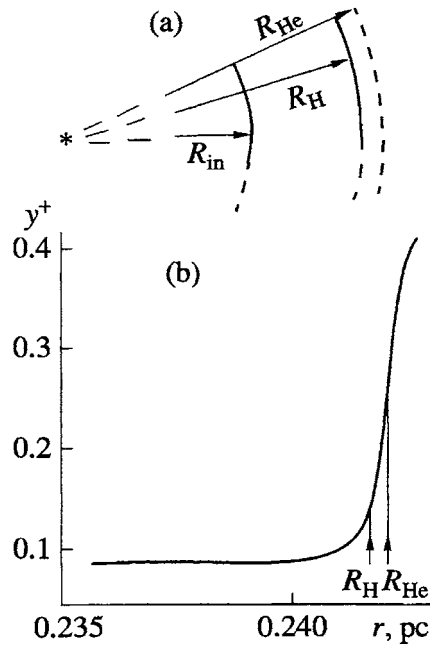
Still another possibility for the large values of y^+ is a localized enrichment by nearby hydrogen-burning stars, producing helium as an “ash,” and later introducing it into the ISM through a combination of convective transport and subsequent helium-enriched stellar winds as the stars evolved.

Most intriguing of all are the results of new calculations. Gulyaev et al. (1997) re-examined the possibility of the diameter of an He II ionization zone exceeding that of an H II zone. Using the best available ionization cross sections, including a 2^3s level of orthohelium, and adopting a shell model for W3A, they found that a narrow zone can exist where the He II would be enhanced relative to H II owing to a hardening of the UV radiation field, as shown in Fig. 3.26. Additional calculations show that the presence of dust would widen this zone. The implication is that the source-averaged value of $y^+ \approx 0.1$ is the characteristic value for W3A best representing y and that higher values of y^+ are local anomalies that depart significantly from the true helium abundance y .

3.2.4.2 Galactocentric Gradient of He II

Early observations of y^+ as a function of Galactocentric distance suggested an astrophysically important relationship: that the ionized helium abundance systematically increased with distance from the Galactic center. Here, it must be noted that the concentration of discrete H II regions in a narrow range of Galactocentric radii (see Fig. 3.14) makes it difficult to determine existence of any Galactocentric gradient of y^+ . There is an unavoidable selection effect. The data are dominated by a few discrete H II regions at the Galactic center and by a clustering of most within the range $4 < R < 8$ kpc. Nonetheless, the possibility of a gradient was intriguing.

Fig. 3.26 (a) Cartoon of a shell model for W3A. R_{in} marks the inner radius of the shell. R_H and R_{He} mark the outer boundaries of the respective Strömgen spheres. (b) The calculated variation of y^+ as a function of radius from the exciting star. From Gulyaev et al. (1997)



To explain this gradient, Mezger et al. (1974) proposed a selective absorption by dust. This absorption would filter the UV radiative field from the embedded stars such that the observed ratio of He II/H II would vary with Galactocentric distance independently of y . To some extent, the observed excess IR emission of H II regions lent support to this idea. However, it proved difficult to model dust grains that could produce this effect (Mathis, 1980) and the proposal was later abandoned.

Alternatively, Panagia (1979) suggested a Galactocentric gradient in the temperature of the O stars producing the discrete H II regions. Because the star formation rate was expected to be much greater in the center of the Galaxy than that in its outer regions, the resulting gradient in metallicity could easily produce a Galactocentric gradient in stellar types of stars exciting the H II regions and, hence, in y^+ .

In time, additional data made it possible to investigate the Galactocentric variation of y^+ more thoroughly. Adding to the radio Hn α observations are the optical values determined from the $\lambda = 4,471$, 5,876, and 6,678 Å lines of singly ionized helium. RRLs made these new optical values possible; the analyses used the T_e s accurately determined from Hn α observations for the same nebulae (Shaver et al., 1983).

Figure 3.27 shows the results: no Galactocentric gradient of y^+ is apparent. The early, low upper limits for the Galactic center are due to a geometrical averaging within large telescope beams; newer observations give a value of ≈ 0.1 depending on resolution and position. Note how well the optical values

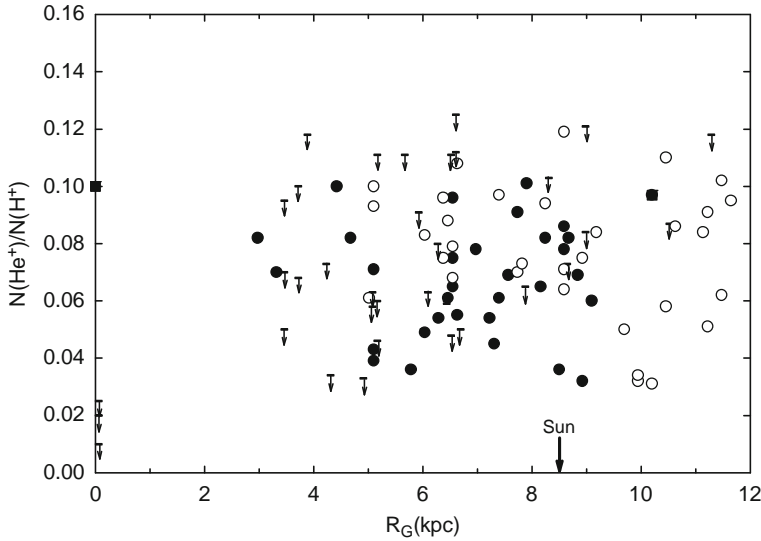


Fig. 3.27 The variation of y^+ with Galactocentric radius, determined from H II regions. The Sun lies at 8.5 kpc. *Filled circles*: measurements with RRLs. *Open circles*: measurements with optical lines. *Filled square*: average of H76 α observations made with a 4'' resolution. *Upper limits* are also indicated. Data from Churchwell et al. (1974), Shaver et al. (1983), Roelfsema et al. (1987), and Gulyaev et al. (1997)

agree with the radio ones – in the range of scatter as well as in the mean value of the data. The radius-averaged value $\langle y^+ \rangle = 0.081$ and the median is 0.074.

This null result is perplexing. Figure 3.8 clearly shows a negative Galactocentric gradient of metal abundance. The higher metallicity probably results from a higher astration rate near the Galactic center, giving a higher percentage of metals in the ISM. Furthermore, Fig. 3.9 shows a corresponding positive Galactocentric gradient of the electron temperatures of compact H II regions, which can be explained by the higher cooling rates facilitated by the high metallicities of the gas nearer the Galactic center. These observations indicate that, on average, stellar types should increase from late (cool, higher metallicity) to early (hot, lower metallicity) with increasing Galactocentric distance. If the helium abundance relative to hydrogen is constant throughout the Galaxy, we would expect that y^+ should increase from the center outward, reflecting the increasing hardening of the stellar UV radiation from the Galactic center outward. Clearly, this is not seen.

The explanation must lie in the relationship of y^+ to y . It is not clear how well y^+ represents y in general. The percentage ionization of helium may well increase from the center outward but perhaps the total amount of ^4He is correspondingly decreasing because of a gradient in the star formation rates. Observations show that some Galactic nebulae have $y^+ \ll y$, some

have $y^+ < y$, and some have $y^+ \approx y$. To understand this situation better, we must await new information on the ionization structure of helium within H II regions and, especially, on the Galactocentric distribution of helium itself in all its forms.

3.2.4.3 Cosmology

By themselves, RRLs have not yet proven useful for cosmological studies. In principle, the lines of helium and hydrogen should provide values of y that might allow separation of the primordial y_p produced by cosmological nucleosynthesis from that produced within hydrogen-burning stars. As we have seen, the best that RRLs can measure is y^+ which is not always a measure of y owing to variations in the UV radiation emitted by the exciting stars.

A more important tool for cosmological studies is the abundance of the isotope ^3He relative to H. Because stars might be net producers of this isotope, the lower limit to this ratio is an upper limit to the primordial value and the baryon-to-photon ratio. This limit constrains models for the chemical evolution of the Galaxy (Burles, Nollett and Turner, 2001). Furthermore, production of ^3He by solar-type stars ($M < 2 M_\odot$) would lead to local enhancements in the ISM that could be detected.

Direct observations of $^3\text{He}^+$ by RRLs are extremely difficult, beside the fact that only the ion can be observed with RRLs. The masses of ^4He and ^3He are about 4.0 and 3.0 amu, respectively. Equation (1.19) shows that this mass difference would give a frequency difference between their RRLs of only 0.005%. To avoid Stark broadening of the RRL profiles while still providing the most favorable line intensities, the observations would have to be made in the centimeter wavelength regime. Since the RRLs would come from H II regions, the typical turbulence broadening of approximately 20 km s^{-1} would exceed the line separation, resulting in a blend of RRLs at these frequencies that would be difficult to separate. Furthermore, the ratio of the number densities of $^3\text{He}^+ / ^4\text{He}^+$ may be approximately 10^{-4} , which would ensure that the target line of $^3\text{He}^+$ would be buried within the line profile of the $^4\text{He}^+$ RRL. Finally, the superposition of $^4\text{He}^+$ RRLs with those of greater mass on the high-frequency side of the lines, such as those from $^{12}\text{C}^+$, would make a decomposition of line profiles nearly impossible to perform to an accuracy that would be useful.

Another avenue to the $^3\text{He}^+ / \text{H}^+$ ratio exists – one that depends upon RRLs in a partnership role. Following the detection of helium RRLs, Sunyaev (1966) and Goldwire and Goss (1967) suggested a search for the hyperfine transition of $^3\text{He II}$ that lies near 8.7 GHz. This line is a parallel to the $\lambda = 21 \text{ cm}$ line of atomic hydrogen. Unlike the $\lambda = 21 \text{ cm}$ line, the helium line is extremely weak.

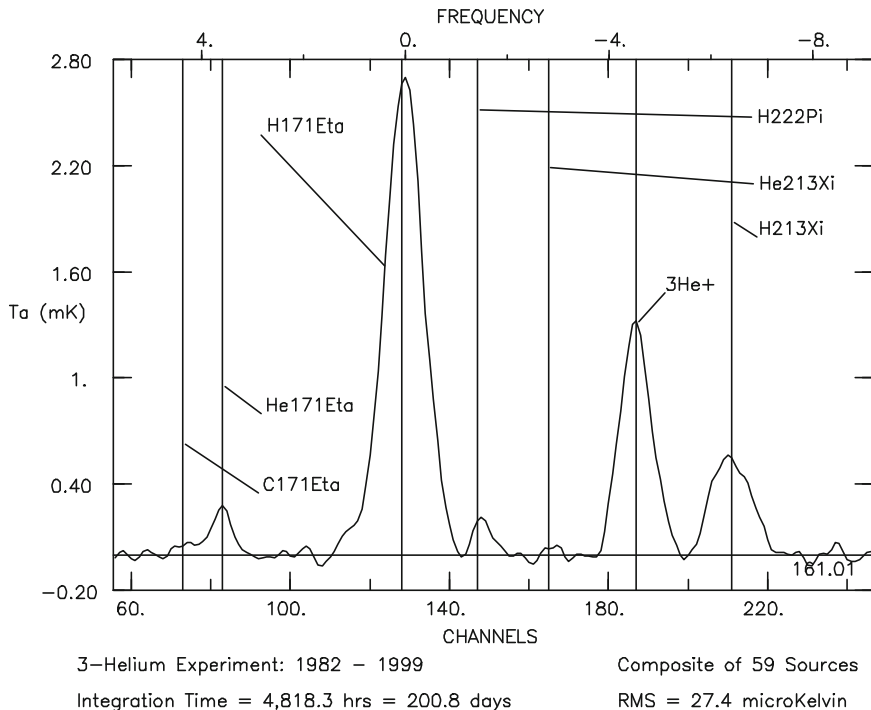


Fig. 3.28 The averaged spectra of 59 Galactic H II regions, aligned to the hyperfine line of $^3\text{HeII}$ at 8.7 GHz. The effective integration time is 200.8 days and the RMS noise of the spectrum is $27\ \mu\text{K}$ ($1\ \text{K} \approx 3\ \text{Jy}$). The *ordinate* is in terms of mK of antenna temperature of the NRAO 140-ft. telescope. The *lower abscissa* is units of autocorrelator channels (78.1 kHz) and the *upper abscissa* is the offset in MHz from the H171 η recombination line. From Bania (2001). See also Bania et al. (2000)

Figure 3.28 shows a spectrum (Bania, 2001) resulting from the averaged spectra of 59 H II regions and planetary nebulae, representing an integration time of about 201 days and an RMS of $27\ \mu\text{K}$. Here, an antenna temperature of $1\ \text{K} \approx 3\ \text{Jy}$. In addition to the detection of the $^3\text{HeII}$ hyperfine-structure line, the spectrum shows the 171 η ($\Delta n = 7$) RRLs of H, He, and C; the 213 ξ , ($\Delta n = 14$) lines of H and He; and possibly the H222 π ($\Delta n = 16$) line.

Converting observations of $^3\text{HeII}$ into an abundance ratio of $^3\text{He}^+/\text{H}^+$ is challenging. The technique consists of direct observations of the hyperfine line, of hydrogen recombination lines, of helium recombination lines, and the underlying free-free continuum. Using electron temperatures determined from the RRLs allows the extraction of the emission measure from the continuum emission. The ratio of the He/H RRLs then corrects this emission measure for electrons contributed by He II. Most difficult of all is the correction for density fluctuations within the discrete H II regions – often known as

clumping. Emission measures involve $N_e^2 d\ell$; observations of clumped nebulae weight high densities much more than the low densities. In contrast, the hyperfine line is a linear function of the electron column density because it is excited by collisions; i.e., the intensity $\propto N_e d\ell$. Combining these measurements on the basis of a uniform sphere model for the clumped nebula would then give too small a ratio of ${}^3\text{He}^+/\text{H}^+$ unless detailed, complex models of the clumped nebulae were made (Balser, Bania, Rood and Wilson, 1999) to correct the ratio.

The solution was serendipitous. These studies discovered that large, diffuse nebulae tended to have little structure. Observations show that there are at least 21 of these simple H II regions in our Galaxy. Consequently, combining results for the emission measure and the hyperfine line from these nebulae would then give accurate results for the ${}^3\text{He}^+/\text{H}^+$ ratio.

The results are interesting (Bania, Rood and Balser, 2002). First, there does not appear to be a Galactocentric gradient of ${}^3\text{He}^+/\text{H}^+$ in the disk from the Galactic center to a radius of 15 kpc. Nor is there any relationship with the metallicity in the Galaxy. This result was unexpected because theory indicates that low mass stars can produce ${}^3\text{He}$ by nucleosynthesis. Possibly, the new material does not circulate widely within the ISM, or the net stellar production of ${}^3\text{He}$ is much lower than predicted either through the generation process itself or from an unknown loss mechanism. Second, the minimum value found – the upper limit on the primordial ratio – is ${}^3\text{He}/\text{H} = (1.1 \pm 0.2) \times 10^{-5}$. This number is consistent with an open universe, i.e., a universe that will expand forever.

3.3 Exploration of the Cold ISM by RRLs

The properties of the cold ISM differ considerably from those of the discrete H II regions surrounding hot stars. The most important and the most obvious is that it is not heated by ionizing radiation from embedded hot stars. Consequently, helium and, with a small exception, hydrogen RRLs are not detected from the cold ISM.

Nevertheless, RRLs have become an important tool to study the cold ISM. Carbon is the leading player in this drama. As described in Sect. 2.4.2, only carbon recombination lines have been detected at high principal quantum numbers. These high- n lines revealed unique information regarding the physics of highly excited Bohr atoms unobtainable in terrestrial laboratories. In addition, the carbon RRLs unveiled new secrets about our Galaxy, by teaching us about the characteristics of the cold interstellar gas that pervades the disk environment.

3.3.1 $C\text{II}$ Regions at the Boundaries of $H\text{II}$ Regions and Molecular Clouds

Historically, carbon RRLs were first detected in hydrogen spectra from the $H\text{II}$ regions NGC2024 (Orion B) and IC1795 (W3) (Palmer, Zuckerman, Penfield, Lilley and Mezger, 1967). In back-to-back papers, Palmer et al. (1967) and Goldberg and Dupree (1967) announced the detection of a new RRL and offered the tentative identification that it was from interstellar carbon, respectively. Figure 3.29 shows the original spectrum from NGC2024.

Initially, there were problems with the identification. Observations of the $H110\alpha$ showed the same line, thereby proving it to be an RRL. However, the exact offset of the new line from the $H109\alpha$ in amu was not certain because of the marginal frequency sampling of only three points across the line profile. All that could be said was that the new RRL was from an element with a mass between 8 and 12 amu. There was also a problem with intensity. If the new RRL was due to interstellar carbon (12 amu) whose cosmic abundance was just below hydrogen and helium, the intensity was wrong. In the Sun, the abundance of carbon is 5×10^{-4} that of hydrogen. Figure 3.29 shows the integrated intensity of the new line to be approximately 3% of hydrogen – approximately 60 times greater than the carbon abundance. Yet, carbon seemed the best candidate on the basis of abundance and frequency.

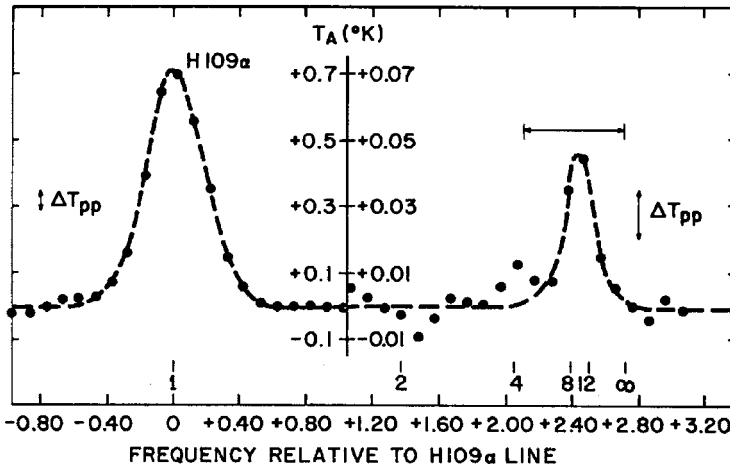


Fig. 3.29 The detection of the $C109\alpha$ line in NGC2024 (Orion B) with the NRAO 140-ft. telescope (Palmer et al., 1967). *Left*: the $H109\alpha$ line at 5 GHz. *Right*: the newly detected line later identified as the $C109\alpha$ line. The *ordinates* are antenna temperature in K. The *outer abscissa* is the frequency offset in MHz from the $H109\alpha$ line. The *inner abscissa* is in units of amu relative to the hydrogen line. Figure from Palmer (1968). Reproduced with permission of Nature

What could be happening? Goldberg and Dupree (1967) supplied the answer. They calculated that the upper quantum levels of carbon were being overpopulated through dielectronic recombination¹³ and that the line intensities were correspondingly amplified above their LTE values. Therefore, they had no problem in identifying the new RRLs to be those of carbon.

Additional observations of the carbon RRLs from several nebulae gave more surprising results. Maps showed the centroid of the carbon line emission to be offset from that of the hydrogen emission, the radial velocities were often different from the hydrogen RRLs, and the line widths were always much narrower. Although the carbon lines were spatially associated, they probably did not come from the region of the H II gas itself. For these reasons, Zuckerman and Palmer (1968) suggested that the lines originated in the outer parts of a dense H I region bounding the discrete H II regions. Particularly for NGC2024, they suggested that the carbon lines arose in the same region where IR emission was observed.

Still more observations made this suggestion a certainty. Ball et al. (1970) found the width of the carbon line in NGC2024 to be only 4 km s^{-1} . This width required the line to be generated in a gas with $T_e < 1,500 \text{ K}$, which excluded origination in the H II region. Furthermore, while the radial velocities of the $Cn\alpha$ lines often differed from the $Hn\alpha$ lines, they usually agreed very well with those of the $\lambda = 21 \text{ cm}$ H I, OH absorption, and H_2CO lines known to be associated with the cold ISM.

All of the observations pointed to the C II regions being formed in the outer layers of molecular clouds at the boundaries with H II regions (Balick, Gammon and Hjellming, 1974; Zuckerman and Ball, 1974; Dupree, 1974). Here, UV photons with $\lambda > 912 \text{ \AA}$ leave the H II region, where they dissociate molecules and ionize atoms with ionization potentials less than hydrogen. The most abundant of these ISM constituents is carbon which has an

¹³ This process of ordinary dielectronic recombination differs from the special one described earlier in Sect. 2.4.2 for low-frequency carbon RRLs in a cold ($\approx 100 \text{ K}$) medium. In the much warmer H II region environment where kT can be large, the process involves excitation of a bound electron of a carbon ion to a different n -state during the recombination of the singly ionized ion and a free electron, temporarily creating a neutral carbon atom with two electrons in excited states. In other words, the ion core becomes excited as well as the recombined outer electron. The newly formed atom then either autoionizes, which is of no interest to us, or stabilizes, usually by spontaneous emission of the inner electron via a resonance line, leaving the outer electron in its high quantum state. Because the rate of dielectronic recombination can be much greater than that of simple radiative recombination that occurs in the one-electron hydrogen atom, the result can be a greatly enhanced number of carbon atoms in highly excited states and, consequently, a great enhancement of the high- n level populations and their db_n/dn gradient. This produces carbon RRL lines with intensities much greater than those expected from the numerical abundance of the carbon atoms.

As described earlier in Sect. 2.4.2, in the cold environment of the low-frequency carbon spectra, the result is similar but the process is different because of the comparatively small value of kT . Stabilization occurs via collisions. To emphasize the differences between the two dielectronic processes, Watson et al. (1980) suggested the term “dielectronic captures” for the low-temperature process.

ionization potential of $V_{ion} = 11.3 \text{ eV}$ for the first electron. Along with carbon, other elements with $V_{ion} < 13.6 \text{ eV}$ are ionized. Among these are sulfur ($V_{ion} = 10.4 \text{ eV}$), magnesium ($V_{ion} = 7.6 \text{ eV}$), iron ($V_{ion} = 7.9 \text{ eV}$), and silicon ($V_{ion} = 8.2 \text{ eV}$) listed in terms of decreasing relative cosmic abundance. The result is the formation of partially ionized regions located between the cold, nonionized material of the parent molecular cloud and the fully ionized, discrete H II region. Such C II regions are also known as “photodissociation regions” (PDRs) because the far-UV photons from the H II region stars play a significant role in their creation.

Improved instrumentation revealed an RRL from another element. Figure 3.30 shows two spectra obtained in 1977 with the MPIfR 100-m telescope toward NGC2024 and W3. These spectra clearly show the H166 α and C166 α lines (Pankonin, Walmsley, Wilson and Thomasson, 1977). No He166 α lines appear because of the late spectral types of the stars producing these H II regions. In addition, both spectra show a weak new line – marked “X” – at a lower velocity (higher frequency) than the carbon line. In the spectrum of NGC2024 where the C and X lines appear more crisply, their velocity

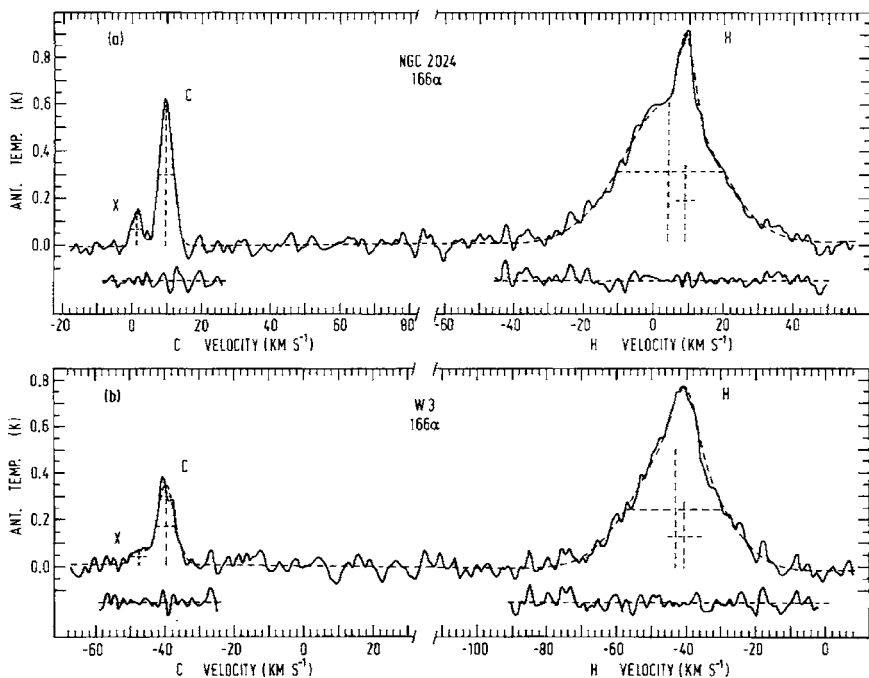


Fig. 3.30 The spectra of 166 α lines from NGC2024 and W3 taken with the 100-m radio telescope of the MPIfR in Germany with a spectral resolution of 0.83 km s^{-1} . The ordinate is antenna temperature and the abscissae are V_{LSR} with respect to C (left) and to H (right). Dashed lines indicate the Gaussian fits. The residuals to the fit are plotted below the spectra. From Pankonin et al. (1977)

separation is 8.5 km s^{-1} . If both C and X have the same radial velocity, i.e., originate from the same gas, the position of the X line corresponds exactly to that of sulfur. Specifically, the difference in the ionization potentials of C and S corresponds to a velocity separation of 8.58 km s^{-1} , in excellent agreement with the observations. This detection of sulfur RRLs confirmed the tentative identification made 2 years earlier in a much noisier H158 α spectrum of the ρ Oph dark cloud (Chaisson, 1975).

Tielens and Hollenbach (1985) analyzed the situation theoretically to see how the composition changed as a function of depth into a cloud. They modeled the structure of a PDR with the most realistic parameters for the H II region–molecular cloud complex available. The UV radiation field was taken to be $1.6 \times 10^2 \text{ erg cm}^{-2} \text{ s}^{-1}$ and the density of hydrogen nuclei $n_0 = 2.3 \times 10^5 \text{ cm}^{-3}$. The depth into the cloud is in terms of visual extinction; specifically, the column density $n_0 L = 2 \times 10^{21} \text{ cm}^{-2}$ at $A_V = 1 \text{ mag}$.

Figure 3.31 shows their results. The geometry is a plane-parallel slab illuminated from the left by the UV radiation. The two panels are the same except for the plotted constituents. At the boundary with the H II region ($A_V = 0$), all of the molecules are dissociated and the PDR consists only of atoms. Moreover, atoms with ionization energies $< 13.6 \text{ eV}$ are completely ionized here. They include C^+ , S^+ , and Mg^+ . Because carbon is dominant, they adopted an electron abundance equal to those released from carbon, $N_e = 3 \times 10^{-4}$.

Moving deeper into the PDR, we find conditions changing further because of the attenuation of the UV radiation from the H II region by scattering, absorption, and other mechanisms. At $A_V \simeq 2$, H_2 molecules form from

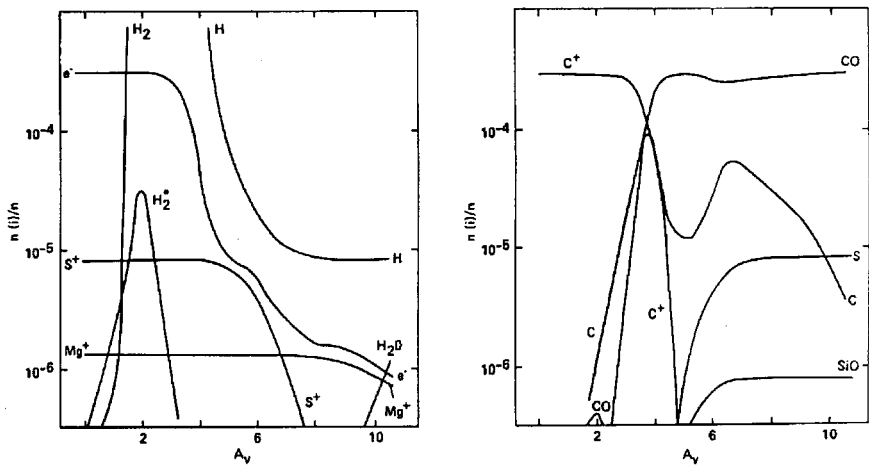


Fig. 3.31 The computed abundance of selected ions, atoms, and molecules, $n(i)/n_0$, as a function of the visual extinction, A_V into a photodissociation region. From Tielens and Hollenbach (1985)

hydrogen atoms, part of them in the vibrationally excited state H_2^* . At greater depths where $A_V > 2$, the density of excited hydrogen molecules H_2^* begins to fall because of a decrease in the UV photons necessary to excite these levels. These molecules return to the ground states through collisions and spontaneous transitions.

Note that C^+ and S^+ also vary from the edge of the PDR toward the center. At $A_V \simeq 2$, the ionized carbon atoms begin to change into neutral carbon and CO molecules. The C II layer begins to end at $A_V \simeq 3.5$ and then falls off sharply. The calculated column density for C^+ is $\simeq 2 \times 10^{18} \text{ cm}^{-2}$ with a weak dependence on the gas density (n_0) and on the incident UV radiation.

The layer of S^+ is thicker than C II, reaching a depth of $A_V \simeq 6$ because of its lower ionization potential relative to carbon. Sulfur in any form is less abundant than carbon; Tielens and Hollenbach (1985) adopted a column density for this constituent $\approx 10^{17} \text{ cm}^{-2}$.

The study of C II regions is important for understanding the complete picture of how stars form. Although the process itself takes place within dense molecular clouds, the secondary processes play a significant role in shaping the environment. The H II region created by the newly formed stars contains density gradients that facilitate expansion and fragmentation of the region's edges. This promotes mixing of enriched material into the ISM, locally at first but ultimately throughout the ISM of the host galaxy. At the same time, the ionization front and associated shock waves from the nascent H II regions create higher densities that, in turn, lead to new star formation (Elmegreen and Lada, 1977) that sequentially amplify the process. The RRLs from carbon and sulfur enable us to watch the part of this rather grand process that takes place at the boundary between H II region and the host molecular cloud, in what is now called the PDR.

3.3.2 C II Regions: Information from Carbon RRLs

Unlike H II regions where hydrogen RRLs can determine temperature and density rather accurately, C II regions are much more difficult to investigate through carbon RRLs. First, it is impossible to separate the continuum emission of electrons contributed by carbon atoms from the strong continuum background of electrons contributed by hydrogen atoms. Accordingly, the ratio T_L/T_C cannot be determined for the carbon RRLs, making the electron temperature for C II regions unavailable by this method (see Sect. 3.1.1). Second, it is not possible to obtain densities from Stark broadening of the carbon RRLs. The electron densities in the C II regions are too small to give detectable broadening except for carbon RRLs with $n > 300$, which are impossible to observe with adequate signal-to-noise ratios.

Consequently, the physical conditions within the C II regions have been determined primarily by comparing the observed intensities of the carbon RRLs, including α and β transitions, over a range of principal quantum numbers with those predicted from models made with various temperatures and densities. Specifically, the calculated brightness intensities at line center T_L result from the expression

$$T_L \approx b_n \tau_L^* T_e - b_n \beta \tau_L^* T_{BG}, \quad (3.12)$$

where τ_L^* is the optical depth of the carbon RRL from the C II region in LTE; b_n and β are the departure coefficient and correction term for stimulated emission at the lower principal quantum number n , respectively; and T_{BG} is the brightness temperature of the continuum emission of the associated H II region at the line frequency.

Equation (3.12) is very simple. The first term is the spontaneous emission in the carbon recombination line corrected for departures from LTE. The second term corrects the first term for emission stimulated by the background free-free emission from an associated H II region when that region lies behind the C II region – which is often the case for observations of carbon RRLs. This equation is similar to (2.142), here expressed in terms of brightness temperature rather than specific intensity. The same assumptions apply: $|\tau_L|$ and τ_C are much less than 1 and $h\nu \ll kT_e$.

Table 3.4 gives the physical characteristics of a few C II regions obtained by fitting models to observed carbon RRLs. Although carbon RRLs were detected from more than 30 H II regions, only nine regions were suitable for analysis by this technique. In these sources, a few carbon RRLs were observed over a wide range of principal quantum numbers $n = 85 \rightarrow 220$. The table lists the source regions, their galactic coordinates, the observed lines, and the values of electron temperature and density resulting from the fits.

Examination of the table shows a large range of the values of temperature and density obtained from the fits, indicating fundamental difficulties with the analysis method. In a majority of the cases, the modeling assumed that the C II region lies both in front of and behind the H II region; i.e., it assumed that some of the photons from the H II region traveled through the C II region en route to the observer and some did not. This model causes large variations in the values derived for temperature and density because of radiation transfer conditions peculiar to the wavelength range ($\lambda > 5$ cm or $\nu < 6$ GHz) where the carbon lines were observed. The problem is that the stimulated emission from the C II region is comparable with the spontaneous emission (see (3.12)). In these circumstances, $\beta < 0$ and the two kinds of emission add. Therefore, a unique specificity of the physical conditions of the C II region is difficult to achieve because different conditions can produce the same line intensities.

There is one other concern regarding the modeling described above. The departure coefficients (b_n) and gradients (β) were assumed to be hydrogenic, calculated from a program identical or similar to the one listed in Appendix D.

Table 3.4 Physical conditions of C II boundary regions from C RRLs

| Source | ℓ ($^{\circ}$) | b ($^{\circ}$) | Carbon RRLs | T_e (K) | N_e (cm^{-3}) | References |
|---------|--------------------------|-----------------------|---|------------------|-------------------------------|--------------------------------|
| S64-W40 | 028.8 | +3.5 | 100 α , 125 α | 50→200 | 0.2→5 | Vallée (1987a) |
| W48 | 035.2 | -1.7 | 140 α , 167 α | 30→100 | 10→100 | Silverglate and Terzian (1978) |
| | 035.2 | -1.7 | 109 α , 125 α , 158 α , 166 α , 167 α | 100 ^a | 10 ^a | Vallée (1987b) |
| S88 | 061.5 | +0.1 | 140 α , 167 α | 30→100 | 15→100 | Silverglate and Terzian (1978) |
| DR-21 | 081.7 | +0.5 | | 10→90 | 300 | Vallée (1987c) |
| S140 | 106.8 | +5.3 | 142 α , 166 α | 10 | 0.15 | Knapp et al. (1976) |
| W3 | 137.7 | +1.2 | 85 α → 220 α (12) | 50→200 | ≈10 | Hoang-Binh and Walmsley (1974) |
| | | | 85 α → 183 α (9) | 100 | 20→40 | Dupree (1974) |
| | | | 157 α , 197 β | ≈100 | ≈20 | Pankonin (1980) |
| S235 | 173.6 | +2.8 | | 100 | 1→8 | Vallée (1987d) |
| NGC2024 | 206.5 | -19.4 | 85 α → 220 α (12) | 50 | 10 | Dupree (1974) |
| | | | 85 α → 220 α (12) | 50→200 | ≈10 | Hoang-Binh and Walmsley (1974) |
| | | | 157 α , 197 β | ≈100 | ≈20 | Pankonin (1980) |
| Orion A | 209.0 | -19.4 | 85 α → 220 α (11) | 10→100 | > 1 | Dupree (1974) |
| | | | 85 α → 220 α (12) | 50→200 | ≈10 | Hoang-Binh and Walmsley (1974) |
| | | | 85 α , 109 α , 137 β | 70→150 | 10→20 | Jaffe and Pankonin (1978) |

The number in parentheses in column 4 indicates the number of RRLs used for the modeling

^aAn error three times the value given

Subsequently, it has become clear that the low-temperature dielectronic recombination is an important process regarding the excitation of carbon in the ISM – a process not included in that program. Considering this process somewhat changes the numerical values of the departure coefficients and their gradients, and could lead to different results for the physical conditions derived for the C II regions bounding H II regions. Nevertheless, the general nature of the C II models would be about the same with regard to RRLs produced from quantum levels $n = 80 \rightarrow 220$.

Despite the large range of temperatures and densities for a specific C II boundary region, the carbon RRLs indicate that these regions are cold ($T_e \approx 100$ K) and dense ($N_e \approx 3 \rightarrow 30 \text{ cm}^{-3}$ and $N_H \approx 10^4 \rightarrow 10^5 \text{ cm}^{-3}$).

3.3.2.1 The S II Boundary Layer

In the nine sources connected to the discrete H II regions where carbon RRLs have been detected, sulfur RRLs have also been detected. As examples, Fig. 3.32 shows spectra of the C166 α and S166 α lines detected toward the H II regions W48 and S87. Table 3.5 gives some numerical details.

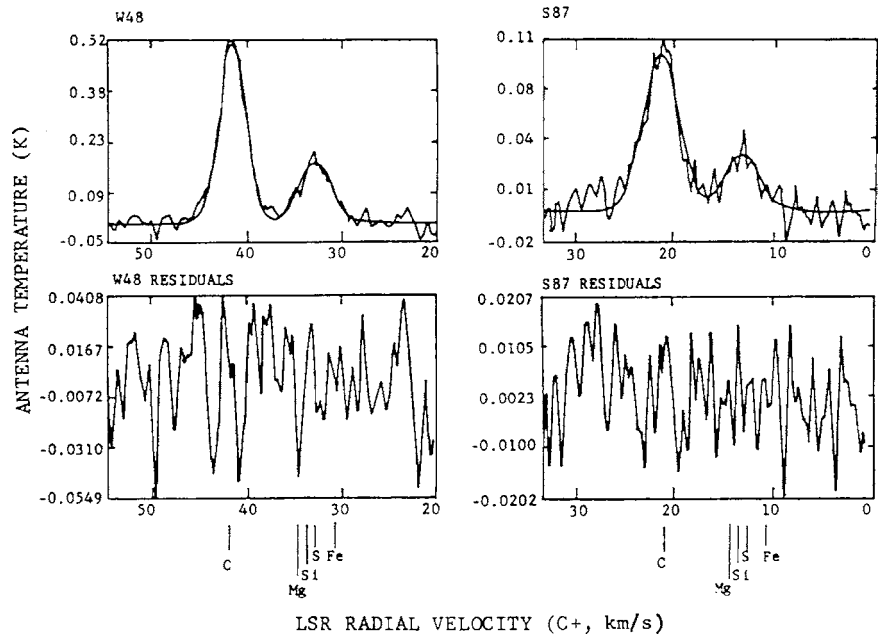


Fig. 3.32 Spectra of C166 α and S166 α lines obtained with the 1,000-ft. (305-m) radio telescope of the US National Astronomy and Ionospheric Center at Arecibo, PR. The radial velocity is referenced to the carbon RRL frequency at the LSR. Gaussians have been fitted to and superposed on the spectra. The residuals resulting from the subtraction of the fits from the spectra are shown below the spectra. From Silvergate (1984)

Table 3.5 Observed parameters of the C166 α and S166 α lines

| Source | RA (1950) | Declination (1950) | Full line width at half-intensity | | V_{LSR} | |
|--------|---|-----------------------|--|--|--|--|
| | | | C166 α (km s ⁻¹) | S166 α (km s ⁻¹) | C166 α (km s ⁻¹) | S166 α (km s ⁻¹) |
| W48 | 18 ^h 59 ^m 14 ^s | 01° 08' 29" | 3.25 ± 0.04 | 3.98 ± 0.09 | 42.52 ± 0.02 | 42.51 ± 0.04 |
| S87 | 19 ^h 44 ^m 16 ^s | 24° 28' 28" | 4.32 ± 0.01 | 4.30 ± 0.23 | 21.43 ± 0.04 | 21.61 ± 0.10 |

From Silvergate (1984)

The high signal-to-noise ratios and spectra resolution of these spectra (0.26 km s^{-1}) ensure the identification of the sulfur recombination line. First, the situation is similar for both sources: the relative radial velocities of both carbon and sulfur RRLs are the nearly the same. The table shows their respective differences to be 0.18 km s^{-1} in S87 and 0.01 km s^{-1} in W48, i.e., nearly within the measurements errors. Second, if the line belonged to silicon or magnesium or iron rather than sulfur, these differences would be 0.71, 1.64, and -2.19 km s^{-1} , respectively – greater than the quoted measurement error. The possibility of a blended line from several elements is also excluded because it would have an asymmetrical profile, which is not seen. It is then easy to conclude that the new line is indeed that of sulfur and, further, has about the same width as the carbon line.

Why do we see RRLs of sulfur rather than those of silicon, magnesium, and iron whose solar abundances are comparable with sulfur (Cameron, 1973)? A likely answer is that the other three elements have condensed onto – or, into – interstellar dust grains, thereby depleting their abundance in the gaseous phase of the ISM (Field, 1974). From the observational limits of his spectral observations, Silverglate (1984) estimated the depletion factors for Mg, Si, and Fe to be ≈ 10 for S87 and ≈ 27 for W48.

At this writing, sulfur lines have been observed in four sources listed in Table 3.4: W48, W3, NGC2024, and Orion A. These observations facilitated modeling of the S II boundary regions similar to that done for the C II regions (Vallée, 1989). The problems were the same as for the C II models. Nevertheless, electron temperatures and densities were determined for the S II regions with an uncertainty factor of about 3. The modeling found that the S II layer extends into the host molecular cloud 2–3 times further than the C II one, that its average temperature is $\approx 40 \text{ K}$, and that its average density is $\approx 6 \text{ cm}^{-3}$ (Vallée, 1989).

These experimentally determined parameters agree well with theory (Tielens and Hollenbach, 1985). Their calculations plotted in Fig. 3.31 predict that the thickness of the S II boundary layer is about twice that of the C II layer in terms of visual extinction. They also predict the temperature to decrease as a function of depth, reaching about 50 K at the middle of the S II layer.

3.3.2.2 The H^0 (H I) Layer

The spectra shown in Fig. 3.30 include one more detail of fundamental importance to our understanding of the boundary layers of discrete H II regions. Unlike most hydrogen RRLs, these profiles are asymmetrical for both sources. They consist of a narrow line blended with a broad line. This effect, first discovered by Ball et al. (1970), reveals the presence of two hydrogen recombination lines in NGC2024. One is a conventional broad hydrogen RRL from the hot H II region and the other is a narrow hydrogen RRL that must

originate in the cool gas ahead of an ionization front. Separation of the two profiles is complicated, requires some judgement, and probably cannot result in unique results. Nevertheless, these spectra establish the presence of a cool hydrogen layer associated with some discrete H II regions as an astronomical fact.

In addition to NGC2024 and W3, seven other sources exhibit these narrow hydrogen lines. These include K3-50 (Roelfsema, Goss and Geballe, 1988); DR21 (Roelfsema et al., 1989); W48, S87, S88 (Onello, Phillips and Terzian, 1991); NRAO584 (Onello and Phillips, 1995); and GGD12-15 (Gómez, Lebron, Rodríguez, Garay, Lizano, Escalante and Canto, 1998). In all of these sources, the width of the “narrow” hydrogen component ranged from 3.6 to 10.5 km s^{-1} , i.e., smaller than the widths of the normal hydrogen RRLs from the H II regions by a factor of 3 or more.

We refer to this gas as the H^0 component of the ISM. As mentioned earlier, the narrow widths of the lines and their spatial association with discrete H II regions require that they arise in very cool hydrogen gas linked to these emission nebulae. Because the lines are RRLs, this gas must contain an ionized component. Therefore, the term *partially ionized component* of the ISM is an appropriate description. Astronomically, it could also be called H I gas, although this specific spectroscopic term usually refers only to the widely distributed neutral component of the ISM that radiates the spin-flip $\lambda = 21 \text{ cm}$ hydrogen line.

The source of this H^0 component is not known but it must be connected to some characteristic of the discrete H II region, such as the exciting stars. Two solutions have been proposed for the ionization. First, the narrow hydrogen line could be generated in the ionization front of the H II region. Calculations suggest that an H^0 line of appropriate intensity could be formed in cold, $T_e \approx 100 \text{ K}$, gas at the outer side of weak D-type ionization fronts.¹⁴ These fronts would be typical for H II regions expanding into the surrounding neutral medium (Hill, 1977).

A second possibility is that a soft X-ray flux from the vicinity of the exciting stars is ionizing the neutral gas of the boundary region. This flux could arise from a stellar wind with a velocity of 500 km s^{-1} and would easily pass through the H II region itself, creating a layer of partially ionized gas ahead of the ionization front adequate to generate RRLs of the observed intensities and widths (Krügel and Tenorio-Tagle, 1978).

The observations are not definitive enough to confirm one theory over the other at this writing. Either model can be fitted to the observed values of the narrow hydrogen RRLs (Onello et al., 1991). To complicate the situation further, VLA observations of two H^0 regions led to conflicting results. One source, W48, has a small H^0 region with characteristics consistent with the ionization front model rather than the soft X-ray model (Onello, Phillips, Benaglia, Goss and Terzian, 1994). The other, S88, has an H^0 region apparently

¹⁴ “Weak D-type ionization front” means that the front propagates through a rather dense medium, causing only relatively small changes in the ambient density (Spitzer, 1978).

with an electron density of approximately 250 cm^{-3} and a depth of 0.3 pc – a few orders of magnitude larger than one might expect from the Hill’s hypothesis of weak ionization fronts (Garay, Lizano, Gómez and Brown, 1998).

The data on the physical conditions within the H^0 regions are rather poor. Based on the initial observations of narrow hydrogen lines, Pankonin (1980) concluded that conditions in such regions are probably close to those of C II regions. The width of the $\text{H}168\alpha$ narrow line in W48 indicated that the temperature of the H^0 region in this source must be $<200 \text{ K}$ (Onello et al., 1991). Based on the $\text{H}92\alpha$, $\text{H}110\alpha$, and $\text{H}166\alpha$ observations (Garay, Lizano, Gómez and Brown, 1998), the temperature of the H^0 region associated with S88 may be $\approx 800 \text{ K}$. This is a large difference in temperature.

3.3.3 The Relationship Between H II , H^0 , and Molecular Gas

The radial velocities of different interstellar components can tell us about the dynamics associated with each H II region, star-forming complex.

Table 3.6 lists the radial velocities observed for spectral lines from seven emission nebulae. These include normal RRLs from the H II gas, the so-called H^0 RRLs from the partially ionized gas, carbon RRLs from the C II gas, sulfur RRLs from the S II gas, and molecular lines from the parent cloud of the discrete H II region. The radial velocities are with respect to the LSR.

Examination of the table entries tells us immediately about the environment of these sources. First, the radial velocities of the partially ionized gas of the H^0 , C^+ , and S^+ layers *and* of the molecular lines are very nearly the same for each source region. These constituents must then be spatially associated, as considered by Tielens and Hollenbach (1985) in their analysis of PDR models.

Second, the radial velocities of the discrete H II regions themselves differ substantially from the velocities of the partially ionized gas. Depending upon the direction of motion, the hot gas moves either toward, away from, or perpendicular to the velocity of the parent cloud. From the table, we see that the sources NRAO584, S87, W3, and NGC2024 correspond to the first case, W48 and S88 correspond to the second case, and K3-50 corresponds to the third case.

These velocity differences tend to confirm the streaming of the newly ionized, hot gas from the vicinity of the stars exciting the nebulae; i.e., the hot H II gas is streaming away from the parent molecular cloud – which would have the same velocity as the newly formed stars – as described by the “champagne” evolutionary model. In the particular case of K3-50, it is also possible that the hot H II gas is not leaving the molecular cloud at all, and a champagne flow has not developed for this nebula.

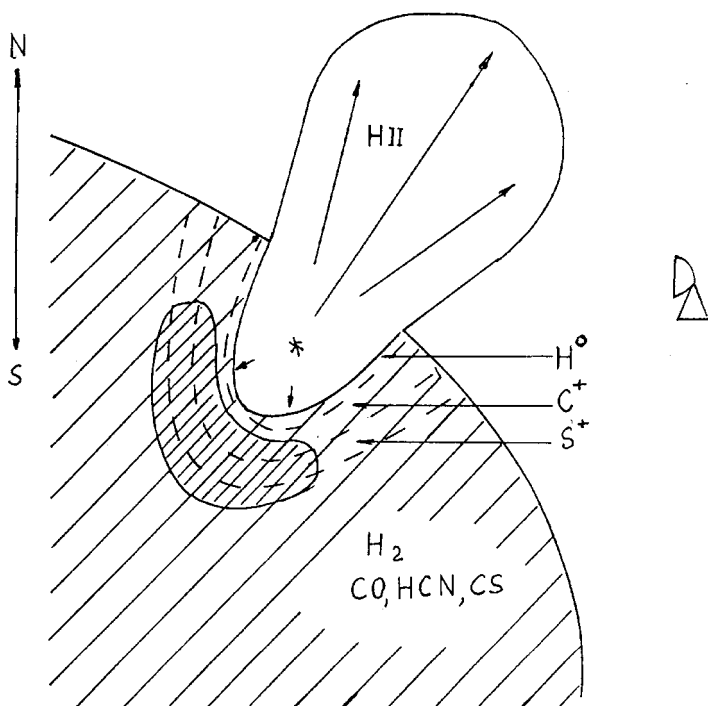


Fig. 3.33 Location and dynamics of the ionized (*bubble*), partly ionized (*dashed iso-chasms*), and molecular gas (*hatched*) components of the NGC2024 star-forming complex. Closely spaced lines indicate the dense core in the molecular cloud where protostars may be forming. An *asterisk* marks the location of the exciting star. A radio telescope shows the direction of the observer

Figure 3.33 illustrates the probable situation for NGC2024, based upon the observed radial velocities. The model is a cut through the nebula at constant right ascension. The partially ionized layers of H^0 , C^+ , and S^+ lie near the boundary of the $H II$ region and the parent molecular cloud. The outflow of $H II$ gas leaves the nebula from the side near the observer at an angle to the line of sight, as the maps of hydrogen and carbon RRLs suggest (Krügel et al., 1982).

The decrease in the radial velocities of the $Hn\alpha$ lines as n increases establishes the kinematics. At lower frequencies (longer wavelengths), the line emission comes from the outer regions of the $H II$ “fountain” because of the greater opacities. At higher frequencies where the opacity is small, the emission comes from the core region of the fountain – the nebula itself.

In principle, the radial velocities of the carbon and sulfur lines can be interpreted similarly. However, the detailed characteristics of these lines are a little different. Figure 3.33 shows these lines to originate from two layers: a far one and a near one with respect to the observer. The lines from the near

layer include emission stimulated by the background H II emission from the fountain. The brightness temperature of this continuum emission decreases with frequency (increases with wavelength), correspondingly increasing the stimulated emission in the carbon and sulfur RRLs. For this reason, the velocity decrease of the carbon and sulfur RRLs occurs at higher quantum numbers. Line emission from the layers nearest the observer plays a more important role in the line profiles.

At the highest frequency lines detected for carbon, spontaneous – not stimulated – emission dominates the line profiles. The radial velocities of these lines, C56 α and C76 α , are approximately 11 km s^{-1} and represent gas deep in the molecular cloud. These velocities also correlate well with those of the HCN, CS, and CO molecular lines because they originate in the same general region.

Using all of these radial velocities, we can determine the general dynamics of NGC2024. Expansion of its H II fountain is primarily toward the north, into the less dense ISM. The expansion of the H II region itself within the molecular cloud is either absent or very small, i.e., with the scatter of the velocity measurements of approximately 1 km s^{-1} .

There is a second possibility for interpreting the dynamics of NGC2024. Observations of the continuum emission at $\lambda = 1.3 \text{ mm}$ with an angular resolution of $10''$ reveal the presence of six condensations of cold gas and dust with a number density of $N_H \approx 10^8 \rightarrow 10^9 \text{ cm}^{-3}$. These lie in the compressed core of the molecular cloud close to the discrete H II region. Mezger (1988) suggested that these condensations have masses of about $60 M_\odot$ and could be protostars. Subsequently, the Infrared Space Observatory (ISO) confirmed the presence of these condensations by detecting in them a large number of CO molecular lines in the FIR range of $45 \rightarrow 200 \mu\text{m}$ (Giannani, Nisino, Lorenzetti, DiGiorgio, Spinoglio, Benedettini, Saraceno, Smith and White, 2000). It seems possible that sequential star formation is taking place in the host molecular cloud, perhaps triggered by the star-forming event of the NGC2024 H II region.

There are similar stories to be deduced from the other sources listed in Table 3.6. The positive or negative sign of $V_{HII} - V_{H^0}$ (or $V_{HII} - V_{C+}$) tells us whether a listed H II region lies at the far side of its parent molecular cloud and is moving away from us, or lies on the near side and is moving toward us. On the other hand, if $V_{HII} - V_{C+} > 0$ and $V_{C+} - V_{mol} < 0$, or if $V_{HII} - V_{C+} < 0$ and $V_{C+} - V_{mol} > 0$, then the expansion of the H II region is taking place deep within the molecular cloud where the exciting stars formed.

All of this information regarding partially ionized regions came from RRL observations with high angular resolution. These observations led to our understanding the location and characteristics of the C II layers with respect to the H II region ionization fronts and to the host molecular gas (Wyrowski, Schilke, Hofner and Walmsley, 1997). These relationships are particularly apparent in the Orion nebula complex. There, the discrete H II region (Orion A) was formed in the near side of the molecular cloud, having a cup-like shape

with its symmetry axis slightly inclined to the direction of the observer. It penetrates about 0.6 pc into the molecular cloud (Hogerheijde, Jansen and van Dishoeck, 1995). At about 2' southeast of the star¹⁵ θ^1 C Ori in the region of the Orion Bar, the plane of the C II layer surrounding the discrete H II region is almost parallel to the line of sight. The C II layer, the ionization front, and the boundary with the molecular cloud are seen almost edge on. This geometry is ideal for observations that can test the model.

Figure 3.34 shows just such observations. This image juxtaposes the $\lambda = 3.5$ cm C91 α and free-free continuum emission against the $^{13}\text{CO}(3 \rightarrow 2)$ emission from the molecular gas. The $\lambda = 3.5$ cm emission has an angular resolution of 10"; the CO emission, 20".

There is a distinct spatial separation between the ionized H II gas (the continuum), the partially ionized C II gas (the C91 α emission), and the molecular gas (CO emission). The distance between the ionization front and the center of the C II emission is about 20" (0.05 pc), which is approximately the same distance between the C II region and the molecular gas. The thickness of the C II layer is about 28" (0.07 pc).

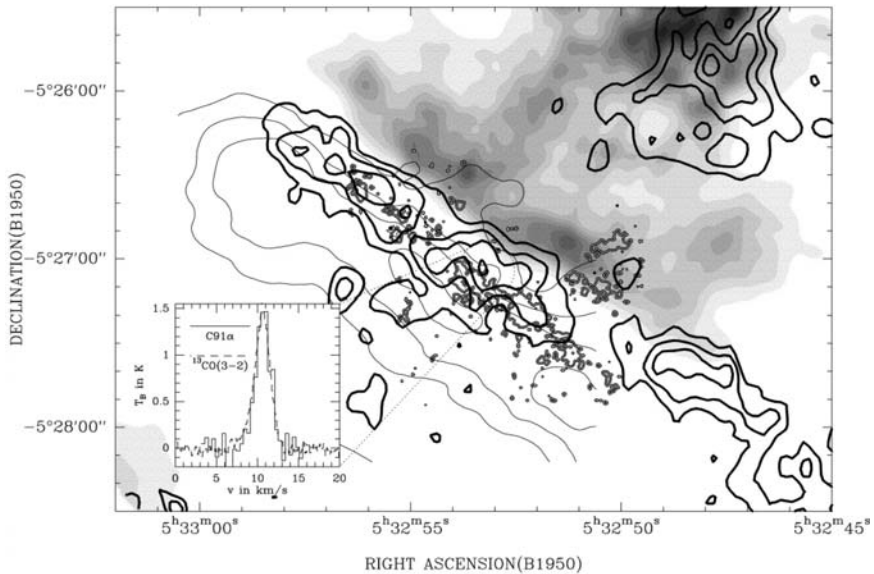


Fig. 3.34 A comparison of continuum emission, carbon RRL emission, and CO molecular emission in the vicinity of the “Orion Bar.” The *gray scale* marks the $\lambda = 3.5$ cm emission observed with the VLA. The *thick lines* mark the 30, 50, 70, and 90% contours of the peak intensity of 5.5 K km s^{-1} of the C91 α line. The *thin lines* mark the $^{13}\text{CO}(3 \rightarrow 2)$ emission in the immediate vicinity of the bar (Lis et al., 1997). The *gray lines* mark contours (small, busy ones) of $\text{H}_2(1 \rightarrow 0 \text{ S}(1))$ emission (van der Werf et al., 1996). The *inset* at the bottom left compares a C91 α line (smoothed to 20") with $^{13}\text{CO}(\text{peak } 29 \text{ K})$ emission toward the position in the Orion Bar marked with *dotted lines*. From Wyrowski et al. (1997)

¹⁵ The stellar coordinates ($\alpha_{1950}, \delta_{1950}$) are ($5^{\text{h}}32^{\text{m}}49^{\text{s}}, -5^{\circ}25'16''$).

From these observations, we can calculate the column density of the C II region. If the number density of hydrogen ranges from 5×10^4 to $2.5 \times 10^5 \text{ cm}^{-3}$ (Wyrowski et al., 1997), the corresponding column density of the C II layer, $N_{\text{H}L}$, has to range from 10^{22} to $5 \times 10^{23} \text{ cm}^{-2}$. Such values correspond to an opacity range of $A_V = 5 \rightarrow 25$, which exceeds the range calculated in the PDR model shown in Fig. 3.31.

What could be the problem? The answer lies in the model used for the calculations. The real PDR medium is inhomogeneous, whereas the theoretical calculations assumed a homogeneous one (Stutzki, Stacey, Harris, Jaffe and Lugten, 1988; Howe, Jaffe, Genzel and Stacey, 1991). The inhomogeneities allow the stellar UV radiation, causing the molecular dissociation and the ionization of carbon to penetrate considerably further into the molecular cloud. The inset within Fig. 3.34 shows a nearly perfect correspondence between the profiles of the C91 α and ^{13}CO line emission, implying that these constituents are well mixed in the PDR (Wyrowski et al., 1997).

VLA observations of hydrogen, carbon, and sulfur RRLs from S88 provided some interesting results. At angular resolutions ranging from $3''$ to $10''$, these observations showed the C II source region to consist of two components of dimensions $6''$ and $16''$, sandwiched between the molecular cloud and two ionized hydrogen regions. Evidently, two discrete H II regions were formed nearly simultaneously in S88 and are now in the champagne phase in which ionized gas is leaving the host molecular cloud (Garay, Gómez, Lisano and Brown, 1998; Garay, Lizano, Gómez and Brown, 1998).

Similar VLA observations of NGC2024 but with a larger angular resolution of about $50''$ have produced equally interesting results. These showed that the C II and H 0 regions overlap spatially. The intensity peaks of the C166 α and the narrow H166 α lines coincide with the location of maximum free-free continuum emission, thereby confirming the importance of stimulated emission in the line formation process (Anantharamaiah, Goss and Dewdney, 1990).

To summarize, RRLs – mainly from carbon – have provided information crucial to our understanding the interfaces (PDRs) between discrete H II regions and their host molecular clouds. The characteristics of these PDRs correspond well to the theoretical model produced by Tielens and Hollenbach (1985), modified to account for inhomogeneities in the ambient ISM. Taken together, the observations and the model have provided us with a well-grounded picture of the structure and interconnection of these transition regions.

However, the observations still need to improve to allow refinement of the models for the gas interfaces. Table 3.4 cites values of temperature and density for C II regions that have uncertainties of multiples of the observed values.

Is it possible to measure these parameters more precisely and more accurately? Which parameters have the dominant effect upon making more detailed models of the PDRs? The answers to these questions appeared only recently, and we discuss them in the next section.

3.3.4 Physical Conditions from Carbon RRLs, IR Fine-Structure Lines of C⁺, and O I Lines

The difficulty in analytically determining the physical conditions in C II regions from carbon RRLs alone rests upon one important restriction. Their intensities are functions of *three* unknown quantities: the temperature, local volume density, and column density of C⁺ ions along the lines of sight. This multiparameter dependence is the reason astronomers build models which they fit to imperfect observations to derive the physical characteristics of the PDRs.

Incorporating data from other observables could simplify the process and greatly increase the accuracy of the results. For example, we could include data derived from the fine-structure line of carbon ions, $^2P_{3/2} \rightarrow ^2P_{1/2}$, at a wavelength of 158 μm in the IR. The probable existence of this line toward some H II regions became evident after the detection of the carbon RRL (Palmer et al., 1967), and it was detected soon after the construction of an appropriate high-resolution IR spectrometer (Russell, Melnick, Gull and Harwit, 1980). Both line types involve the same carbon ions. After recombination into highly excited levels, the subsequent cascades result in RRLs while transitions between fine-structure levels emit the 158 μm line. The importance here is that, while both lines originate from the same region, their intensities have different dependencies on the ambient physical conditions.

The physical conditions transmitted by these lines can be easily determined. In the optically thin case, the intensity of the 158 μm line is proportional to the *first* power of the density, whereas the intensity of the RRL is proportional to the *square* of the density. Consequently, the ratio of the line intensities enables us to determine the local hydrogen density as a function of temperature in the emission region (Natta, Walmsley and Tielens, 1994; Smirnov, Sorochenko and Walmsley, 1995; Wyrowski et al., 1997).

In the general case with no assumptions as to the optical depth, the mathematics are somewhat complicated. Smirnov et al. (1995) give the hydrogen density as a function of temperature to be

$$N_H = \frac{2.33 \times 10^5 \Delta\nu_L T_L \alpha_{1/2} \beta_{158} T^{1.5} \exp(-1.58 \times 10^5/n^2 T)}{b_n \Delta\nu_{158} \left(1 - \frac{\beta T_{bg}}{T}\right) \ln \left[\frac{\exp(91.2/T_{158}) - 1}{\exp(91.2/T_{158}) - \exp(91.2/T_{ex,158})} \right]}, \quad (3.13)$$

where the parameters $\Delta\nu_L$ and T_L are the full width at half-intensity and the brightness temperature of the carbon RRL in Hz and K, respectively, and $\Delta\nu_{158}$ and T_{158} are similar values for the carbon 158 μm fine-structure line. The background temperature $T_{bg} = 3.55 + T_C$ includes the continuum emission from the H II emission behind the C II region. The b_n and β factors correct the quantum population levels of carbon for departures from LTE, including the effects of dielectronic recombination (Ponomarev and Sorochenko, 1992) extended to higher values of T and N_H .

In addition, the equation contains parameters defined as

$$\alpha_{1/2} = \frac{1 + R_{158}}{(1 + R_{158}) + 2 \exp(-91.2/T)}, \quad (3.14)$$

$$\beta_{158} = 1 - \frac{\exp(-91.2/T)}{(1 + R_{158})}, \quad (3.15)$$

and

$$T_{ex,158} = \frac{91.2T}{91.2 + T \ln(1 + R_{158})}, \quad (3.16)$$

which, respectively, determine the fraction of carbon atoms in the lower level, $^2P_{1/2}$, relative to their total number density N_{C+} ; the correction for stimulated emission; and the excitation temperature of the fine-structure levels.

All of the parameters above depend upon R_{158} , which is defined as the ratio of the radiative transition rate to the deactivation rate of the $^2P_{3/2}$ level by collisions with electrons, hydrogen atoms, and molecules. Considering that collisions with hydrogen atoms are the dominant deactivation process, we can write

$$R_{158} = \frac{A_{3/2-1/2}}{N_H \gamma_H} = \frac{4.1 \times 10^3}{N_H T^{0.02}}, \quad (3.17)$$

where $A_{3/2} = 2.4 \times 10^{-5} \text{ s}^{-1}$ is the probability of spontaneous transitions between the $^2P_{3/2}$ and $^2P_{1/2}$ levels, and $\gamma_H = 5.8 \times 10^{-10} T^{0.02}$ is the deactivation rate of the $^2P_{3/2}$ level by collisions with hydrogen atoms (Tielens and Hollenbach, 1985).

The derivation of (3.13) assumes that $N_C = 3 \times 10^{-4} N_H$, that all the carbon in the CII region is completely (but singly) ionized, and that all electrons result from this ionization, i.e., $N_C = N_{C+} = N_e$.

There was one small observational obstacle that had to be overcome. The spectral resolution of the IR spectrograph was too small to resolve the $158 \mu\text{m}$ fine-structure line adequately. Only the integral of the line profile could be measured. Assuming the line width to be identical to that of the carbon RRL, Sorochenko and Tsvilev (2000) derived the brightness temperature of the fine-structure line in K from

$$T_{158} = \frac{91.2}{\ln(1 + 1.08 \times 10^{-10} \Delta\nu_{158}/I_{158})}, \quad (3.18)$$

where they applied a slight correction for broadening as a consequence of the optical depth τ_{158} .

There, remaining parameter required for evaluation of (3.13) is the gas temperature T . This can be obtained from our third source, the OI lines. In the PDR, oxygen exists as neutral atoms because its ionization potential of 13.62 eV slightly exceeds that of hydrogen, i.e., most of the photons below that energy ionize hydrogen. Figure 3.31 shows that oxygen atoms begin to bind into CO molecules only when the visual extinction $A_V > 4$. The ground

state of atomic oxygen is split into three fine-structure levels: 3P_2 (lower), 3P_1 , and 3P_0 (upper). Therefore, oxygen atoms radiate the fine-structure lines $^3P_1 \rightarrow ^3P_2$ and $^3P_0 \rightarrow ^3P_1$ at 63 and 146 μm , respectively, which can be observed in the IR.

The fine-structure lines of carbon and oxygen have greatly different dependencies on density, which makes them useful for determining the density of the PDR. The cross section for collisional excitation of the C^+ fine-structure levels by hydrogen atoms (Launay and Roueff, 1977a) is one to two orders of magnitude greater than those of O I (Launay and Roueff, 1977b). Consequently, for PDR environments where the density $N_H \geq 3 \times 10^4 \text{ cm}^{-3}$, the fine-structure levels of carbon are already thermalized and the intensity of the carbon 158 μm line is independent of density. In contrast, there is no thermalization of the O I IR lines under such conditions; their intensities are direct functions of the hydrogen density (Tielens and Hollenbach, 1985). Therefore, the ratios $\text{I}(\text{C II } 158 \mu\text{m})/\text{I}(\text{O I } 63 \mu\text{m})$ and $\text{I}(\text{C II } 158 \mu\text{m})/\text{I}(\text{O I } 146 \mu\text{m})$ depend only on the density and can be used to resolve ambiguities regarding the physical conditions within the C II layers of the PDR.

For comparison with the fine-structure lines, it is best to use carbon RRLs at frequencies above 20 GHz ($\lambda < 1.5 \text{ cm}$). In this range, the opacity is small and stimulated emission (the $\beta T_{bg}/T$ term in (3.13)) can be neglected. This approximation substantially simplifies the interpretation of the observations. Figure 3.35 shows the RRL spectra used for the analysis. These include H, He, and $\text{C } 56\alpha$ lines observed toward NGC2024, W3, and the Orion Bar region of the Orion nebula (Sorochenko and Tsivilev, 2000).

The corresponding fine-structure lines came from observations made from the Kuiper Airborne Observatory (KAO). These are the C^+ 158 μm line from NGC2024 (Jaffe, Zhou, Howe and Stacey, 1994), the O I 63 μm line from NGC2024 (Luhman, Jaffe, Sternberg, Herrmann and Poglitsch, 1997), and the O I lines at 158 and 146 μm from W3 (Howe, 1999) and from the Orion Bar (Herrmann, Madden, Nikola, Poglitsch, Timmermann, Geis, Townes and Stacey, 1997).

Results. Figure 3.36 shows the intensity ratios plotted as a function of volume density and temperature for the nebulae NGC2024 and W3. The ratios for $\text{C } 56\alpha/\text{C II}(158)$ as a function of N_H and T resulted from (3.13). The other ratio, $\text{C II}(158)/\text{OI}(63)$ for the case of NGC2024, was calculated iteratively from the observations of $\text{I}(\text{O II } 158 \mu\text{m})/\text{I}(\text{O I } 63 \mu\text{m}) = 0.4$. A similar calculation was performed for the ratio $\text{C II}(158)/\text{OI}(146)$ for W3. Sorochenko and Tsivilev (2000) give details of these calculations.

To summarize: The essence of this analysis is that these spectral lines have different dependencies upon temperature and density.

RRLs are based upon an ion–electron recombination process, for which the line intensities (related to the populations of the bound levels) are proportional to the square of the density – as noted above. Furthermore, since the partition of electrons between bound and unbound states is a function of temperature, the level populations and the line intensities decrease as

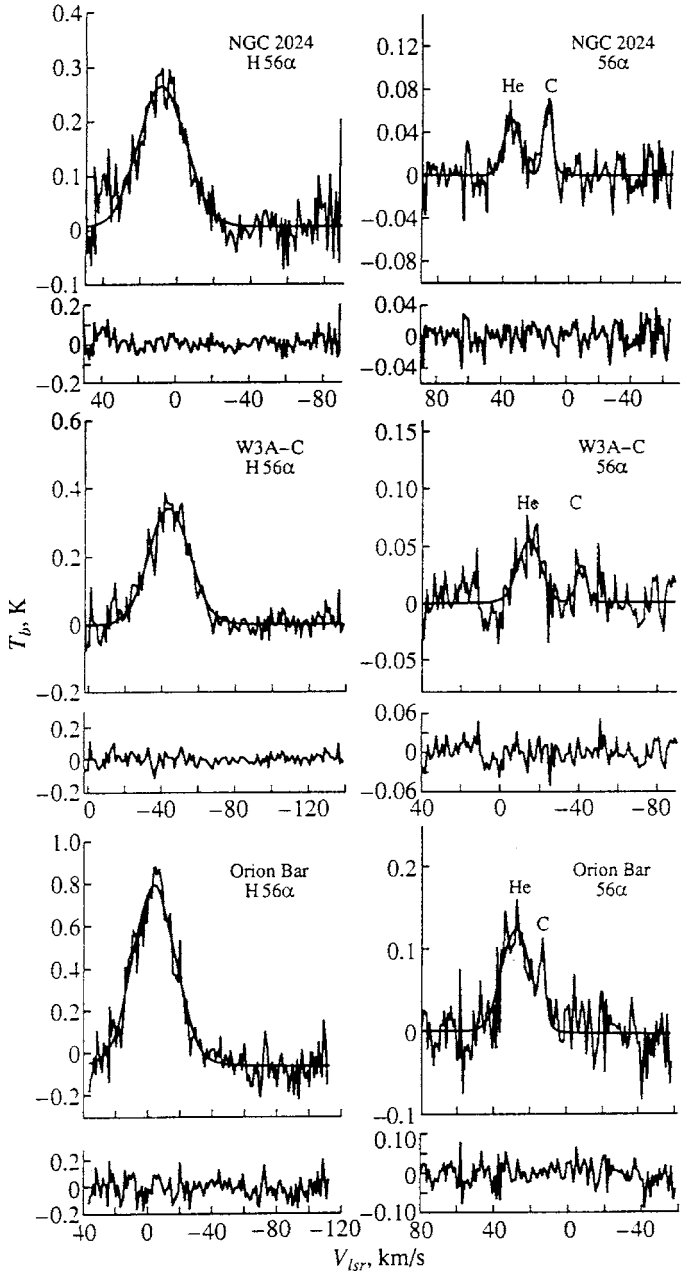


Fig. 3.35 56α lines of hydrogen, helium, and carbon observed with the 22-m telescope at Pushchino, Russia. *Smooth lines* indicate the Gaussians fitted the lines. The spectra below the lines gives the residuals from the fits. From Sorochenko and Tsvilev (2000)

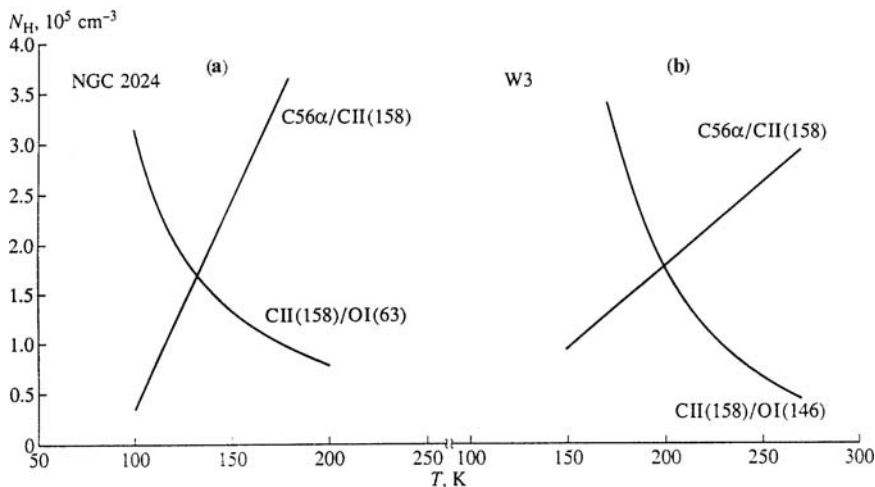


Fig. 3.36 Ratios formed from the observed intensities of C56 α RRLs and the IR fine-structure lines of carbon 158 μm , O I 146 μm , and O I 63 μm plotted against volume density (N_H) and temperature. Their *intersections* mark the solutions for the temperature and density of the C II PDRs in the nebulae NGC2024 and W3

temperature increases, actually $\propto T^{1.5}$. These RRL dependencies on N_e and T are shown by the Saha-Boltzmann relationship of (2.113). On the other hand, the intensities of the fine-structure lines *increase* with temperature. Therefore, the intensity ratio C56 α /C II(158) has a positive slope in Fig. 3.36; higher densities correspond to higher temperatures.

The intensity ratio of the IR fine-structure lines of carbon and oxygen has a different dependence on density and temperature. Both IR lines have their levels populated by collisions with hydrogen atoms. For this reason, their intensities increase with temperature (more frequent collisions) and density (more colliders). However, the different physical “sizes” of C and O atoms mean different excitation rates: slower for carbon and faster for oxygen. Consequently, a I(C)/I(O) ratio will have a negative slope in Fig. 3.36.

Because of these different behaviors, the two ratio curves of Fig. 3.36 intersect, enabling the fixing of the density and temperature of the C II region within a PDR. Table 3.7 gives the results (Sorochenko and Tsivilev, 2000). The table also gives results for the source S140/L1204, where IR observations were compared with the C165 α (C166 α) lines. The first three columns on the left give the characteristics of the discrete H II regions and the exciting star, and the columns on the right give the lines used and the results derived from them for the abutting C II regions.

The physical conditions for the C II regions derived in Table 3.7 generally agree with those obtained earlier in Table 3.4, obtained exclusively from RRLs. However, the parameters obtained by comparing RRLs and IR

Table 3.7 Characteristics of C II regions

| Source | H II region | | C II region | | | | |
|------------|--------------|--------------------------------|---------------|-------------------------|------------|-------------------------------|-------------------------------|
| | Stellar type | N_e (cm^{-3}) | RRL | IR (μm) | T (K) | N_e (cm^{-3}) | N_H (cm^{-3}) |
| Orion Bar | O6 | 10^4 ^a | C56 α | 158, 146 | 215 | 39 | 1.3×10^5 |
| W3 | O5–O6 | 1.7×10^4 ^b | C56 α | 158, 146 | 200 | 54 | 1.8×10^5 |
| NGC2024 | O9.5 | 1.9×10^3 ^c | C56 α | 158, 63 | 132 | 51 | 1.7×10^5 |
| S140/L1204 | BO V | 10^d | C165 α | 158, 63 | 67–85 | 3 | $\approx 10^4$ |

^aSmirnov et al. (1984), ^bColley (1980), ^cBerulis and Sorochenko (1973), and ^dSmirnov et al. (1995)

fine-structure lines are much more accurate. Sorochenko and Tsivilev (2000) concluded that the uncertainties from the newer method are about 20–30% for temperature and 30–50% for density.

Even with only four regions, the data in Table 3.7 reveal new information regarding the C II regions. Higher temperatures should be expected for C II regions lying at the boundaries of H II regions excited by earlier-type (hotter) stars. This result seems reasonable. The early stars would have a harder UV emission that, in turn, would provide more energetic photons entering the surrounding PDR.

Finally, these data generally agree with the theory of PDRs (Tielens and Hollenbach, 1985). According to this theory, the temperature of the C II layer should decrease from about 1,000 K at the boundary with the H II region to ≤ 100 K at $A_V \approx 4$ near the outer boundaries of the layer. The carbon RRLs are actually averaged over a range of depths of the C II layers and, therefore, give average temperatures. The only discrepancy with theory seems to be the linear dimensions – and that is probably the result of the simplifying but unrealistic assumption of a homogeneous medium within the PDR.

3.3.5 Carbon RRLs from Atomic and Molecular Clouds

C II regions occur not only in complexes of discrete H II regions but also in cold clouds in the ISM that are exposed to the general interstellar UV radiation field. There, the flux density of the UV is much less and the correspondingly weaker intensity of the carbon RRLs in these regions makes them more difficult to observe. Nonetheless, astronomers have been able to detect the carbon RRLs toward a few strong background sources.

3.3.5.1 Cold Interstellar Clouds Observed Toward Cassiopeia A

The radio source Cassiopeia A or, simply, Cas A is an interesting object. Lying about 2.8 kpc from the Sun in the plane of our Galaxy, it is a remnant of a supernova that occurred in the latter part of the seventeenth century. It is an intense source of radio waves.

For us, however, Cas A is an important tool for determining the characteristics of the interstellar gas. Serendipitously, cold interstellar clouds lie along the line of sight to this object. In fact, the majority of information about C II regions in cold interstellar clouds was obtained by observing this cosmic radio beacon, much like determining the characteristics of fog by viewing a distant street light. Carbon RRLs have been detected toward this object over a wide range of frequencies and principal quantum numbers, from $n = 766 \rightarrow 166$ and from 15 MHz to 1.5 GHz – a frequency range of two decades (Konovalenko, 1990; Sorochenko and Walmsley, 1991).

Figure 3.37 shows two examples of carbon spectra obtained toward Cas A. The top spectra are two components of the C221 α line ($\nu \approx 600$ MHz) in emission

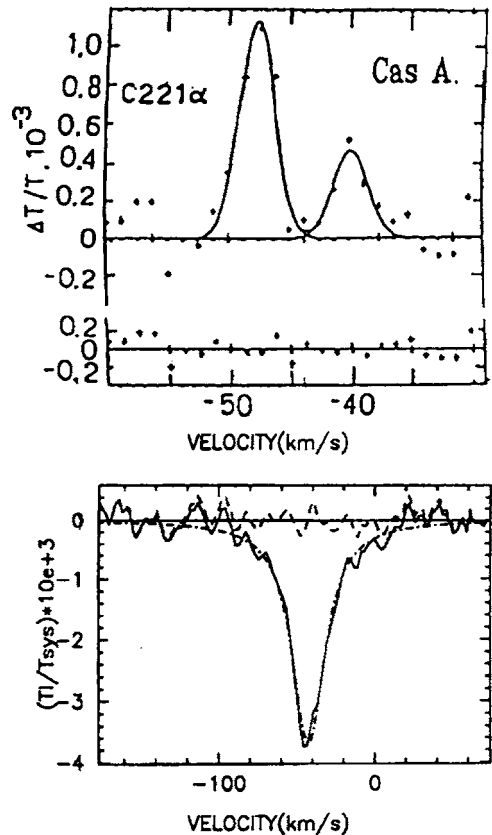


Fig. 3.37 *Top*: two components of the C221 α line ($\nu \approx 600$ MHz) in emission obtained toward Cas A with the RT-22 radio telescope at Pushchino, Russia (Sorochenko et al., 1991). The integration time was about 170 h. Residuals from the Gaussian fits are shown immediately below the spectrum. *Bottom*: the average of eight transitions (C571 $\alpha \rightarrow$ C578 α at $\nu \approx 34.5$ MHz) in absorption toward Cas A obtained with the E-W arm of the T-shaped radio telescope at Gauribidanur, India (Kantharia et al., 1998). The total integration time for one transition was about 400 h. The *broken line* shows the best-fitting Voigt profile and the *dashed line* shows the residuals after fitting. The *abscissae* are V_{LSR}

emission at the radial velocities -40 and -48 km s^{-1} . The bottom spectrum is the average of eight transitions ($\text{C}571\alpha \rightarrow \text{C}578\alpha$ at $\nu \approx 34.5 \text{ MHz}$) seen in absorption. The population of the quantum levels of carbon determines whether the lines appear in emission or absorption, as described in Sect. 2.4.2. At low frequencies, the two velocity components blend into a single Voigt absorption profile. In general, the line width of the carbon α -type RRLs toward Cas A increases with n . Figure 3.38 clearly shows this relationship in which all known observations have been plotted.

One has to be careful when comparing a two-component emission spectrum with a single-component absorption spectrum. In Fig. 3.38, the widths of these carbon RRLs were referenced to the width of the emission component at $V_{LSR} = -48 \text{ km s}^{-1}$ seen in Fig. 3.37 and measured to be 3.7 km s^{-1} . Because the absorption spectra are a blend of the two components seen in emission, the widths of these absorption blends were decreased by the separation of the emission components, 8 km s^{-1} to facilitate comparison with the emission profiles. The final data set includes spectra collected Sorochenko and

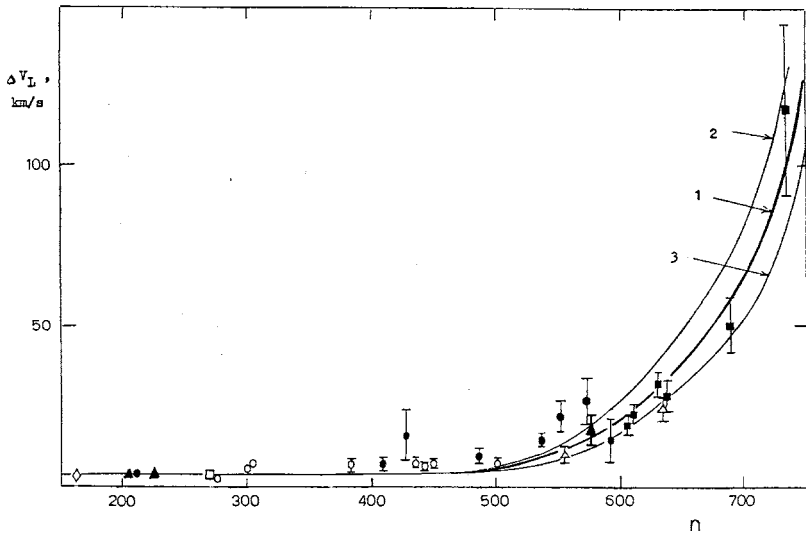


Fig. 3.38 Width of carbon RRLs toward Cas A plotted against principal quantum number. *Filled squares and circles* show respective data from Kharkov (Konovalenko, 1984) and Pushchino (Ershov et al., 1984; Ershov et al., 1987; Lekht et al., 1989; Sorochenko et al., 1991; Kitaev et al., 1994). *Open triangles* (Anantharamaiah et al., 1985) and *open circles* (Payne et al., 1989) represent observations from Green Bank, *open squares* represent observations from the VLA (Anantharamaiah et al., 1994), and *diamonds* from Effelsberg (Sorochenko and Walmsley, 1991). The data on the C640 α line have been corrected (Konovalenko, 1995). *Curve 1* shows the widths $\Delta V_L(n)$ calculated for $N_e(T_e/100)^{0.62} = 0.1$, and *curves 2 and 3* show the same but with $N_e(T_e/100)^{0.62}$ increased and decreased by a factor of 1.5, respectively. From Sorochenko (1996) (recent observational data in three frequency ranges (Kantharia et al., 1998) have been added to this figure as *filled triangles*)

Smirnov (1990) augmented by more recent spectra on the lines C165 α and C166 α (Sorochenko and Walmsley, 1991), C220 α (Sorochenko et al., 1991), C270 α (Anantharamaiah et al., 1994), C537 α \rightarrow C540 α (Kitaev et al., 1994), C201 α \rightarrow C206 α , C223 α \rightarrow C229, and C571 α \rightarrow C578 α (Kantharia et al., 1998).

Following (2.72), we can describe the full velocity width of the Voigt line profile at half-intensity as

$$\Delta V_V = 0.53L + \sqrt{0.22L^2 + D^2} \quad \text{km s}^{-1}, \quad (3.19)$$

with an accuracy better than 1%. Here, L and D are the full widths of the Lorentz and Doppler components of the profile at half-intensity. The Lorentz component itself consists of two parameters:

$$L = (\delta\nu_{col} + \delta\nu_{em}) \frac{3 \times 10^5}{\nu} \quad \text{km s}^{-1}, \quad (3.20)$$

where $\delta\nu_{col}$ and $\delta\nu_{em}$ represent the broadening contributed by collisions and emission, respectively. In the conditions of the cold ISM, (2.160) and (2.181) give these parameters as

$$\delta\nu_{col} = 1.16 \left(\frac{n}{100} \right)^{5.1} \left(\frac{T_e}{100} \right)^{0.62} N_e, \quad \text{Hz} \quad (3.21)$$

and

$$\delta\nu_{em} = 3.1 \times 10^{-2} \left(\frac{n}{100} \right)^{5.65}, \quad \text{Hz}, \quad (3.22)$$

where n , T_e , and N_e are the lower principal quantum number of the line, the electron temperature in K, and the electron density in cm^{-3} , respectively.

The best agreement of the measured and observed values occurs at

$$N_e \left(\frac{T_e}{100} \right)^{0.62} = (0.1 \pm 0.02) \quad \text{cm}^{-3} \text{K}^{0.62}. \quad (3.23)$$

Figure 3.38 shows these values as a thick line.

Observations of the width of the carbon RRLs over a broad range of frequency give important information about the nature of C II regions in the general ISM. Unfortunately, they do not give sufficient information to determine the physical conditions within these regions. Equation (3.23) provides only a functional relationship between electron temperature and density of the C II regions.

One way to specify conditions in the C II regions is to make use of the intensities of the carbon RRLs observed toward Cas A. Two extreme models have been considered: “cold” ones with $T_e = 16 \rightarrow 20$ K and $N_e = 0.27 \rightarrow 0.4 \text{ cm}^{-3}$ and “warm” ones with $T_e = 50 \rightarrow 100$ K and $N_e = 0.05 \rightarrow 0.15 \text{ cm}^{-3}$ (Walmsley and Watson, 1982; Ershov et al., 1982; Ershov et al., 1984; Er-

shov et al., 1987; Konovalenko, 1984; Anantharamaiah et al., 1985; Payne et al., 1989). A “cold” model with $T_e = 18$ K and $N_e = 0.3 \text{ cm}^{-3}$ and a “hot” model with $T_e = 50$ K and $N_e = 0.15 \text{ cm}^{-3}$ both gave satisfactory agreement between the calculated values and the observational data.

The details of these models are somewhat different. The “cold” model assumed hydrogen-like carbon atoms with the usual hydrogen-like mechanisms of populating the quantum levels. In contrast, the “hot” model included dielectronic recombination of carbon atoms, described in Sect. 2.4.2 (Sorochenko and Smirnov, 1990).

Distinguishing these models required the expansion of the carbon RRL observations into the decimeter wavelength range. There, the intensity of the lines is a strong function of the temperature of the C II regions.

Figure 3.39 compares the calculated and observed values of the integrated line-to-continuum ratios for four models. The same data shown in Fig. 3.38 have been used but with corrections to the intensities of the C603 α , C611 α , C621 α , and C640 α lines (Konovalenko, 1984; Payne, Anantharamaiah and Erickson, 1994). In the formation of carbon RRLs along the line of sight to a strong background source like Cas A, stimulated transitions play a major role in the observed intensity of the lines. Specifically, the rightmost term of (3.12) becomes important. The ratio of the integrated line to the underlying continuum emission is

$$\int \frac{T_L}{T_C} d\nu = - \int \tau_L d\nu = - \frac{2.0 \times 10^6 EM_{CII} b_n \beta}{T_e^{5/2}} \text{ Hz}, \quad (3.24)$$

where EM_{CII} is the emission measure of the C II region and $\tau_L = \tau_L^* b_n \beta = k_L^* \ell b_n \beta$ is the line optical depth, where k_L^* is the LTE line absorption coefficient derived from 2.116 and ℓ is the thickness of the C II region.

The solid curves of Fig. 3.39 show curves for the line ratio calculated for temperatures of 25, 50, and 75 K and corresponding densities derived from (3.23). The values of b_n and β include dielectronic recombination (Ponomarev and Sorochenko, 1992). All curves are referenced to the observations of the C537 α –C540 α lines, where the measurement error is small owing to an integration time of 1,224 h (Kitaev et al., 1994).

The calculations give a solution for the characteristics of the C II regions along the sight line to Cas A. The best agreement of observations and calculations occurs for $T_e = 50$ K and $N_e = 0.15 \text{ cm}^{-3}$ – thereby excluding the “cold” models. With these conditions, the emission measure for the C II region, EM_{CII} , is $1.7 \times 10^{-2} \text{ cm}^{-6} \text{ pc}$. The critical observations determining the most appropriate model were the high-frequency carbon RRLs: C165 α and C166 α (Sorochenko and Walmsley, 1991); C220 α (Sorochenko et al., 1991); C270 α (Anantharamaiah et al., 1994); and C300 α –C303 α , C308 α , and C310 α lines (Payne et al., 1994). As before, the additional observations involving $n \approx 205$, 225, and 575 (Kantharia et al., 1998) have been added to the figure and agree with the solution.

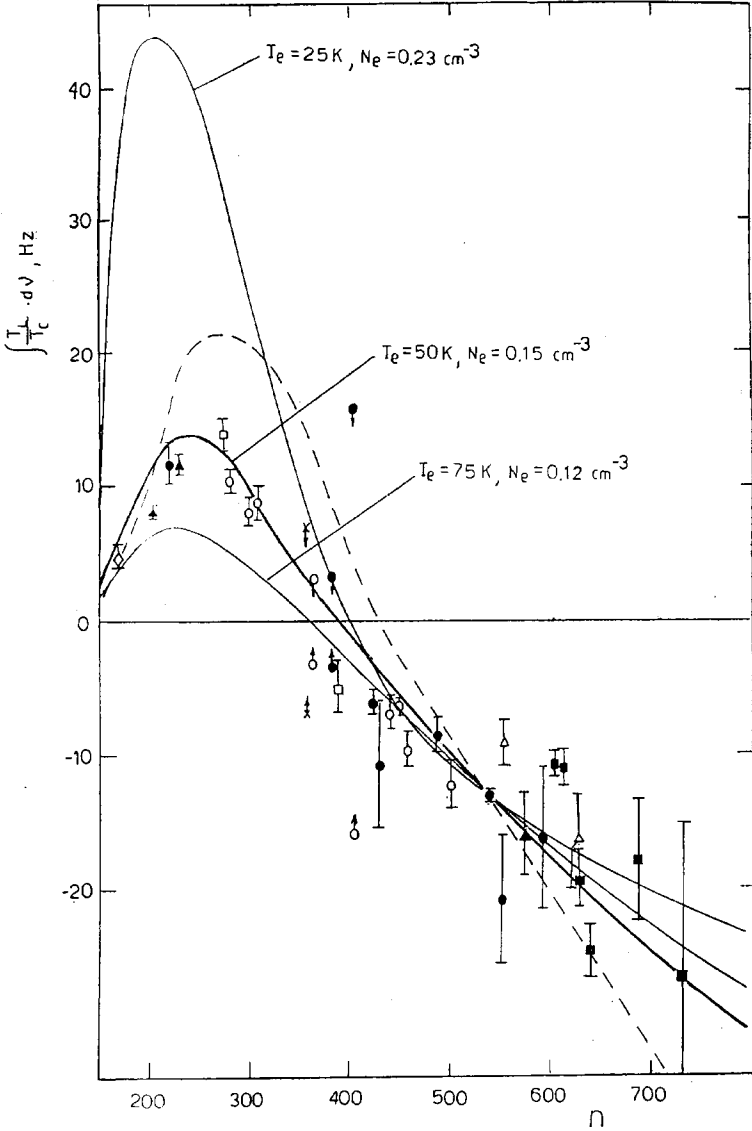


Fig. 3.39 The observed ratios of the line integral to the continuum toward Cas A compared with values calculated from different models. The emission profiles of two components at $V_{LSR} \approx -40 \text{ km s}^{-1}$ and $V_{LSR} \approx 48 \text{ km s}^{-1}$ have been summed. The *solid curves* indicate models which do not consider the influence of Cas A radiation on the width of the carbon lines. The *dashed curve* indicates the model where $T_e = 50 \text{ K}$ and $N_e = 0.05 \text{ cm}^{-3}$, and $I_{Cas} = 0.83 I_{bgr}$

Despite this great success in fitting the observations of the $Cn\alpha$ RRLs from the general ISM, the theory could still be improved. In the calculations, the only radiation field considered for line broadening was the nonthermal

background radiation, I_{bgr} , involved in the derivation of broadening parameter given by (3.22). At 100 MHz, this isotropic radiation has a brightness temperature $\approx 1,000$ K, varying $\propto \nu^{-2.55}$. However, along the line of sight toward Cas A, clouds lying close to it would be exposed to the additional nonthermal radiation from that supernova remnant, which would be significant at low frequencies. Low-frequency carbon RRLs from these clouds would have additional radiation broadening, the required broadening by electron collisions would be less, and the analysis described above would overestimate the derived value of the electron density (Payne et al., 1994; Kantharia et al., 1998).

Kantharia et al. (1998) examined this effect in more detail. At the temperature $T_e = 75$ K, they considered three combinations of radiation intensity (I) and density: $N_e = 0.15 \text{ cm}^{-3}$ and $I_{Cas} = 0$, $N_e = 0.11 \text{ cm}^{-3}$ and $I_{Cas} = I_{bgr}$, and $N_e = 0.02 \text{ cm}^{-3}$ and $I_{Cas} = 3I_{bgr}$. Despite better agreement of the first combination with the observations, they preferred the third combination because the gas pressure within the C II regions better conformed to the pressure of the surrounding ISM. For example, the first combination would give a gas pressure¹⁶ of $P/k = N_H \cdot T_e = 3.75 \times 10^4 \text{ K cm}^{-3}$, which about one order of magnitude larger than the value of $3,700 \text{ K cm}^{-3}$ proposed for the ISM by McKee and Ostriker (1977). In contrast, the third combination produced a gas pressure of $5,000 \text{ K cm}^{-3}$. To resolve this disagreement, Kantharia et al. (1998) suggested that additional refinements to the cloud models might produce a pressure that agreed better with the $3,700\text{-K cm}^{-3}$ value.

To achieve the $3,700\text{-K cm}^{-3}$ pressure, one might imagine that it would be possible to increase the contribution of the Cas A radiation to the carbon line broadening to allow a corresponding reduction of the density of the C II region models and, correspondingly, of the pressure (see $\Delta V_L = f(n)$ in Fig. 3.38).

But, it is not possible. Collision and radiation broadening have similar dependencies on quantum number and cannot be separated given the measurement errors of the observations, but the intensity of the carbon RRL is a tight function of the electron density. Decreasing N_e results in increasing the β_n coefficient and its dependence on the principal quantum number n , which changes the carbon line intensity in the wrong sense.

One curve in Fig. 3.39 does include emission from Cas A in the radiation broadening of the carbon lines. This model is not a particularly good fit to the data. Its radiation component was taken to be $I_{Cas} = 0.83I_{bgr}$, which corresponds to locating the C II cloud about 150 pc from the supernova remnant. The cloud characteristics were assumed to be $T_e = 50$ K and $N_e = 0.05 \text{ cm}^{-3}$. Inspection shows that these conditions cause this model to overestimate the line emission at quantum numbers near $n \approx 200 \rightarrow 300$ and, although less strikingly, to underestimate the emission for $n \geq 650$. Furthermore, the curve crosses the 0 point on the ordinate (the transition from emission to absorption) at a value of n greater than that suggested by the observations.

¹⁶ The assumption is that $N_e = N_{c+} = 3 \times 10^{-4} N_H \text{ cm}^{-3}$ and that all of the carbon is in the gas phase.

There is also a practical restriction to the density – the observations. At most, the observations would allow a minimum density of 0.1 cm^{-3} within the observational errors, as shown by Fig. 3.39. This density is equivalent to setting the minimum distance of the C II region toward Cas A to about 250 pc, compared with the known 2.8-kpc distance of Cas A from the Sun. The value $N_e = 0.05 \text{ cm}^{-3}$ is inconsistent with these observations.

At the temperature of 50 K and electron density of 0.15 cm^{-3} , the thermal pressure of the C II region is $P/k = T_e N_H = 2.5 \times 10^3 \text{ K cm}^{-3}$, significantly higher than that of the intercloud gas of the ISM. Because we want the model to agree with observations, we will re-examine it to try to explain why the pressure of the C II region exceeds that of the general ISM.

The above analysis shows that the carbon lines observed toward Cas A originate in rather dense regions. At $N_e = 0.15 \text{ cm}^{-3}$, the hydrogen density has to be about 500 cm^{-3} even without considering the partial depletion of carbon. Observations of CO lines give approximately the same number ($N_{H_2} = 300 \text{ cm}^{-3}$) for the density of molecular clouds (Goldreich and Kwan, 1974).

3.3.5.2 The Nature of C II Regions Toward Cas A

Another method of investigating the characteristics of the interstellar C II regions is to compare the $Cn\alpha$ lines with the profiles of molecular spectra seen in the same direction, in this case toward Cas A. Figure 3.40 compares the C221 α recombination line (Sorochenko et al., 1991) with observations of the $1_{10} \rightarrow 1_{11}$ formaldehyde H_2CO line (Goss, Kalberla and Dickel, 1984) and CO. The observations were taken of 16 molecular clouds in the direction of the Perseus arm of our Galaxy – which contains Cas A – at an angular resolution of $10''$. The H_2CO is an average over all of the clouds to facilitate comparison with the carbon RRL.

The velocity structure of the C221 α and H_2CO lines agrees well even though the recombination line appears in emission and the other, in absorption. There are two velocity components in each profile: $V_{LSR} = -49$ and -46 km s^{-1} and $V_{LSR} = -42$ and -36 km s^{-1} , where we list the subcomponents visible in the H_2CO profile but which are blended together in the C221 α profile. The H_2CO is intrinsic to the dense material in the molecular clouds, because only there can collisions invert its level populations so that the molecule can absorb the background radiation. In the C II regions, the carbon line is enhanced by emission stimulated by radiation from Cas A. Thus, the similarity of the two velocity profiles tells us that the C II regions correspond spatially to the dense molecular clouds.

Also shown in Fig. 3.40 are contour maps of ^{13}CO emission toward Cas A obtained with the 30-m IRAM radio telescope in southern Spain (Wilson, Mauersberger, Muters, Przewodnik and Olano, 1993). The angular resolution was $21''$. These velocity slices through the CO lines tell us where the cloud

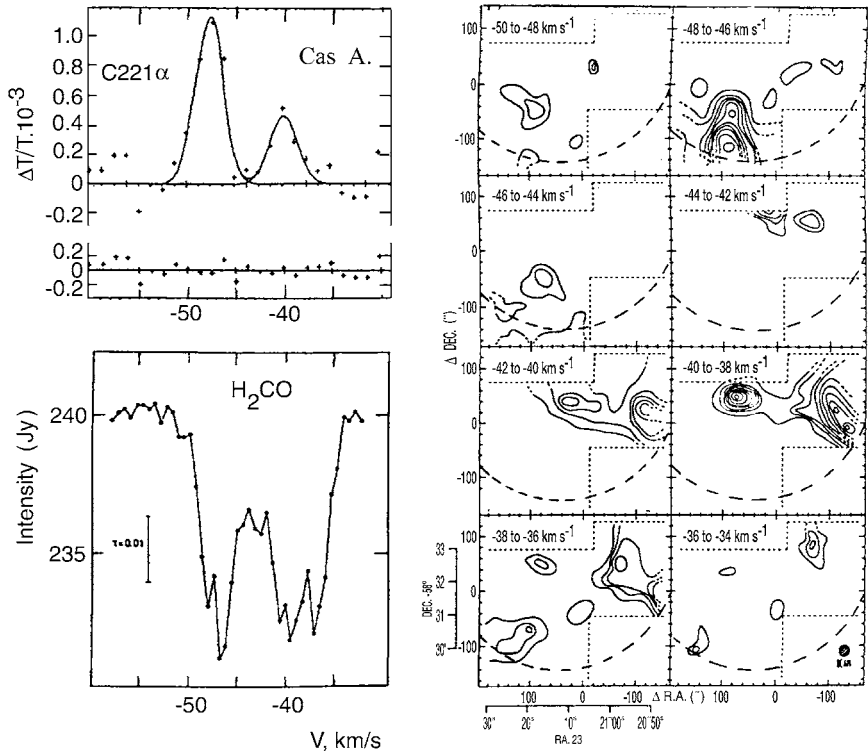


Fig. 3.40 Comparison of the C221 α and molecular emission spectra toward Cas A. *Left:* spectra of the C221 α line (*above*) and the $1_{10} - 1_{11}$ H_2CO line (*below*). *Right:* a contour map of the ^{13}CO line for eight V_{LSR} intervals summed over 2 km s^{-1} . The *thin dashed line* indicates the map borders and the *thick dashed line* indicates the boundary of Cas A. The *text* gives the references

components lie. Toward the south lies, a cloud with a radial velocity in the interval $V_{LSR} = -48 \rightarrow -46 \text{ km s}^{-1}$, exactly the velocity range of the most intense component of the C221 α profile. Furthermore, the central and western parts of the maps indicate intense CO emission in the velocity interval $V_{LSR} = -42 \rightarrow -36 \text{ km s}^{-1}$, agreeing well with the other velocity component of the C221 α profile. Finally, in the velocity ranges $V_{LSR} < -48 \text{ km s}^{-1}$ and $V_{LSR} > -36 \text{ km s}^{-1}$ where there is no C221 α emission, there is also very little CO emission.

This spatial correspondence between the C II region and the molecular clouds is certainly not accidental. We expect the carbon RRLs to arise in the interfaces of the molecular clouds and the general ISM, regions illuminated by the ambient UV radiation field of the Galaxy (Ershov et al., 1984; Ershov et al., 1987). Calculations predict that the dense molecular clouds are surrounded by a layer of atomic hydrogen that protects the H_2 molecules of the cloud from dissociation by the interstellar UV radiation. This envelope also

contains ionized carbon. The length scale or thickness of the H I/H₂ transition region is

$$L_{tr} = \frac{9.5 \times 10^{-5} \epsilon^{-1.4}}{\langle N_H \rangle} \text{ pc}, \quad (3.25)$$

where the dimensionless parameter ϵ ranges from 6×10^{-5} to 2×10^{-4} and is defined by the ratio of the formation and destruction rates of the H₂ molecules and by the column density of hydrogen in the transition region (Federman, Glassgold and Kwan, 1979). The units of the space-averaged hydrogen volume density, $\langle N_H \rangle$, are cm^{-3} .

In fact, observations generally confirm the presence of H I shells surrounding molecular clouds. Comparison of $\lambda = 21$ cm and formaldehyde maps in 12 out of 16 clouds detected with high probability toward Cas A suggests the presence of H I envelopes. The average thickness of these shells (transition regions) was 0.19 pc, the average hydrogen number density was 300 cm^{-3} , and the average hydrogen column density was $3.4 \times 10^{20} \text{ cm}^{-2}$ (Goss et al., 1984). Wilson et al. (1993) found similar characteristics by comparing observations of $\lambda = 21$ cm and CO emission lines in ten molecular clouds toward Cas A. These H I transition layers surrounding the cloud cores had hydrogen column densities $\geq 10^{20} \text{ cm}^{-2}$.

The temperature and volume density of the molecular clouds themselves can be determined from CO emission lines (Troland, Crutcher and Heiles, 1985; Wilson et al., 1993) and from NH₃ (ammonia) absorption lines (Gaume, Wilson and Johnston, 1994). For ¹²CO, collisions with H₂ excite these optically thick lines, thereby providing a gas temperature from the line intensities. The optically thin lines from ¹³CO give the column densities and, by dividing by a cloud length, the corresponding hydrogen volume densities. For the NH₃ observations, the temperatures are derived from the relative population of the rotational states when multiple rotational transitions are observed. Taken together, these observations give $T \approx 20$ K and $N_{H_2} = 1 \rightarrow 4 \times 10^3 \text{ cm}^{-3}$. These values are averaged over the clouds; in fact, generally the temperature increases and the density decreases outward from the cloud centers (Sorochenko and Walmsley, 1991). Only a small portion of H I lies within the shells. The column density of atomic hydrogen in the Perseus arm toward Cas A, summed over both velocity features, exceeds $3.5 \times 10^{21} \text{ cm}^{-2}$ (Troland et al., 1985).

Carbon RRLs originate mainly in the narrow transition layer of molecular clouds, i.e., their contiguous outer envelopes. In general, these ISM atoms are ionized by the background (ambient) UV radiation escaping from discrete H II regions.¹⁷ The integrated optical depth of the line emitting region $\propto N_e N_{C^+} / T^{2.5}$. Therefore, the contributions from the outer regions H I layer fall off rapidly as the density decreases and the temperature increases. For

¹⁷ Carbon is also ionized in the surface layer of the molecular cloud core because the borders of C II region and those of transition layer between H I and H₂ overlap somewhat (see Sect. 3.3.1).

this reason, the carbon RRLs are restricted to only the small envelope volume in which the ionized medium is both dense and cool.

For this reason, the two velocity components observed toward Cas A in the C221 α profile of Fig. 3.40 tell us something about the associated molecular clouds. The separate radial velocities indicate that there must be two distinct C II regions: one surrounding a cloud at $V_{LSR} \approx -48 \text{ km s}^{-1}$ and another surrounding a cloud at $V_{LSR} \approx -40 \text{ km s}^{-1}$. Furthermore, the narrowness of the components indicates fairly small dimensions for the host molecular clouds, because the C221 α is not expected to exhibit either collision or radiative broadening because of its (high) frequency. The respective line widths are $\Delta V_{-48} \approx 3.7 \text{ km s}^{-1}$ and $\Delta V_{-40} = 5 \rightarrow 8 \text{ km s}^{-1}$.

With these velocity characteristics, it is possible that these clouds indicate the passage of interstellar gas through a spiral density wave associated with the Perseus arm. Spiral density waves provide a mechanism for compressing Galactic gas to stimulate star formation, thereby creating the “arms” of spiral-type galaxies like our Milky Way (see the review by Roberts (1975)). First proposed by Lindblad and Langebartel (1953), revised by Lin and Shu (1964), and further revised in a series of papers largely but not exclusively by C.C. Lin and his students, the density-wave theory envisions a quasistationary wave pattern rotating around the center of a galaxy at a fixed angular velocity. The differentially rotating Galactic gas¹⁸ would flow through this pattern, being compressed at passage by a factor of 4 or less, thereby inducing gravity forces to create dense molecular clouds. In turn, regions within these clouds would form massive early-type stars, ultimately creating the luminous spiral arms characteristic of spiral galaxies. Of all the improvements to the theory, the most significant was the nonlinear mathematical treatment. This revision predicted the density increase and corresponding shock to occur in the ambient gas over smaller distances, stimulating star formation over equally small distances, and creating a crisper (more contrasty) spiral arm.

There has been an interesting refinement to the density-wave theory that seems to explain the two velocity components of the C221 α profile of Fig. 3.40 observed toward Cas A. Roberts (1972) has examined the observational data – both optical and radio – in this direction in terms of a nonlinear, two-armed spiral shock (TASS). He envisions the Perseus arm to result from this Galactic shock wave embedded in the background density wave. Unlike standard density waves, the TASS can compress density by as much as a factor of 15, very probably accelerating star formation of that expected from the nonlinear density-wave theory.

Specifically, this TASS model predicts that the two (and sometimes more) distinct velocity components observed in the Cas A carbon RRLs and H I emission (and in optical objects) could arise in very nearly the same place within the Perseus arm, i.e., from molecular clouds that spatially lie very close to each other.

¹⁸ The Galactic gas is not rigidly bound together and rotates differentially (see Fig. 3.11).

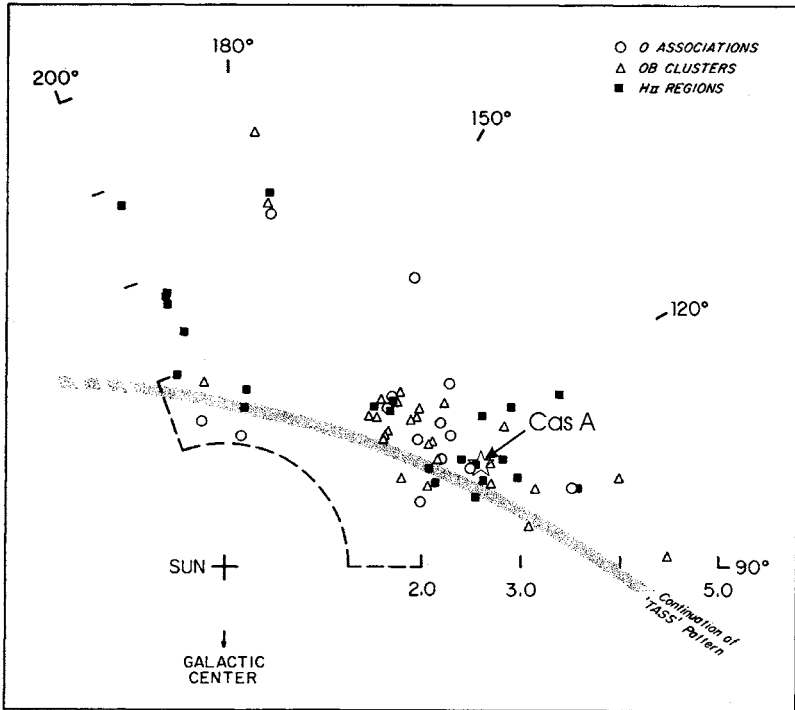


Fig. 3.41 The space distribution of young astronomical objects in the Perseus arm, based upon radial velocities and the Schmidt rotation curve (HII regions and O associations), and color-magnitude diagrams (Young Open (OB) Clusters). A *star* indicates the position of Cas A. The Galactic gas flows clockwise through the two-arm spiral shock marked by *hatching*. After Roberts (1972)

Figure 3.41 illustrates the situation. Plotted as a function of linear coordinates are the locations of Population I objects in the Perseus arm of the Galaxy. The data include O star associations with distances determined from the radial velocities of absorption spectra; OB clusters, from their color-magnitude characteristics (reddening); and HII regions, from the radial velocities of their RRLs. The Schmidt (1965) rotation model was used to convert radial velocity into distance from the Sun. An important result of this plot is the large spread of distances determined from radial velocities.

According to Roberts (1972), determining Galactic distances from a simple rotation curve, say, from the Schmidt, can give incorrect results. Much of the large distance spread shown in Fig. 3.41 could arise naturally from the local velocity perturbations imparted by the TASS wave. Figure 3.42 gives the details. The passage of the TASS through the Perseus arm creates a large velocity dispersion immediately following the leading edge of the shock. Gas located in this vicinity would have at least a 20-km s^{-1} range depending

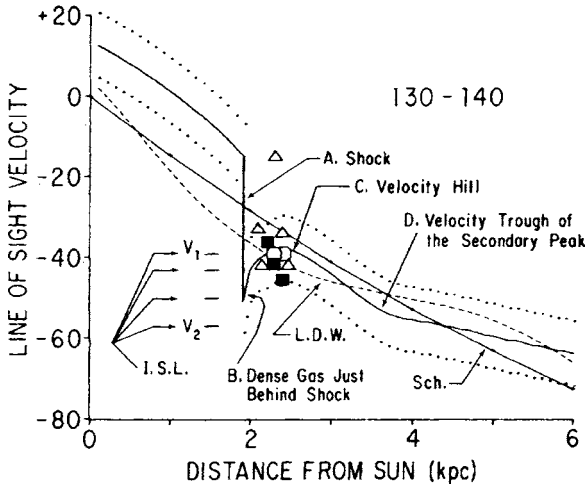


Fig. 3.42 The line-of-sight velocities in km s^{-1} created by a TASS wave within Galactic longitudes $130 \leq \ell \leq 140$. In this plot, the Sun lies 10 kpc from the Galactic center. Curve A marks the radial velocities created by the TASS wave, as are the velocity “hill” and “trough” following the shock front. The Schmidt (1965) and linear density-wave (L.D.W) rotation curves are also shown. On the left dashes indicate the velocities of Ca II and Na I interstellar absorption lines (I.S.L) observed toward O star associations in the Perseus arm (Münch, 1957). The symbols mark probable locations of the Population I objects considered in Fig. 3.41. From Roberts (1972)

upon its exact location with respect to the front. A similar picture¹⁹ obtains for the longitude range containing Cas A. This model easily explains the large velocity dispersion of interstellar absorption lines observed against more distance O star associations.

More importantly for us, the model also explains the two-component velocity structure seen in the carbon RRLs and in the H I absorption observed toward Cas A. Interstellar gas (with its molecular cloud) piled up just following the shock front would contribute the emission at $V_{LSR} \simeq -48 \text{ km s}^{-1}$. As would be expected from the large compression of the TASS, this H I gas is observed to have a large optical depth of $\tau > 8$ (Greisen, 1973). Furthermore, this model also explains the large longitude–velocity gradient of $0.7 \text{ km s}^{-1} \text{ pc}^{-1}$ observed for this gas with the VLA: 20 times greater than the velocity gradient expected from Galactic rotation alone (Bieging, Goss and Wilcots, 1991). The second gas component, at $V_{LSR} \simeq -40 \text{ km s}^{-1}$, would lie close by but in the relaxed, more diffuse gas slightly downstream of the shock. The distance separation might be $\approx 500 \text{ pc}$ but it could also be much smaller. Because of the unknowns regarding the detailed characteristics of the interstellar gas, the

¹⁹ We reproduce the $\ell = 130 - 140^\circ$ figure of Roberts (1972) rather than the one dealing specifically with the Cas A range, $\ell = 110 - 115^\circ$, because of its better annotation.

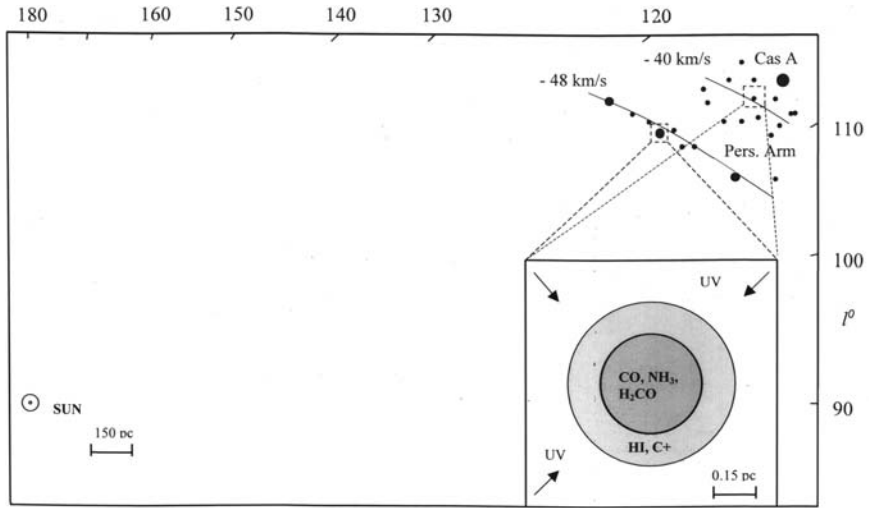


Fig. 3.43 Perseus arm based upon spiral density waves (Roberts, 1972). The *small filled circles* indicate the locations of atomic and molecular clouds formed in the possible two branches of the Perseus arm. Both kinds of clouds would exist within a tenuous medium of HI not shown in the figure. The *inset* shows a model of a molecular cloud surrounded by an HI shell containing the CII layer where the carbon RRLs originate

TASS model is more illustrative than quantitatively rigorous.²⁰ The molecular clouds formed in the two gas components would then contribute the carbon RRLs from their CII envelopes.

For our purpose, the density-wave theory augmented by the TASS is able to explain the multiple velocity components seen in carbon RRLs and HI spectra observed toward Cas A. It is also able to explain the high pressure in the CII regions seen in the same direction. Figure 3.43 shows a possible model for the situation, although the size scale is uncertain for reasons discussed above. The gas clouds emitting these velocity components lie at a distance of approximately 3 kpc within the Perseus arm of our Galaxy, the higher velocity one being less dense and lying somewhat further from the Sun than the lower velocity, denser one. The carbon RRLs irrefutably identify the sources as molecular clouds because they would be enveloped in CII envelopes produced by the ambient interstellar UV radiation field. The theory indicates that these clouds would have been formed as a consequence of the passage of the interstellar gas through the shock front. In time, both of these molecular clouds will likely serve as parents to OB stars and others.

Analysis indicates that the masses of all the molecular clouds detected toward Cas A fall in the range of one to tens of M_{\odot} and, moreover, are less than the virial masses calculated from their velocity structures (Goss

²⁰ Roberts (2001) notes that adding density inhomogeneities to the TASS model would reduce the distance dimensions of the compression wave.

et al., 1984; Wilson et al., 1993). Such clouds may be in a dynamic evolutionary process, in which they are re-expanding after being created in the denser regions immediately following passage through the shock front. Such expansion would extend to the H I and C II envelopes surrounding them. This expansion can explain the higher pressure of the C II gas relative to the ambient ISM.

3.3.5.3 Carbon RRLs from Other Cold Clouds in the ISM

The detection of low-frequency carbon RRLs in cold clouds toward Cas A suggested that such lines might be found elsewhere in the Galaxy as well. Soon, the C443 α and C447 α lines near 75-MHz range were detected toward the Galactic center with the 140-ft. (43-m) NRAO telescope in Green Bank (Anantharamaiah, Payne and Erickson, 1988). In the same direction but at still lower frequencies, the C537 α , C538 α , and C539 α lines were detected near 42 MHz with the E-W arm of the synthesis telescope DKR-1000 in Pushchino, Russia (Smirnov, Kitaev, Sorochenko and Schegolev, 1996). Both sets of spectra shown in Fig. 3.44 appear in absorption against the nonthermal emission of Sgr A.

Additional observations of carbon RRLs indicated that the C II emission could be spatially extended. A survey made of the C441 α line (76.5 MHz) with the 64-m Parkes telescope (HPBW=4 $^\circ$) in Australia showed emission over the range of Galactic longitude of $\ell = \pm 20^\circ$ at $b = 0^\circ$ (Erickson, McConnell and Anantharamaiah, 1995). The line intensity decreased with distance away from the Galactic center and, because of the large beamwidth, was not detected reliably along the Galactic equator except at

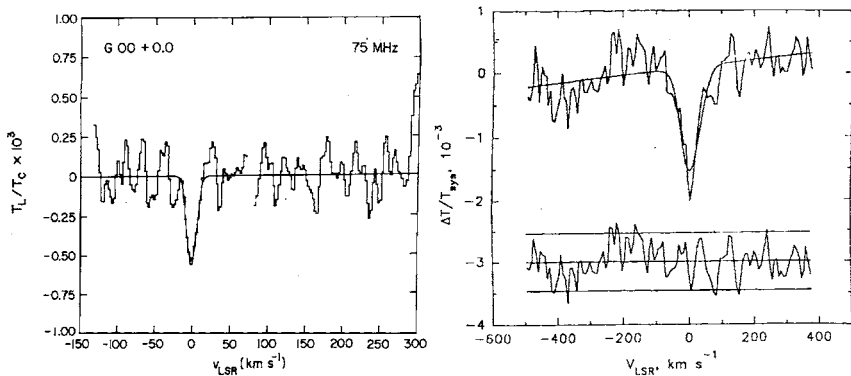


Fig. 3.44 *Left:* averaged C443 α and C447 α spectra fitted with a Gaussian. *Points to the left of 75 km s $^{-1}$* represent all observations and to the *right* represent only observations free of radio interference (Anantharamaiah et al., 1988). *Right:* averaged C537 α , C538 α , and C539 α spectra fitted to a Gaussian. The residuals lie below (Smirnov et al., 1996)

the longitude 312° . However, at $\ell = 352^\circ, 358^\circ, 0^\circ, 2^\circ$, and 14° , the survey detected C441 α emission at $b = \pm 2^\circ$. These observations showed that C II regions in the central part of the Galaxy can extend as much as 4° in Galactic latitude.

In subsequent observations near 34.5 MHz made with the T-shaped telescope at Gauribidanur, India, Kantharia and Anantharamaiah (2000) detected eight carbon RRLs with $n \approx 575$ toward the Galactic center – a direction similar to the 76-MHz observations noted above. The angular resolution of this telescope was $21' \times 25'$. The lines clearly appeared in absorption in six directions at $\ell < 17^\circ$.

In these same directions, searches for carbon lines were carried out near 328 MHz with the radio telescope at Ooty, India (Anantharamaiah and Kantharia, 1999). Four carbon RRLs with $n \approx 271$ were detected from all six directions. Figure 3.45 shows 328- and 34.5-MHz spectra observed from India.

As a group, these lines tell us about the C II regions in which they arise. First, all the line widths are about the same; in other words, Stark broadening is absent. This absence means that the upper limit on the electron density in the C II regions must be $N_e \leq 0.1 \text{ cm}^{-3}$. Furthermore, comparison of the

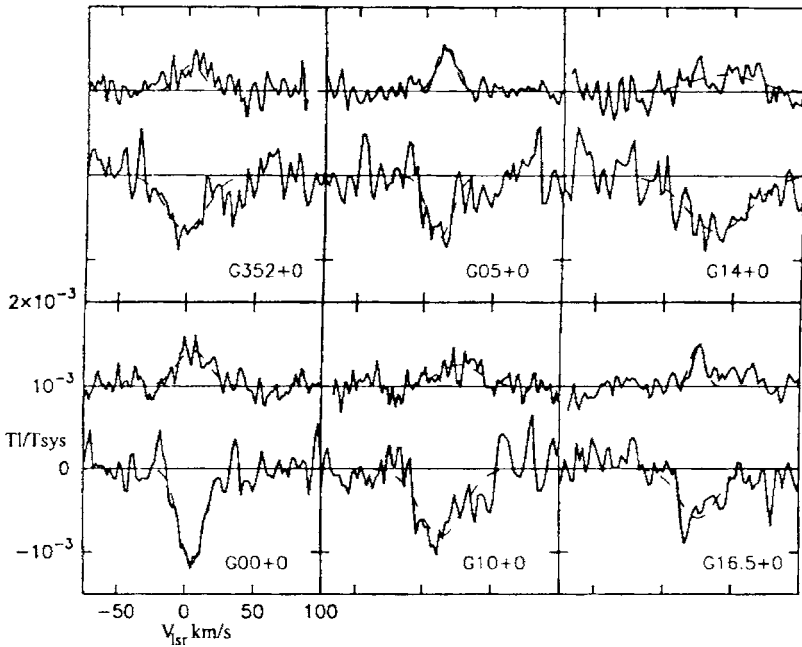


Fig. 3.45 Above: spectra near 328 MHz averaged over four transitions with $n \approx 271$. Velocity resolution of 1.8 km s^{-1} and angular resolution of $2^\circ \times 2^\circ$. Below: spectra near 34.5 MHz averaged over eight transitions with $n \approx 575$. Velocity resolution of 4.5 km s^{-1} and angular resolution of $21' \times 25'$. From Anantharamaiah and Kantharia (1999). Reproduced with permission of The Astronomical Society of the Pacific

line intensities at the three frequencies (34.5, 75, and 328 MHz) limits the temperature of the C II regions to $20 < T_e < 300$ K. The radial velocities of these lines place the C II regions at Galactocentric distances of $4 < R_G < 8$ kpc, in the Scutum or Sagittarius arms of the Galaxy, shown in the spiral model of Fig. 3.13 (Erickson et al., 1995; Kantharia and Anantharamaiah, 2000).

What are the detailed characteristics of these C II regions? Is ionized carbon located only in the central regions of the Galaxy? Or, is it present everywhere but requires highly sensitive telescopes for its detection?

To answer these questions, we turn to observations of the fine-structure C⁺ line at $158\ \mu\text{m}$. Section 3.3.1 described how the formation of the fine-structure line and carbon RRLs takes place in the same regions. Therefore, studies of the $\lambda = 158\ \mu\text{m}$ line may give us information about the formation of carbon RRLs in the Galaxy and allow us to predict the results of additional searches for these RRLs.

Figure 3.46 shows a contour map of the carbon fine-structure line, $\lambda = 158\ \mu\text{m}$, toward the Galactic center. The observations were made with the Balloon-borne Infrared Carbon Explorer (BICE) with a resolution of $15'$. The data show bright sources within an extended, diffuse component. Many of these sources correspond to well-known bright H II regions, so the carbon emission probably comes from the PDRs enveloping these regions and molecular clouds as discussed earlier in Sect. 3.3.1.

The separation of one of these discrete sources, NGC6334, from the underlying background is shown clearly in the $\lambda = 158\ \mu\text{m}$ C⁺ emission of Fig. 3.47. These spectral data are a cut across the Galactic plane made with the Balloon-borne Infrared Telescope (BIRT), which has a higher angular resolution ($3'4$) than BICE. The peak of the C⁺ emission coincides with a maximum of the 5-GHz continuum emission. We note that McGee and Newton (1981) detected the C76 α RRL from NGC6334 a decade earlier.

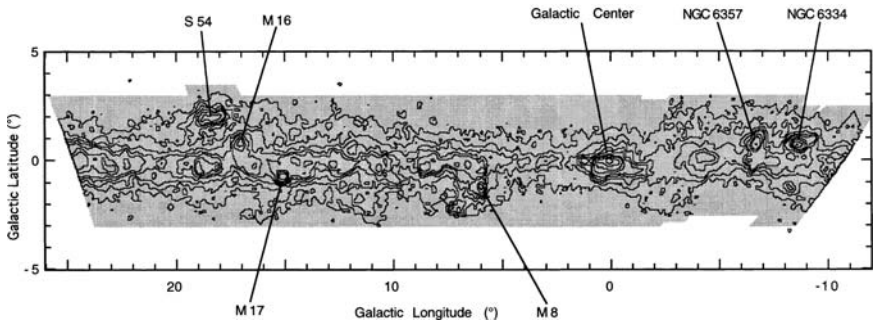
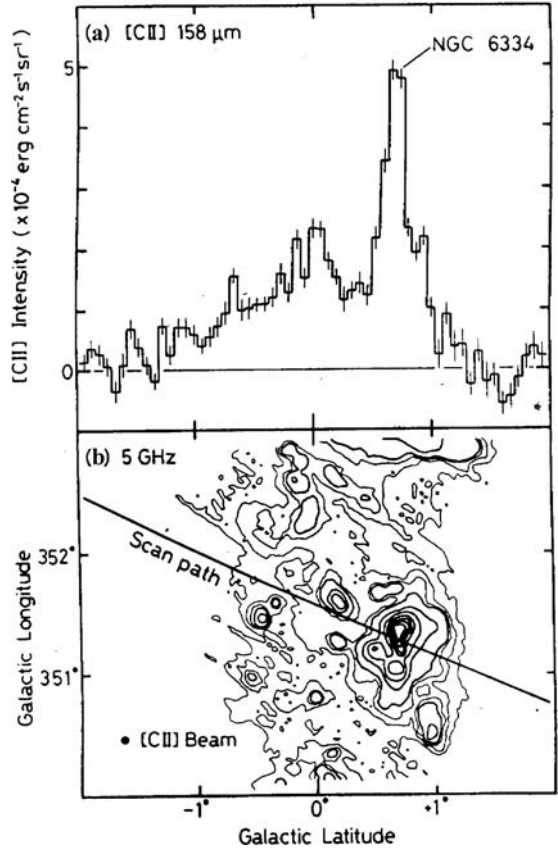


Fig. 3.46 A map of the $\lambda = 158\ \mu\text{m}$ fine-structure line of C⁺ obtained by the Balloon-borne Infrared Carbon Explorer (BICE) with a resolution of $15'$. The contour levels are 0.3, 0.6, 1, 1.5, 2, 3, 4, 5, 6, and 9×10^{-4} ergs s⁻¹ cm⁻² sr⁻¹. From Nakagawa et al. (1998)

Fig. 3.47 (a) The line intensity profile of the $\lambda = 158 \mu\text{m}$ fine-structure line of C^+ across NGC6334. The angular resolution is $4'$. (b) The 5-GHz radio continuum emission from the vicinity of the spectral scan. From Shibai et al. (1991)



If one were to remove the $\lambda = 158 \mu\text{m}$ emission associated with NGC6334, a strong extended component of C^+ emission from the Galactic plane over $b \approx \pm 1^\circ$ would remain. Such an extended component of CII emission would agree with the observations of the low-frequency carbon RRL data toward this region. Unlike the carbon RRLs whose emission is confined to $\ell < 20^\circ$, the $\lambda = 158 \mu\text{m}$ emission extends to greater longitudes, from $\ell = 30^\circ$ to $\ell = 51^\circ$ at $|b| = 1 \rightarrow 2^\circ$ (Shibai, Okuda, Nakagawa, Matsuhara, Maihara, Mizutani, Kobayashi, Hiromoto, Nishimura and Low, 1991). On the other hand, the C^+ emission in this region is weak at these extended longitudes, which might explain the uncertainty or failure of the searches for the corresponding carbon RRLs at $\ell > 20^\circ$ along the Galactic plane. For example, Makuti et al. (1996) measured a differential of $I_{158} = (2 \rightarrow 3) \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1}$ in this region, which is a few times less than the brightness observed in directions closer to the Galactic center.

Figure 3.48 shows the distribution of the C^+ $\lambda = 158 \mu\text{m}$ emission along the Galactic plane with respect to other kinds of emission. The C^+ observa-

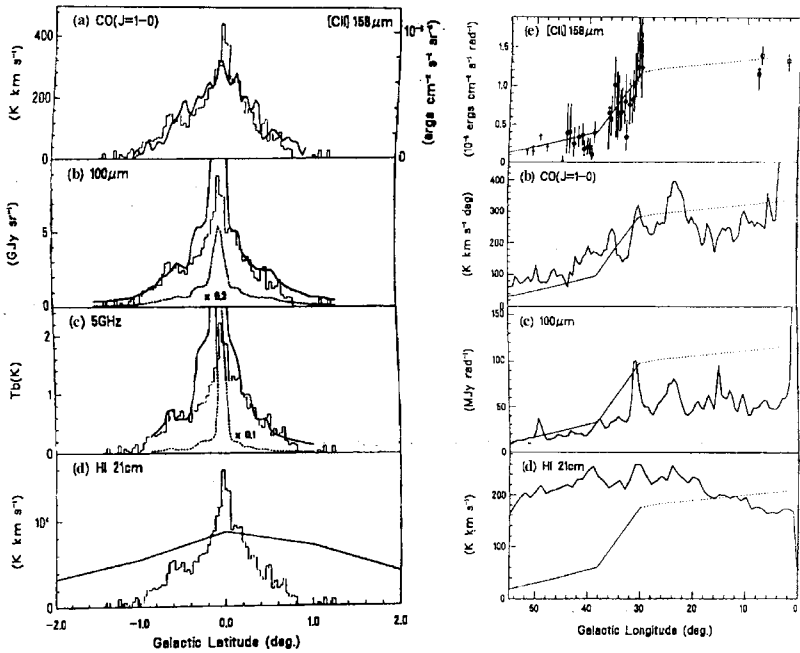


Fig. 3.48 The intensity distribution of C^+ $\lambda = 158 \mu\text{m}$ emission in Galactic plane compared with other types of radiation. *Left:* (a) The C^+ profile across W43 superimposed on the latitudinal profile of the integrated ^{12}CO ($J = 1 \rightarrow 0$) intensity. (b) The C^+ scan superimposed on the $\lambda = 100 \mu\text{m}$ continuum emission observed by IRAS. (c) The same scan superimposed on the 5-GHz continuum emission. (d) The same scan superimposed on the integrated $\lambda = 21 \text{ cm}$ emission of H I. *Right:* (a) The longitude distribution of the integrated C^+ emission. The solid and dashed lines represent a best-fit linear longitudinal profile, where the dashed line indicates interpolation between the data obtained with BIRT and the data obtained with the NASA Lear Jet (Stacey et al., 1985). (b) The same data plotted with the ^{12}CO distribution. (c) The same data and the $\lambda = 100 \mu\text{m}$ continuum emission observed by IRAS. (d) The same data and $\lambda = 21 \text{ cm}$ emission of H I. From Shibai et al. (1991)

tions, made by BIRT, show the line intensity along a cut passing through the region of the H II region W43 ($\ell = 30^\circ 8$). Latitude profiles of CO, of H I and of the $\lambda = 100 \mu\text{m}$ and 5-GHz continuum emission are superimposed upon the C^+ emission for comparison. Longitude profiles of the same emission are shown on the right-hand side of the figure.

Both similarities and dissimilarities are striking. The latitude profiles of the C^+ and CO emission almost coincide except for the narrow peak of the $\lambda = 158 \mu\text{m}$ emission at $b = 0^\circ$ toward the W43 source. This agreement contrasts sharply with the large discrepancy between the C^+ and H I line profiles. The latitude profile of the carbon emission is much narrower than the H I profile. The longitude profile exhibits a sharp decrease in intensity at $\ell > 30^\circ$ that is not seen in the corresponding H I profile.

These observational data – the C^+ and CO emission – testify to the spatial association of the C II regions and the molecular clouds. On this basis, one might conclude that the PDRs form on the surfaces of the molecular clouds. The ionizing agent must be the interstellar background UV radiation that, in turn, would cause the $\lambda = 158 \mu\text{m}$ emission from singly ionized carbon atoms. Note that these circumstances may be very different from the C II emission observed from the PDRs surrounding discrete H II regions. For one thing, the angular extent of the carbon emission is more diffuse than one would expect from the distribution of the discrete nebulae (Shibai et al., 1991). Here, the ionizing UV radiation may leak from within the H II regions.

Heiles (1994) offered an alternative explanation for the origin of the diffuse carbon emission in the inner part of the Galaxy. He suggested that the ELDWIM, which contains the major part of Galactic ionized hydrogen, may be connected with the $\lambda = 158 \mu\text{m}$ line emission (see Sect. 3.2.2). The efficiency of excitation of the upper level of the carbon fine-structure line is higher in H II regions than in H I regions. The collision cross section for electron excitation of the $C^+ \ ^2P_{3/2}$ level is larger than that for hydrogen atoms. That is why the $\lambda = 158 \mu\text{m}$ brightness per nucleon is higher in H II regions than in the H I gas (Spitzer, 1978). As a result, even though the total mass of H II is much less than H I in the ISM and, moreover, the main part of the C^+ gas lies in the neutral component of the ISM, most of the carbon $\lambda = 158 \mu\text{m}$ emission comes from regions of ionized hydrogen.

Support for this suggestion lies in the excellent correlation between the $\lambda = 158 \mu\text{m}$ emission and the 5-GHz continuum shown in Fig. 3.48 on the left. However, the spatial resolution of the carbon line is insufficient for examination of the detailed correspondence between the C II regions and ELDWIM.

And so, from these data, we have two possible models for the origin of the C^+ fine-structure emission. In the first one, the source of the $\lambda = 158 \mu\text{m}$ emission is a cold, neutral medium where envelopes of atomic carbon surround molecular clouds, where they are subsequently ionized by the ambient UV background radiation. In the second model, the $\lambda = 158 \mu\text{m}$ emission originates in a warm, ionized medium; i.e., in the extended, low-density H II gas known as ELDWIM.

Can observations of carbon RRLs contribute to the viability of either of these models? These lines occur in cold regions only. The absence of Stark broadening in the carbon RRLs requires a low electron density that would be characteristic of neutral hydrogen regions. The intensity of the carbon RRLs is strongly dependent upon transitions stimulated by the background radiation field and has complex dependence upon principal quantum number. High-frequency (low quantum number) lines are observed in emission and low-frequency (high quantum number) lines are observed in absorption as described by Sects. 2.4.1 and 2.4.2. This dependence upon principal quantum number was observed for carbon RRLs detected toward Cas A. From observations of these RRLs at 328, 75, and 34.5 MHz, Kantharia and Anantharamaiah (2000) found the “Cas A” C II regions to have temperatures ranging from 20 to 300 K.

The “warm” model in which carbon lines form in H II regions is a different situation. At the approximately 3 cm^{-3} electron densities of ELDWIM, the low-frequency carbon lines would exhibit strong Stark broadening. At these densities, (2.74) gives a line width of $\Delta V_L = 435 \text{ km s}^{-1}$ for $n \approx 565$, which is an order of magnitude larger than $20\text{--}54 \text{ km s}^{-1}$ observed (Kantharia and Anantharamaiah, 2000) for the widths of Galactic carbon lines.

The line intensities of the “warm” model would also conflict with observations. Because of the recombination nature of the populations of the highly excited levels of carbon, the optical depth of these lines decreases sharply with temperature; i.e., τ varies approximately as $T_e^{-2.5}$. At conditions typical for ELDWIM where $T_e \approx 7,000 \text{ K}$, $N_e \approx 3 \text{ cm}^{-3}$, $N_{C^+} \approx 10^{-3} \text{ cm}^{-3}$, and a path length of approximately 100 pc, the optical depth of carbon RRLs would be determined primarily by spontaneous transitions and would be very small. For example, at a frequency of 75 MHz, the LTE line optical depth τ_L^* would be approximately 10^{-5} for the carbon lines and maser effects would not be important. The carbon line would be observed in emission with a brightness temperature ($T_e \tau_L^*$) less than 0.1 K. Yet, observations at 75 MHz show the C441 α RRL to be in absorption with an intensity of approximately 10 K (Erickson et al., 1995).

Because the spatial association between the C II $\lambda = 158 \mu\text{m}$ emission and the carbon RRL has not been clearly established, one might suppose that these lines are formed in different regions. The carbon RRL might be formed in the H I part of the ISM, whereas the $\lambda = 158 \mu\text{m}$ carbon line might be formed in H II regions.

Mochizuki and Nakagawa (2000) revisited the question of the origin of PDRs in molecular clouds under the influence of the UV background radiation. The earlier analysis (Tielens and Hollenbach, 1985) considered only plane-parallel geometry, whereas the new analysis considered a spherical model of a molecular cloud located within an isotropic UV radiation field. In addition to the UV radiation ($6 \text{ eV} < h\nu < 13.6 \text{ eV}$), this model also considered the IR radiation ($0.15 \text{ eV} < h\nu < 6 \text{ eV}$) associated with the interstellar dust.

The spherical geometry revealed that a much weaker UV radiation field could account for C II component of the PDRs. New calculations showed that a UV flux density which was an order of magnitude less than that of the plane-parallel model could account for the $\lambda = 158 \mu\text{m}$ carbon line emission in the Galaxy. This removed the problem in recognizing the PDRs as the fundamental source of this emission. If the Galactic UV flux density²¹ $G_0 \approx 30$ – only a few times higher than believed earlier – then the observations of the fine-structure line would be consistent with the theoretical model.

In addition to examining the radiative energies involved, Mochizuki and Nakagawa (2000) also considered one more important factor that pointed toward the “cold” model. They noted that the survey (Nakagawa, Yui, Doi,

²¹ G_0 is the incident far-UV flux density relative to that of the solar neighborhood value of $1.6 \times 10^{-3} \text{ ergs cm}^{-2} \text{ s}^{-1}$.

Okuda, Shibai, Mochizuki, Nishimura and Low, 1998) of the C II $\lambda = 158 \mu\text{m}$ line emission spatially correlated well with observations of the IR continuum emission. Shibai et al. (1991) mentioned such a correlation, which can be seen in the left panel of Fig. 3.48.

Observations indicate that the majority of the IR continuum is emitted by the neutral rather than the ionized medium. Therefore, Mochizuki and Nakagawa (2000) concluded that the main part of the Galactic C II $\lambda = 158 \mu\text{m}$ is formed in the neutral ISM, in PDRs associated with molecular clouds and H I gas rather than with H II regions.

Moreover, the temperatures calculated for the C II regions within the modeled PDRs agreed with those derived from the carbon RRL observations. The spherical-model calculations indicated that the gas temperatures within the PDRs decreased from 39 K at the outer surface of the C^+ envelope to as low as 20 K at the border of the PDR and the spherical, inner molecular cloud. This temperature range generally agrees with those measured from carbon RRLs emitted by C II regions; i.e., they fall within the range of 20–300 K derived from the carbon recombination lines.

The entire complex of data on both types of carbon lines indicates that the C II regions that emit or absorb carbon RRLs *and* simultaneously emit the $\lambda = 158 \mu\text{m}$ fine-structure line lie in the inner region of the Galaxy within $|b| \approx 1^\circ$ of the Galactic plane. At the angular resolution of the observations, these regions are unresolved and contour maps of their emission indicate the presence of a spatially diffuse component. The source of the carbon ionization in these clouds is the interstellar UV radiation with an energy range of $6 \text{ eV} < h\nu < 13.6 \text{ eV}$. Under its influence, carbon is ionized in atomic clouds and on the surfaces of molecular clouds in PDRs. In both cases, most of the carbon RRL and fine-structure radiation comes from the atomic medium but some also comes from isolated clouds of H I as well as from PDR envelopes around molecular clouds. It is worth noting that H I envelopes, with thicknesses from 0.5 to a few pc, were detected with $\lambda = 21 \text{ cm}$ and CO emission from a number of specific molecular clouds. In particular, these included L1599, S255, Per OB2, Mon OB1 (Wannier, Lichten and Morris, 1983), and L134 (van der Werf, Goss and Vanden Bout, 1988).

The dependence of the line intensities on electron density determines where the emission is found. Carbon RRLs are proportional to N_e^2 . They are more apt to be detected toward dense atomic clouds or near the surface of the PDRs surrounding molecular clouds. On the other hand, the C II $\lambda = 158 \mu\text{m}$ fine-structure line is proportional to N_e and may be expected to be found over a wider area of the ISM where atomic density can be lower.

The location and morphology of the discrete C II regions in the Galaxy is a good subject for future research. The most reliable and detailed data for such research might be obtained from the superposition of observations of carbon RRLs and of the fine-structure line.

3.3.6 Estimates of the Galactic Cosmic Ray Intensity

In the cold ISM, soft cosmic rays (CR) partially ionize the neutral hydrogen. Protons from Type 1 supernovae²² spread through the Galaxy with typical energies of about 2 MeV, corresponding to their velocities of up to 20,000 km s⁻¹ (Spitzer and Tomasko, 1968). The ionization cross section of hydrogen by protons, σ_{ion} , increases with decreasing energy. In the energy range of 1–10 MeV, $\sigma_{ion} \propto 1/E$. At 2 MeV, $\sigma_{ion} \approx 10^{-17}$ cm² (Fowler, Reeves and Silk, 1970).

These 2-MeV cosmic rays can penetrate deep into the ISM. They are absorbed by ionization losses only when the hydrogen column density along their trajectory reaches 9×10^{20} cm⁻² (Bochkarev, 1988). Because of this penetration, it was once proposed that they caused an ionization rate of $\zeta_H = 1.17 \times 10^{-15}$ s⁻¹ in the ISM, which was sufficient to account for the dispersion of pulsar signals outside of H II regions. However, further study did not confirm this hypothesis (see Sect. 3.2.2).

It became apparent that RRLs could provide measurements of the hydrogen ionization rate in the ISM and, correspondingly, the intensities of cosmic rays in the Galaxy. The ionization equilibrium can be described to be

$$\zeta_H N_H = \alpha^{(2)} N_e N_{H+}, \quad (3.26)$$

where N_e , N_H and N_{H+} are the volume densities of electrons, neutral atoms and ions of hydrogen, respectively. The hydrogen recombination coefficient $\alpha^{(2)} = 2.06 \times 10^{-11} \Phi_2 / T^{1/2}$ and includes recombination to all levels except the first one. The factor Φ_2 has weak dependence on the temperature and is tabulated by Spitzer (1978).

Combining (3.26) with the definition of the optical depth of an RRL, Shaver (1976a) derived a simple expression for the hydrogen ionization rate:

$$\zeta_H = 5.7 \times 10^{-14} \Phi_2 \left(\frac{\tau_H}{\tau_{HI}} \right) \cdot \left(\frac{\nu_{m,n}}{100 \text{ MHz}} \right) \cdot \left(\frac{T_e}{T_s} \right) \cdot \frac{T_e}{(b_n \beta)_H}, \quad (3.27)$$

where τ_H and τ_{HI} are the optical depths of the hydrogen RRL and $\lambda = 21$ cm line, respectively; T_s is the hydrogen spin temperature in K; and $\nu_{m,n}$ is the frequency of the hydrogen RRL in MHz emitted from the transition from the upper principal quantum number m to n . As usual, the factor $(b_n \beta)_H$ corrects for deviations of the hydrogen-level populations from LTE and for the contribution of stimulated emission to the line intensity.

²² Supernovae are the cataclysmic explosions of massive stars that occur with a frequency of about 1 every 30 years in large spiral galaxies like our Milky Way. Type 1 refers to those whose optical spectra show no hydrogen emission lines during the “maximum light” phase of their light curves. In contrast, the spectra of Type 2 supernovae include broad, intense hydrogen emission lines. There are also subclassifications. See Reynolds (1988).

The results were astonishing. Using (3.27) and the upper limit for the optical depth of the H352 α line observed toward Cas A of $\tau_H \leq 1.7 \times 10^{-4}$, Shaver et al. (1976) calculated a hydrogen ionization rate of $\zeta_H < 3.3 \times 10^{-17} \text{ s}^{-1}$. They used a value of $\tau_{HI} > 5$ for the optical depth of the $\lambda = 21 \text{ cm}$ line (Clark, 1965). The departure coefficients were calculated by Shaver (1975) for $T_e = 50 \text{ K}$ and $N_e = 0.05 \text{ cm}^{-3}$, values accepted for the ISM toward Cas A at that time. Later, a similar value of $\zeta_H < 2 \times 10^{-17} \text{ s}^{-1}$ was obtained by the same method but for the H300 α line (Casse and Shaver, 1977). These limits were about two orders of magnitude smaller than estimated above from the soft cosmic rays.

To refine these results, it seemed worthwhile to re-examine the assumptions of average temperature and density of the ISM in the direction of Cas A. These parameters are essential to calculate appropriate values of b_n and β , which are factors in (3.27) used to determine ζ_H . Trying different values of temperature and density, Payne et al. (1989) analyzed new observations of the H308 α lines toward Cas A to obtain $\zeta_H < 3.9 \times 10^{-17} \text{ s}^{-1}$ to $\zeta_H < 2.6 \times 10^{-16} \text{ s}^{-1}$. The range of these calculated results is about an order of magnitude, much larger than desired.

Fortunately, this range of uncertainty for ζ_H can be reduced further by using another method of analysis proposed by Sorochenko and Smirnov (1987) based upon simultaneous observations of low-frequency carbon and hydrogen RRLs in cold clouds within the ISM.

Consider the model of the C II shells surrounding molecular clouds that was discussed in Sect. 3.3.5. Interstellar UV background radiation with $\lambda > 912 \text{ \AA}$ ionizes the carbon in these shells while keeping the hydrogen largely neutral. However, some of the hydrogen in these shells must be ionized by soft interstellar cosmic rays. As noted above, the penetration of 2-MeV cosmic rays through the ISM corresponds to a column density of about $9 \times 10^{-20} \text{ cm}^{-2}$, which corresponds approximately to the thickness of the C II–HI shells surrounding the molecular clouds. The intensity ratio of the hydrogen and carbon RRLs will facilitate the determination of ζ_H with a smaller dependence upon the local temperature and local electron volume density.

First, we determine the ionization ratio of hydrogen in the shells of the molecular clouds. If all carbon in the shells is ionized, $N_{C+} = N_C$. Using (2.116), we factor this ratio as

$$\frac{N_{H+}}{N_H} = \frac{N_C}{N_H} \frac{N_{H+}}{N_{C+}} = 3.3 \times 10^{-4} \left(\frac{\tau_H}{\tau_C} \right)_{LTE} \frac{\phi_C(\nu)}{\phi_H(\nu)}, \quad (3.28)$$

where τ_H and τ_C are the LTE optical depths, and $\phi_H(\nu)$ and $\phi_C(\nu)$ are the RRL profiles for hydrogen and carbon, respectively.

Second, we determine the ratio of the line optical depths required by (3.28). In C II regions, the populations of the excited levels are not in LTE and the line optical depths have to be corrected accordingly. Using (2.132), we write

$$\left(\frac{\tau_H}{\tau_C}\right)_{LTE} = \left(\frac{\tau_H}{\tau_C}\right)_{obs} \frac{(b_n\beta)_C}{(b_n\beta)_H}. \quad (3.29)$$

Determining the remaining factor in (3.28) is easy. At low frequencies, the line widths are determined by Stark broadening rather than by the atomic mass of the emitters. For this reason, the line profiles $\phi(\nu)$ are identical for both hydrogen and carbon lines and $\phi_C(\nu)/\phi_H(\nu) = 1$.

Substituting these into (3.26), Sorochenko and Smirnov (1990) obtained

$$\zeta_H = \frac{6.8 \times 10^{-15} \Phi_2 N_e}{T^{0.5}} \left(\frac{\tau_H}{\tau_C}\right)_{obs} \frac{(b_n\beta)_C}{(b_n\beta)_H}, \quad (3.30)$$

where the values of optical depths may be both positive and negative. At low frequencies, these RRLs can be observed in absorption and emission.

We can now calculate the ionization rate. Observations of the C486 α line toward Cas A give $(\tau_C)_{obs} = (2.35 \pm 0.22) \times 10^{-3}$; and, of the H486 α line, $(\tau_H)_{obs} < 6.3 \times 10^{-4}$ (Ershov et al., 1987). In the C II regions, $T_e = 50$ K and $N_e = 0.15 \text{ cm}^{-3}$. For these conditions, $(b_n\beta)_H = -1.05$, $(b_n\beta)_C = 5.3$ (Ponomarev and Sorochenko, 1992), and $\Phi_2 = 3.6$. Substituting these values into (3.30) gives an upper limit of $\zeta_H = 7 \times 10^{-16} \text{ s}^{-1}$. This limit lies near the middle of the range found by Payne et al. (1989).

An even lower estimate resulted from a program of observing low-frequency RRLs especially designed to determine the interstellar hydrogen ionization rate by cosmic rays (Kitaev et al., 1994). This program used the DKR-1000 array telescope in Pushchino, configured into two parts to facilitate fast switching between Cas A and a reference region. The 42-MHz observing frequency allowed observations of both carbon and hydrogen recombination lines involving the lower principal quantum numbers 537, 538, 539, and 540. The total integration time was 1,224 h.

Figure 3.49 shows the average of the four recombination lines resulting from these 42-MHz observations of Cas A. The carbon line profile has two absorption components corresponding to velocity features from the Perseus and Orion spiral arms of the Galaxy. The Gaussian fit to the Perseus component gives $\tau_C = (-3.18 \pm 0.05) \times 10^{-3}$ and $\Delta V = 22.1 \pm 0.3 \text{ km s}^{-1}$. On the figure is marked the expected position for the corresponding hydrogen RRL. Because the detection is unreliable, the optical depth of the hydrogen line can be only an upper limit, $\tau_H \leq 1.1 \times 10^{-4}$.

Substituting these parameters as well as values of $b_n\beta$ calculated for $T_e = 50$ K and $N_e = 0.15 \text{ cm}^{-3}$ gives an upper limit for the hydrogen ionization rate of $\zeta_H = 2.75 \times 10^{-16} \text{ s}^{-1}$ – very close to the lower limit of the range of rates found by Payne et al. (1989).

There is a convergence of the values found for the hydrogen ionization rate ζ_H from the two different techniques. If one accepts the temperatures and volume densities obtained from the carbon RRLs as valid for the calculations of appropriate departure coefficients, then the limits on the ionization rate obtained from the hydrogen optical depth ratios τ_H/τ_{HI} in (3.27) agree well

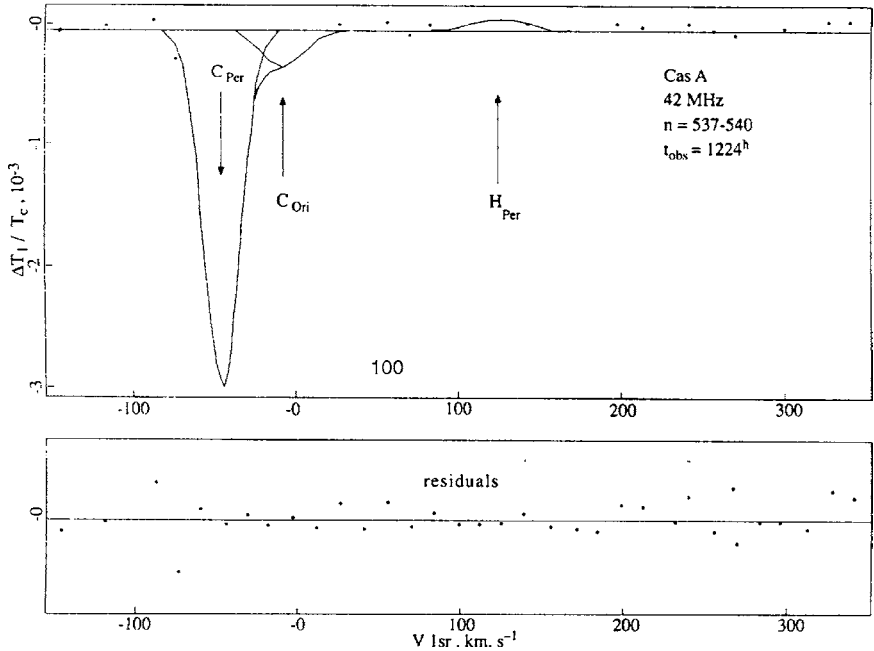


Fig. 3.49 The spectrum of Cas A at 42 MHz with a velocity resolution of 14 km s^{-1} . This spectrum is the average of four recombination lines. The *ordinate* is the ratio of the line to continuum emission with negative numbers indicating absorption, and the *abscissa* is the velocity with respect to the LSR. The *lower plot* shows the residuals remaining after fitting the line components. The integration time is 1,224 h. From Kitaev et al. (1994)

with the inherently more accurate result obtained from the observed ratio of τ_H/τ_C given by (3.30). The hydrogen-based limits of the ionization rates are then $\zeta_H < 4.5 \times 10^{-16} \text{ s}^{-1}$ from the $\text{H}300\alpha-\lambda = 21 \text{ cm}$ combination, $< 3.8 \times 10^{-16} \text{ s}^{-1}$ from the $\text{H}308\alpha-\lambda = 21 \text{ cm}$ combination, and $< 1.2 \times 10^{-16} \text{ s}^{-1}$ from the $\text{H}352\alpha-\lambda = 21 \text{ cm}$ combination.

To date, the attempts to use RRLs to determine a fixed value for the hydrogen ionization rate in the cold ISM have not yet been successful. Observations have determined only an upper limit; i.e., $\zeta_H \approx 3 \times 10^{-16} \text{ s}^{-1}$ or less.

It is interesting to compare this limit to the ionization rates obtained from observations of interstellar molecules. Many of these molecules form from reactions involving hydrogen ions either as intermediate or as fundamental building blocks. For example, the dependence of the OH abundance on the hydrogen ionization rate enables independent determinations of ζ_H (Black and Dalgarno, 1973). Comparing theoretical calculations with observations of OH abundances in the diffuse molecular clouds surrounding $\zeta \text{ Per}$, $\zeta \text{ Oph}$, and o Per , van Dishoeck and Black (1986) found the value of the hydrogen ionization rate, $\zeta_H = (7 \pm 3) \times 10^{-17} \text{ s}^{-1}$.

This value of ζ_H is within a factor of 5 of the upper limit obtained from RRLs detected from the PDR shells surrounding dense molecular clouds. Someday, the sensitivity of radio telescopes will increase sufficiently to detect hydrogen RRLs from the general ISM and, thereby, determine a value of ζ_H that can be directly compared to the one found through interstellar chemistry. Comparison of the two results should then pin down this parameter that is essential to understanding the nature of the ISM within the Galaxy.

3.4 RRLs from Stars and Stellar Envelopes

RRLs offer physical insight to stars and their environments, in addition to their contributions regarding the nature of H II and C II regions and the broad interstellar medium. In particular, the stellar topics include the hot stellar envelopes known as planetary nebulae, our own Sun, and a peculiar and as of this writing a unique stellar system known as MWC349.

3.4.1 *Planetary Nebulae*

As the name implies, planetary nebulae are ionized regions immediately surrounding individual stars. There are no planets, however. Typically, their masses range from approximately 1 to $8 M_\odot$, including the mass of their central star. As a class, they are much smaller than the discrete H II regions and molecular clouds that have been discussed earlier in this book. These masses range from a few to more than $10,000 M_\odot$, depending upon where one chooses to draw the line between entity and association.

The shapes, sizes, color temperatures, and luminosities of planetary nebulae are varied. Despite this heterogeneity, most are easily recognizable as belonging to a distinct astronomical class. The hallmark is a void separating a thin nebular halo(s) from a centrally located bright star. In general, the “typical” morphology is that of a spherical (annular) envelope or shell surrounding a star, separated by a visual void between the star and the envelope. However, multiple concentric rings or shells can often be seen, as well as non-circular shapes like dumbbells. They are believed to represent a late stage in stellar evolution, connecting the red giant branch of the Hertzsprung–Russell diagram with the white dwarf branch by an unobserved process in which an older, evolved star separates into a hot ionized envelope and a core which, in turn, quickly becomes a white dwarf.

In studying them, the most important parameter is their distance (see the monographs by Pottasch (1984) and Kwok (1999)). It not only locates them with respect to other constituents of the Galaxy, but also allows conversion of their observed intensities into physically useful units at the nebulae.

Unfortunately, this is difficult to obtain. There is no direct kinematical method of determining their distances accurately. Trigonometric parallax does not work because, in general, the planetary nebulae are too far from Earth. Spectroscopic parallax does not work because the emission spectra of their central stars are so different that it is impossible to establish a standard “candle” for calibration. As a result, distances listed for planetary nebulae come from a variety of techniques, including interstellar extinction, angular expansion rates of their shells, associations with nearby normal stars, etc. The most commonly used method combined spectral estimates of electron density with a presumption that all planetaries have about the same mass – the “Shklovsky” method (Shklovsky, 1956a) – to allow angular sizes to be converted into distances. This turned out to be a flawed assumption but the method with refinements was used for more than 25 years. Consequently, one must consider quoted distances as the best available but having significant uncertainties.

Spatially, planetary nebulae are distributed within a sphere surrounding the Galaxy rather than confined to the Galactic plane. Because of this, they are often called Population II objects to distinguish them from the younger, earlier-type stars (Population I) confined mainly to the Galactic plane. Based upon a convergence of theory and observation, the stars associated with planetary nebulae are considered to mark an early phase of the development of a galaxy.

The physical characteristics of planetary nebulae vary widely. Figure 3.50 shows the continuum spectra for three compact planetary nebulae. The right-most part of the spectra arises from heated dust in the envelope with a temperature of about one or two hundreds of Kelvins, and the leftmost arises from free-free emission from hot electrons of roughly 10,000 K in the same general location. Note the wide variation in the turnover frequency, from 2 to 200 GHz, which indicates a correspondingly wide range of large emission measures (see (2.95) and (2.105)).

Because the envelopes are composed of hot, tenuous ionized gas, they emit also recombination lines in the radio wavelength regime. However, the angular sizes of the ionized shells are small and, consequently, the RRLs are weak and difficult to detect with single element radio telescopes. For example, Terzian (1990) wrote “Although the number of identified planetary nebulae in the Galaxy is more than one thousand, only about ten have shown detectable radio recombination lines.” At this writing, the number of catalogued planetary nebulae has grown to about 1,500 (Terzian, 2002) and the RRL detections have grown proportionally.

Table 3.8 lists most of the planetary nebulae for which RRLs have been detected up to the year 2001. The table includes full line widths at half-intensity (ΔV), LTE electron temperatures (T_e^*), non-LTE electron temperatures (T_e) where available, and the derived emission measures (EM).

Although not all RRL observations are listed, the listings are sufficient to identify distinctive characteristics of planetary nebulae. First, the derived

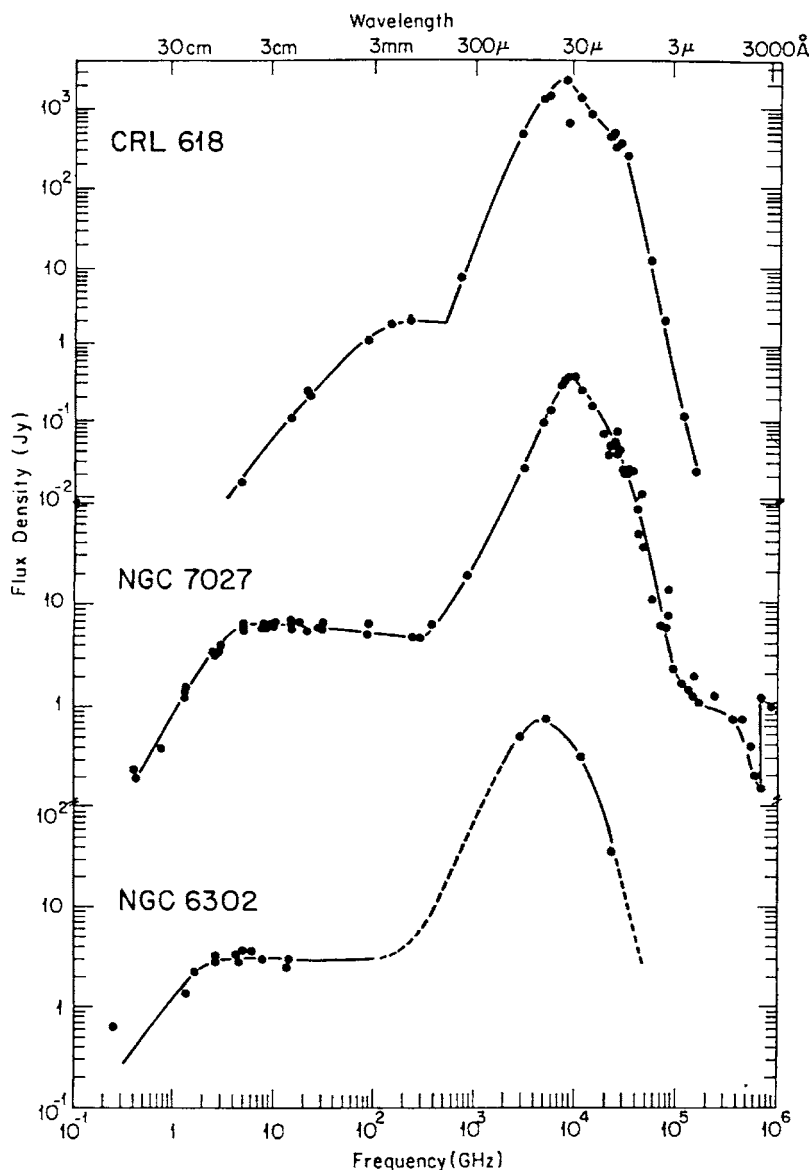


Fig. 3.50 The continuum spectra of the compact planetary nebulae CRL618, NGC7027, and NGC302. The lines have been fitted to the observations. *Broken segments* indicate uncertainty. From Terzian (1990)

LTE temperatures tend to be significantly greater than those observed for discrete H II regions. These temperatures agree well with those determined optically (Terzian, 1990). Second, the emission measures are often one or two orders of magnitude larger than those derived for most discrete H II regions.

Table 3.8 Some planetary nebulae with detected RRLs

| Nebula | Dist. (kpc) | Diam. ($''$) | RRL | ΔV (km s^{-1}) | T_e^* (K) | T_e (K) | EM (cm^{-6} pc) | References |
|-----------|----------------|-------------------|---------------|--------------------------------------|----------------|--------------|------------------------------|--|
| IC418 | 1.8 | 12 | H76 α | 26 \pm 2 | 8,400 | 9,800 | 2×10^7 | Walmsley et al. (1981) |
| CRL618 | 1.7 | ≈ 0.7 | H76 α | 27 \pm 1 | 9,600 | 9,500 | 6.0×10^6 | Garay et al. (1989) |
| | | | H41 α | 30 \pm 9 | 17,800 | | $4 - 10 \times 10^{10}$ | Martín-Pintado et al. (1988) |
| | | | H35 α | 32 \pm 1 | 12,400 | | $4 - 10 \times 10^{10}$ | Martín-Pintado et al. (1988) |
| | | | H30 α | 48 \pm 8 | 13,500 | | $4 - 10 \times 10^{10}$ | Martín-Pintado et al. (1988) |
| NGC1514 | 0.2 | 100 | H140 α | 43 \pm 1 | (14,000) | | $4 - 10 \times 10^{10}$ | Terzian (1990) |
| NGC2440 | 1. | 11 | H92 α | 60 \pm 5 | 16,000 | | 8×10^5 | Vázquez et al. (1999) |
| NGC6302 | 2.2 | 25 | H76 α | 40 \pm 3 | 18,000 | | (7.5×10^7) | Gómez et al. (1989) |
| | | | H110 α | 56 \pm 2 | 21,000 | | (7.5×10^7) | Gómez et al. (1989) |
| NGC6369 | 2.0 | 12 | H76 α | 43 \pm 6 | 12,100 | | 1.6×10^6 | Terzian et al. (1974) |
| NGC6572 | 1.2 | 12 | H76 α | 32 \pm 3 | 12,900 | 15,000 | 3×10^7 | Walmsley et al. (1981) and Kawamura and Masson (1996) |
| BD30+3639 | 1.5 | 11 | H76 α | 50 \pm 5 | 5,900 | 7,600 | 3×10^7 | Walmsley et al. (1981) and Kawamura and Masson (1996) |
| NGC7009 | 0.1 | 29 | H76 α | 44 \pm 6 | 9,700 | 12,000 | 1.6×10^6 | Garay et al. (1989) |
| NGC7027 | 1.0 | 14 | H76 α | 48 \pm 2 | 11,700 | 14,000 | 7×10^7 | Walmsley et al. (1981) |
| M1-78 | 5.0 | 12 | H76 α | 37 \pm 5 | 16,200 | 17,500 | 5×10^7 | Walmsley et al. (1981) |
| | | | H85 α | 54 \pm 12 | 12,800 | | | Terzian et al. (1974) |
| IC5117 | 3 | 1.2 | H109 α | 65 \pm 12 | 8,200 | 11,800 | 2.9×10^7 | Churchwell et al. (1976) |
| | | | H92 α | 49 | 11,000 | | 1×10^8 | Miranda et al. (1995) |
| NGC7662 | 1.7 | 26 | H76 α | 27 \pm 5 | (19,000) | (19,000) | 4×10^6 | Walmsley et al. (1981) |
| Hb12 | 2.1 | 1 | H92 α | 35 | 14,000 | | 3×10^8 | Miranda et al. (1995) |

Note: Parentheses indicate approximate results. This table may not list all detections

Such large values of EM also imply high electron densities and, in fact, Stark broadening has been detected for the low-frequency RRLs from planetary nebulae. Walmsley et al. (1981) analyzed a series of recombination lines observed from NGC7027. While the width data had smaller signal-to-noise ratios than desired, the corresponding variation of the T_L/T_C ratios with frequency also implied a systematically increasing line width as a function of principal quantum number. On this basis, Walmsley et al. cautiously implied the presence of Stark broadening consistent with $N_e \approx 10^5 \text{ cm}^{-3}$ for NGC7027. More recently, Ershov and Berulis (1989) found an electron density of $(6.7 \pm 0.5) \times 10^4 \text{ cm}^{-3}$ for NGC7027 using six RRLs from H56 α to H110 α . VLA observations of IC418 showed the width of the H110 α (5 GHz) line to be significantly larger than that of the H76 α (15 GHz) line (Garay, Gathier and Rodríguez, 1989). These widths implied $N_e = 1.8 \times 10^4 \text{ cm}^{-3}$ when assumed to involve Stark broadening.

The presence of Stark broadening has been used to determine the distance for at least three planetary nebulae. VLA observations of the H76 α and H110 α lines from NGC6302 also showed dissimilar line widths likely due to the presence of Stark broadening (Gómez, Moran, Rodríguez and Garay, 1989). Determining the Doppler profile and electron temperature from the H76 α profile, Gómez et al. fitted a Voigt profile to the H110 α line and derived an increase in the width of $20 \pm 4 \text{ km s}^{-1}$ due to electron impacts. This width implied a mean electron density for the planetary nebula of $(2.5 \pm 0.5) \times 10^4 \text{ cm}^{-3}$. Further, it agreed well with the density of $2 \times 10^4 \text{ cm}^{-3}$ derived for the center of the nebula from optical data (Meaburn and Walsh, 1980). Using this density and temperature, Gómez et al. adjusted the assumed distance to the planetary nebula until the model flux density matched the observations, thereby determining a distance to NGC6302 of $2.2 \pm 1.1 \text{ kpc}$. Applying this technique to similar observations resulted in distances of 1.0 and 0.2 kpc for NGC7027 and IC418, respectively, with comparably large uncertainties.

While none of these distances have as high precision as might be desired, nevertheless the technique points the way to a way of determining distances for planetary nebulae from RRLs. What is needed are higher accuracy line profiles to better establish N_e from Stark broadening as well as sophisticated models with which to calculate the radio flux densities.

One especially interesting way of using RRLs to determine the characteristics of planetary nebulae is to consider the effects of optical depth upon the observed radial velocities. Ershov and Berulis (1989) noted that the radial velocities of RRLs emitted by NGC7027 should vary as a function of frequency. The idea is that the line opacity would vary inversely with frequency and, because of the expansion gradient of the planetary nebula, would allow the expansion velocity to be determined from observations of RRLs over a large range of principal quantum numbers. The dependence of the velocity variation would also tell whether the nebula was expanding or contracting. An increase of radial velocity with frequency would imply expansion. Combining

observations of the H56 α , H66 α , H76 α , H85 α , H90/94 α , and H110 α RRLs, Ershov and Berulis fitted a model to NGC7027 with $N_e \approx 6 \times 10^4 \text{ cm}^{-3}$, $T_e \approx 15,000 \text{ K}$, $V_{LSR} \approx 26 \text{ km s}^{-1}$, and $V_{Exp} \approx 21 \text{ km s}^{-1}$. Combining this linear expansion rate with the measured angular expansion of $0''.0047 \text{ yr}^{-1}$ gives the distance to NGC7027 of $940 \text{ pc} \pm 20\%$ (Masson, 1986).²³

At present, the parameters derived from RRL observations have shown excellent agreement with the physical conditions and distances determined by other techniques. This is a good foundation from which to grow as the sensitivities of radio telescopes increase. In particular, unlike optical lines, radio recombination lines are unaffected by interstellar extinction and would have an advantage for determining the physical characteristics of obscured planetary nebula lying in the Galactic plane.

3.4.2 The Sun

In principle, RRLs can be detected from the Sun. Dupree (1968) noted that highly stripped ions in the solar atmosphere have an overpopulation of their high quantum states. This overpopulation can lead to enhanced intensities – hopefully, detectable – of RRLs emitted by the Sun.

The n -dependencies of the rates describe the situation. According to Dupree, radiative recombination from the continuum is the principal process populating the upper principal quantum levels of hydrogenic atoms in the solar corona with a rate $\propto n^{-3}$. On the other hand, dielectronic recombination is the principal means of forming complex ions, with a rate $\propto n^{-1}$ for principal quantum numbers $20 \leq n \leq 200$. Because the depopulation rate by spontaneous emission is $\propto n^{-5}$, the level population of the complex ions in this range of quantum numbers can vary as n^4 , giving a significant overpopulation of the principal quantum levels populated $\propto n^2$ in LTE. This gives a corresponding enhancement of the intensities of RRLs from the complex ions over the LTE values.

The initial calculations were made for FeXV because of the availability of these rates for ordinary dielectronic recombination. Added to these were the contributions of cascade transitions between levels. Figure 3.51 shows the results for $T_e = 3 \times 10^6 \text{ K}$ and $N_e = 10^8 \text{ cm}^{-3}$. As the quantum levels increase beyond 10, the departure coefficients increase because of dielectronic recombination. Note that the coefficients can reach values of several hundred. When the ion becomes so large that collisional de-excitation becomes important, the departure coefficients begin to decrease sharply. The quantum number at

²³ The method of combining linear and angular expansion rates is a well-known method of determining distances to planetary nebula. In fact, Masson combined older determinations of the linear expansion velocity with new, precise VLA observations of the angular expansion rate. Later, Ershov and Berulis independently determined the linear expansion rate from RRL observations, which confirmed the assumed expansion rate adopted by Masson.

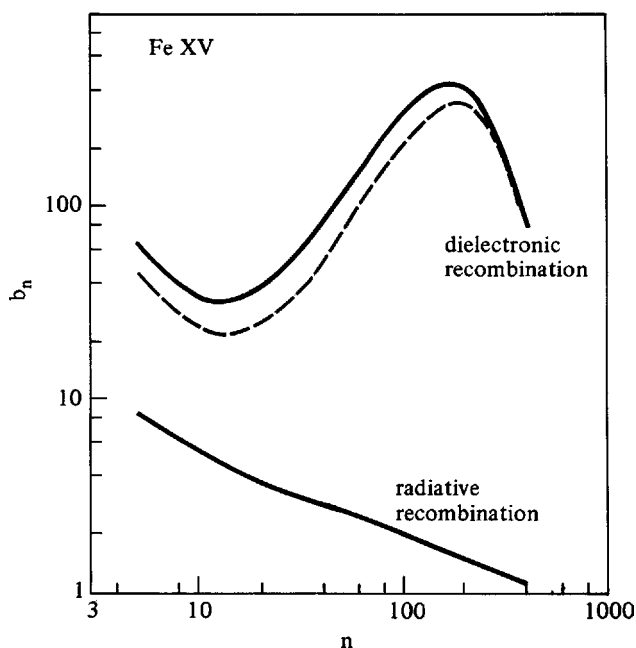


Fig. 3.51 The departure coefficients for principal quantum numbers of Fe XV in the solar corona. *Upper curves* include dielectronic calculation with (*solid line*) and without (*broken line*) cascades between levels. The *lower curve* results from populating only by radiative recombination from the continuum. From Dupree (1968)

which this inflection takes place depends upon the electron density. Higher densities move the inflection toward lower quantum numbers, because of the increased frequency of collisional de-excitation.

Similar calculations can be made for other ions in the Sun. Dupree considered recombination lines from O VI good possibilities for detection in the Sun because of its abundance. Using the departure coefficients calculated for sublevels of Fe XV, she found – very roughly – that the central intensity of its 100α recombination line ($\lambda = 1.28$ mm or 235 GHz) could occur in emission with an intensity of about 2% of the background continuum at the center of the solar disk. On the other hand, the 200α line ($\lambda = 10.2$ mm or 29 GHz) could occur in absorption because of the negative gradient of the departure coefficients of the principal quantum numbers. For large quantum numbers like this one, Stark broadening could smear out the line profile, rendering the RRL unobservable.

Such calculations were quite approximate because, except possibly for the less abundant Fe XV ion, many details regarding the population rates for ions were not known. Nevertheless, the possibility of a new tool for the study of the solar atmosphere generated a great deal of interest.

The initial searches proved disappointing. Dravskikh and Dravskikh (1969) looked unsuccessfully for solar RRLs near $\lambda = 6$ cm (5 GHz). Berger and Simon (1972) tried to detect them at frequencies from 85 to 92 GHz for ions from $Z = 1 \rightarrow 15$ with the 36-ft. (11-m) telescope of the National Radio Astronomy Observatory on Kitt Peak, AZ. No lines were detected. Revisiting the theory, they recalculated the departure coefficients as a function of principal quantum number and, based partly on relative abundance, estimated that solar RRLs from C III, O IV, O V, Ne VII, and Si XI were good candidates for detection. Furthermore, barring unexpectedly large Stark broadening, they showed that the best wavelength range for the searches would be $3 \text{ mm} < \lambda < 3 \text{ cm}$ based upon the calculated line intensities.

Additional searches also failed to detect RRLs from the Sun. Shimabukuro and Wilson (1973) searched at 110–115 GHz for lines from ions with $Z = 3$ –14, more or less following the recommendations of Berger and Simon (1972) to avoid principal quantum numbers that might be subject to Stark broadening. The lines just were not there.

It was again time to scrutinize the theory. Possibly, Zeeman splitting from intense solar magnetic fields could broaden the lines, thereby reducing their intensities (Greve, 1975). Finally, carefully reconsidering the details of radiation transfer and including new estimates of Stark broadening, Greve (1977) calculated that the previous searches for solar recombination lines were at least a factor of 10^2 – 10^3 too insensitive to detect the lines because of pressure broadening by electrons. He suggested that observers would need to establish a limit of at least 10^{-5} for the line to continuum ratio to detect solar RRLs in the millimeter wavelength range. In other words, the previous observing limits were insufficient for detection by a very large factor.

Because of the difficulties in detecting RRLs from coronal ions, Khersonskii and Varshalovich (1980) examined the detection prospects for $Hn\alpha$ lines for $n = 24$ –36 (1,747–135 GHz or $\lambda = 0.63$ –2.13 mm) originating in the solar chromosphere. In this range of n , the collision rates would determine the level populations such that the line intensities would be near LTE and, as such, might be only marginally detectable. Within the range considered, they suggested that the best candidates would lie near $H24\alpha$ where $T_L/T_C \approx 10^{-2}$, with the intensities of higher- n lines sharply decreasing because of increasing Stark broadening. For example, they predicted $T_L/T_C \approx 10^{-4}$ for the $H36\alpha$ line. They concluded that the lower- n lines would be detectable and, in special circumstances such as a flare, might actually be enhanced.

Similarly, Hoang-Binh (1982) also looked at the possibility of detecting normal hydrogen RRLs from the solar chromosphere but in the far infrared and submillimeter wavelength regimes. His calculations indicated that the $Hn\alpha$ RRLs in the range $5 \leq n \leq 20$ should be much stronger than RRLs from coronal ions, making them better candidates for detection. Like Khersonskii and Varshalovich, he found pressure broadening to be an important consideration. The high electron densities of the chromosphere, $N_e \approx 6 \times 10^{10}$

to 10^{11} cm^{-3} , would manifest themselves in detectable pressure broadening even at the chromospheric temperatures of approximately 6,000 K.

These two wavelength ranges proved successful. Solar emission lines from ions were first detected in the mid-infrared (Murcray, Goldman, Murcray, Bradford, Murcray, Coffey and Mankin, 1981; Brault and Noyes, 1983) and later identified as hydrogenic ($n = 7$) transitions from Mg I, Al I, and possibly Ca I (Chang and Noyes, 1983). A year later, Chang (1984) identified the remaining solar lines as due to hydrogenic transitions from Si I. Since then, astronomers have observed the H15 α (1,770 GHz or 59.0 cm^{-1}) and H13 α (2,680 GHz or 89.4 cm^{-1}) (Boreiko and Clark, 1986), H19 α (888 GHz or 29.6 cm^{-1}) (Clark, Naylor and Davis, 2000a), and H21 α (662 GHz or 22.1 cm^{-1}) (Clark, Naylor and Davis, 2000b) lines in the solar chromosphere. Figure 3.52 shows probably the highest frequency *radio* recombination line detected in the Sun.

Although challenging to observe, these lines not only are useful for probing the thermodynamic conditions within the chromosphere, but also provide another cosmic laboratory for investigating Stark broadening and other physics associated with Rydberg atoms.

Although many of these solar recombination lines lie well outside the radio range, the search for them surely originated with the theoretical work of Dupree (1968), stimulated by the detection of centimeter wave RRLs from discrete H II regions about 20 years earlier. They begin a new avenue of astronomical research involving the solar chromosphere.

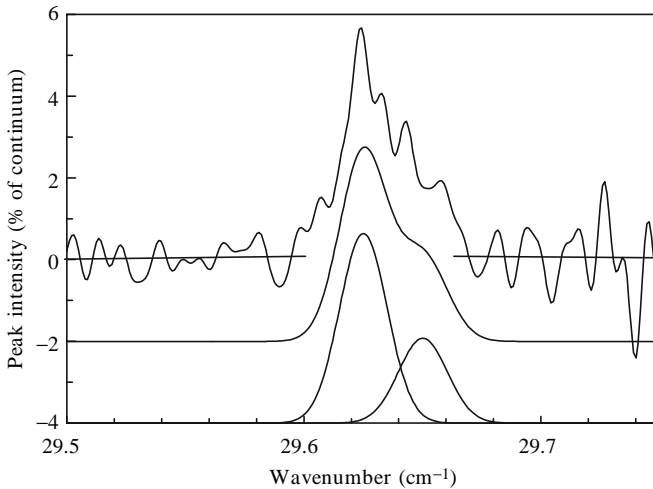


Fig. 3.52 The H19 α (888 GHz or 29.6 cm^{-1}) radio recombination line detected in the solar limb with a polarizing interferometer and the James Clerk Maxwell radio telescope on Mauna Kea, HI. The two fitted profiles and their sum are shown below the spectrum. The weaker profile is probably the Mg19 α line. From Clark et al. (2000a)

3.4.3 MWC349

One of the most intriguing sources of RRLs is MWC349. This is a binary system lying about 1.2 kpc from the Sun. One member, MWC349A, has been classified as a Be star (Merrill and Burwell, 1933); the other, MWC349B, a B0 III star. The optical and infrared spectra from MWC349A are complicated (Cohen, Biegging, Dreher and Welch, 1985). The luminosity of its Lyman continuum is about 10^{48} s^{-1} , comparable with that of some O stars. Many of its IR emission lines have double peaks as might be expected from a circumstellar disk viewed edge on. In the discussion below, we will refer to MWC349A as MWC349 for simplicity.

The search for RRLs from MWC349 was based upon its unusual nature. Its continuum spectra had long been known to be peculiar. Figure 3.53 shows the spectrum determined by combining radio and infrared observations (Harvey, Thronson and Gatley, 1979). First, note the $\lambda = 100 \mu\text{m}$ flux density is more than an order of magnitude lower than the peak emission, indicating that the source of the infrared emission – presumably dust – is extremely hot. Second, note the slowly varying flux density ($\propto \nu^{0.6}$) in the radio wavelength region, unlike the ν^{-2} dependence seen in the optically thick region of spectra from standard discrete H II regions, such as those of Figs. 2.16 and 2.17. On the basis of its continuum spectrum, the accepted model for MWC349 was a hot star with a stellar wind of about 50 km s^{-1} (Olmon, 1975).

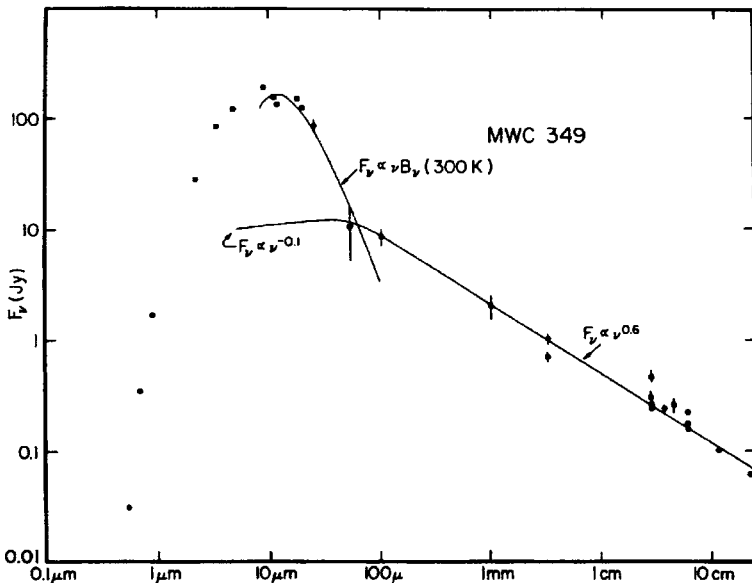


Fig. 3.53 The continuum spectrum of MWC349 fitted with a two-component model. Spectral flux density is plotted against wavelength. From Harvey et al. (1979)

Consequently, groups searching for RRLs from stars placed MWC349 high on their list of candidates. The source had the necessary characteristics: high-intensity radio emission for easy detectability, and indications of copious amounts of moderate-density ionized gas to generate RRLs with line widths less than the spectrometer bandwidths. And, in fact, Altenhoff et al. (1981) had been able to detect $H76\alpha$ (15 GHz) and $H66\alpha$ (23 GHz) RRLs from this system with the 100-m radio telescope at Effelsberg but they were very weak.

Using the 30-m IRAM telescope on Pico Veleta, Spain, Martín-Pintado et al. (1989) pressed the search for stellar RRLs to much higher frequencies. They were able to detect the $H29\alpha$ (256 GHz), $H30\alpha$ (232 GHz), $H31\alpha$ (211 GHz), and $H41\alpha$ (92 GHz) RRLs from MWC349 with high signal-to-noise ratios.

These new lines have unusual characteristics, which the more recent spectra shown in Fig. 3.54 illustrate. This figure compares the high-frequency line profiles from MWC349 with those from the normal H II region DR21. The higher-frequency lines of MWC349 have two principal components and the lower-frequency line has one. Below these, the DR21 spectra exhibit single Gaussian profiles with full widths at half-intensity of approximately 40 km s^{-1} . Of these widths, about half is due to thermal broadening from the $9 \times 10^3 \text{ K}$ gas and the remainder is due to microturbulence within the beamwidth (see (2.18), (2.22), and (2.26)). In contrast, the widths of each of the high-frequency MWC349 components are approximately 10 km s^{-1} – a situation one would expect from, say, a 2,000 K gas with a quarter of the microturbulence normally observed in H II regions. Furthermore, both components sit on a broad, weak pedestal of approximate width 50 km s^{-1} .

In addition, the T_L/T_C ratios differ from RRLs observed from normal H II regions. For example, those from the H II region DR21 increase with increasing frequency, just as expected (see Fig. 3.3). This behavior is very different for MWC349; the variation of T_L/T_C increases much more sharply with frequency for the high-frequency, double-peaked lines. In comparison, the lower-frequency $H40\alpha$ line appears almost normal.

These are the characteristics one would expect from masering RRLs. Accordingly, Martín-Pintado et al. (1989) suggested this explanation in the detection announcement. Subsequent observations of the intensity ratios of $Hn\alpha$ to $Hn\beta$ lines – interlocking lines – have confirmed that the level populations associated with the peculiar RRLs are far from LTE in a way expected for masering transitions (Gordon, 1994; Thum, Strelinski, Martín-Pintado, Matthews and Smith, 1995).

Following the detection, observations revealed another interesting characteristics of the RRLs from MWC349: time variation of the line intensities and radial velocities. Figure 3.55 shows the intensity variations for the components of the $H30\alpha$ line over 3 years. The line areas are referenced to the continuum emission of the calibrator, DR21, because of its greater intensity and intensity stability. Note that there seems to be a decrease in the relative line emission of the pedestal component but the signal-to-noise ratio is

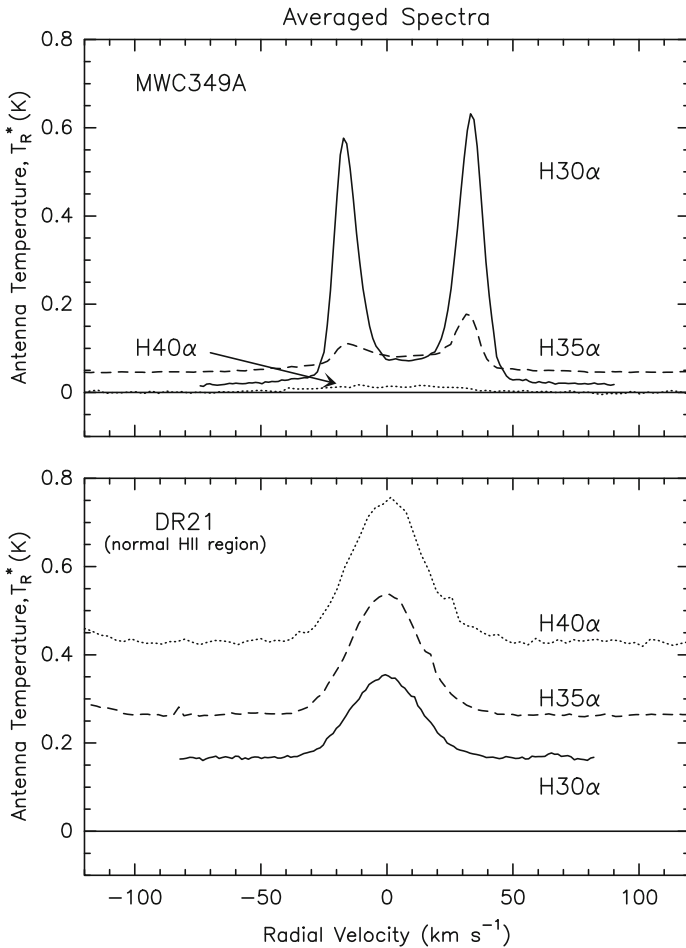


Fig. 3.54 *Top:* $\text{H30}\alpha$ (232 GHz), $\text{H35}\alpha$ (147 GHz), and $\text{H40}\alpha$ (99 GHz) line profiles averaged over 3 years of observations of MWC349. *Bottom:* the same lines from the normal HII region DR21. From Gordon et al. (2001)

marginal. The time variation of the red-shifted component is clearly significant in terms of the observational uncertainties. The intensity varies over a factor of 3 with no monotonic pattern. The same situation obtains for the blue-shifted component.

Figure 3.56 shows the time variations in the radial velocities of Gaussians fitted to the components of the $\text{H30}\alpha$ line. There are significant variations in the radial velocities over the 3-year observing period. It appears that the two red- and blue-shifted components move away each other and then toward each other, as can be seen in the two upper panels and in the lower panel. Meanwhile, the average of the two radial velocities – the “system” velocity – changes very little.

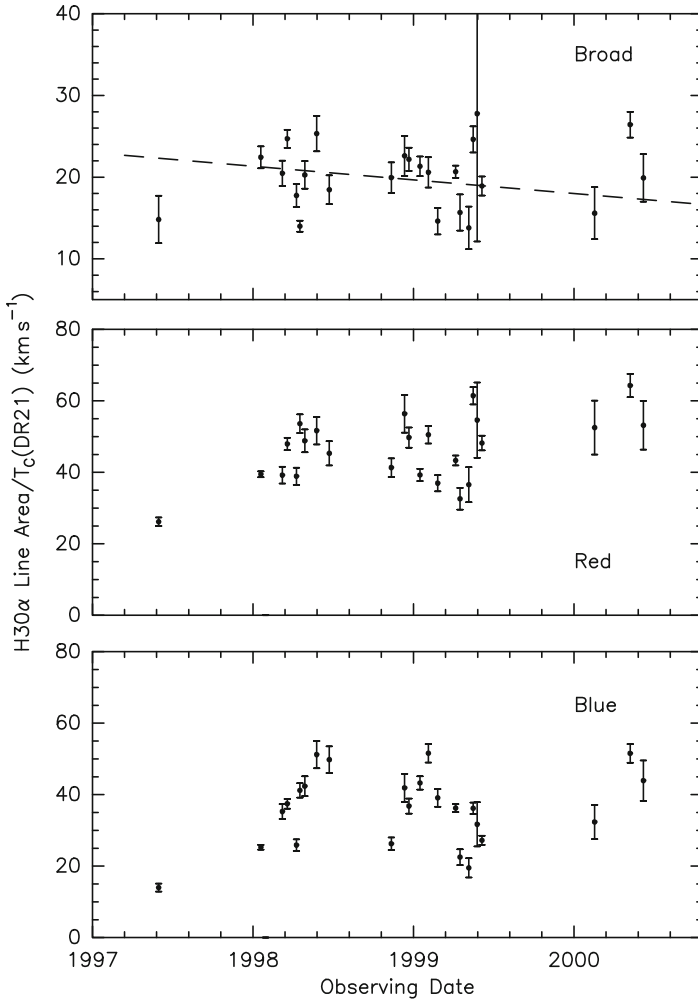


Fig. 3.55 Intensity variations of the H30 α RRL from MWC349 over 3 years. *Top*: the area-to-(DR21)continuum ratio of the underlying pedestal line with a fitted regression line. *Middle*: same, for the red-shifted line component. *Bottom*: same, for the blue-shifted line component. From Gordon et al. (2001)

These characteristics all point to radial movements of the maser emission within a circumstellar disk viewed edge on (Gordon, 1992; Thum et al., 1992), the model originally suggested by Hamann and Simon (1986; 1988) to account for the two-component line profiles observed for IR and FIR lines from MWC349. The red-shifted component would be located on the side of the stellar disk moving away and the blue-shifted component would be located on the side moving toward us.

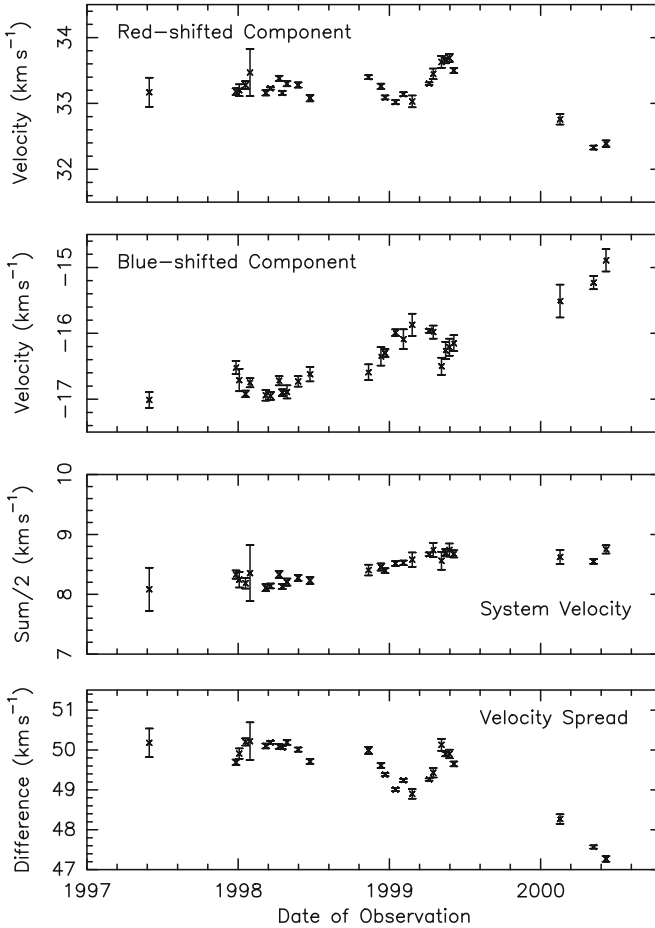


Fig. 3.56 Velocity variations of the H30 α RRL from MWC349 over 3 years. *Top*: the radial velocity of the Gaussian fitted to the red-shifted component. *Upper middle*: same, for the blue-shifted line component. *Lower middle*: one-half of the sum of the velocities; i.e., the radial velocity of the MWC349 RRL “system.” *Bottom*: the difference between the red- and blue-shifted radial velocities. From Gordon et al. (2001)

Observations by high angular resolution synthesis telescopes support the disk model. Planesas et al. (1992) imaged the H30 α line (231 GHz) with the Caltech interferometer in the Owens Valley, CA. They found a separation between the blue- and red-shifted peaks of about 80 AU, if MWC349 lies 1.2 kpc from the Sun. Furthermore, the two peaks lay along a line perpendicular to the axis of the bipolar radio continuum associated with the star. Figure 3.57 shows the approximate geometry.

Since the initial detection of RRLs from MWC349A in 1981, astronomers have observed recombination lines from H76 α (15 GHz) to H6 α

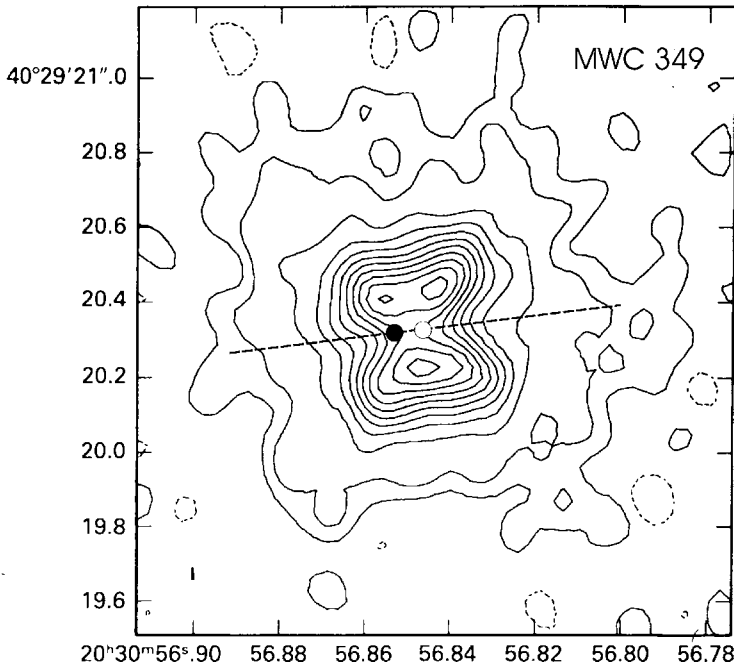


Fig. 3.57 The *circles* mark the relative locations of the *peaks* of the $H30\alpha$ line components observed by Planesas et al. (1992). The *left circle* corresponds to the red-shifted line component and the *right circle* corresponds to the blue-shifted component. The *broken line* marks a position angle of approximately 107° . The *background contours* mark the VLA observations of the MWC346 continuum emission at $\lambda = 2$ cm (White and Becker, 1985). The *ordinate* is 1950.0 declination and the *abscissa* is 1950.0 RA. The absolute location of the circle structure with respect to the contours is a guess

($\approx 20,000$ GHz). Of these, the lines $H40\alpha$ (99 GHz) through $H6\alpha$ are masering to some extent. Furthermore, the average radial velocity of these profiles increases with frequency, as shown in Fig. 3.58. For double-peaked profiles, “average” means the average of the red- and blue-shifted velocity components when both are present.

Very generally, the model of the circumstellar, Keplerian disk viewed edge on accounts for the salient kinematic observations but does not model the masering itself nor the observed time variations. To investigate these, Hollenbach et al. (1994) considered photoevaporation of circumstellar disks around massive O stars to explain some of the details of ultracompact H II regions. Figure 3.59 illustrates their two models of accretion disks involving high-velocity winds. The weak wind model creates a thin, neutral accretion disk with a width that increases significantly with distance from the central star (the disk is not shown in the figure). This disk is sandwiched between an H II atmosphere maintained by Lyman continuum photons from the star. The critical radius r_g is where the sound speed equals the escape velocity from

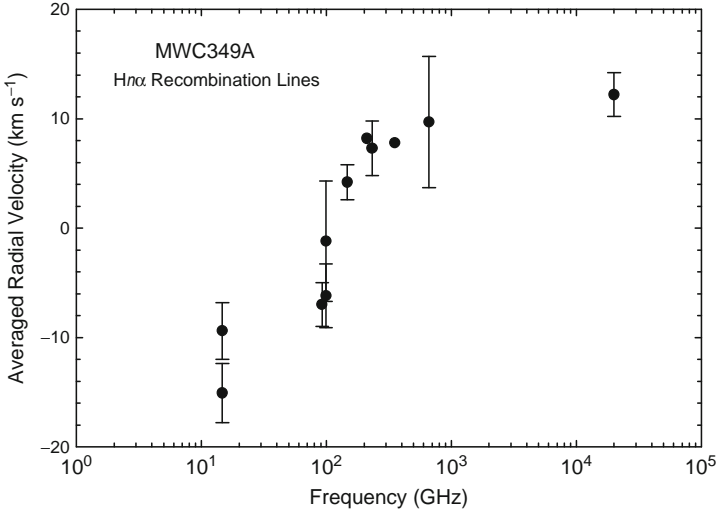


Fig. 3.58 The averaged radial velocity of $Hn\alpha$ recombination lines from MWC349A plotted against their rest frequencies. Data listed by Gordon et al. (2001) and plotted in Gordon (2003)

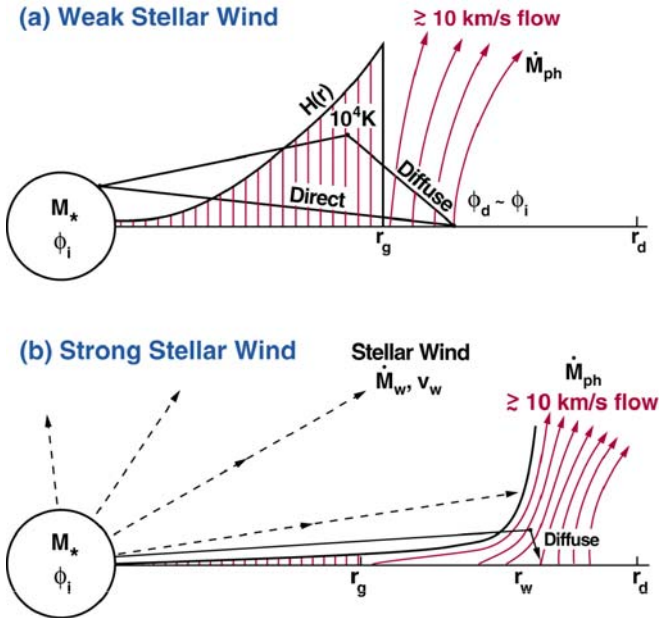


Fig. 3.59 Two models of an evaporating disk viewed edge on surrounding a massive O star, M_* , with a Lyman continuum flux density ϕ_i , stellar mass loss rate of \dot{M}_* , and disk mass loss by photoevaporation of \dot{M}_{ph} . (a) Star with a weak stellar wind, forming a 10^4 K H II disk with a radius r_g and a scale height $H(r)$. The UV photons evaporate the gas outside of r_g . (b) Star with strong stellar wind, forming a more flattened disk and pushing the H II material beyond r_g to where the ram pressure equals the thermal pressure from the ionized flow. Cartoon courtesy of Hollenbach (2002)

the disk, and marks the radius where evaporation begins. The bipolar outflow is probably responsible for the butterfly-shaped continuum emission shown in Fig. 3.57. The strong wind model entails a stellar wind sufficiently strong to blow the atmosphere of the accretion disk very far from the star; the ram pressure thereby narrowing the disk and accentuating the evaporative wind at its outer terminus.

Of these models, MWC349A seemed to fit the weak wind model best. The slow variation of the radio continuum emission with frequency – see Fig. 3.53 – suggests a significant disk component, i.e., one with adequate free-free absorption, must exist that in turn favors the weak wind model. Presumably, the variations in the radial velocities would result from corresponding variations in the locations where the densities are right for masing. Such an opacity variation in the disk could also be responsible for the variation in radial velocities seen in Fig. 3.58. In a differentially rotating disk, the larger optical depths would restrict the observed emission to the near parts of the circumstellar gas.

Figure 3.60 shows a model for an outflow derived (Gordon, 2003) from the RRL observations shown in Fig. 3.58. The lines are presumed to originate at

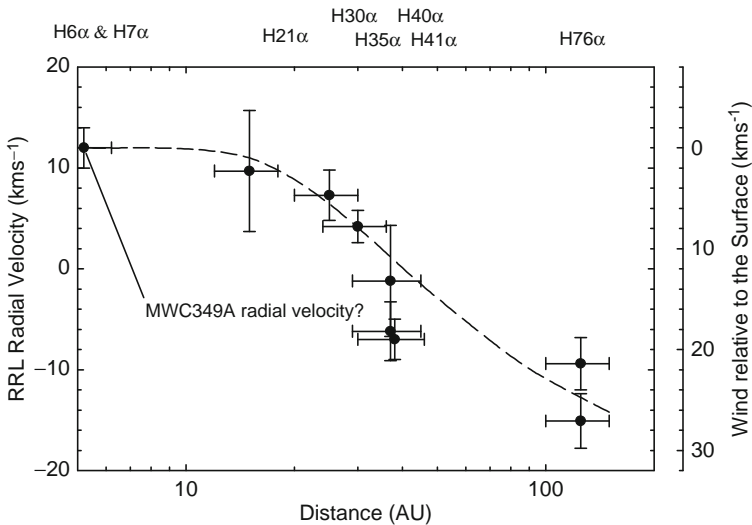


Fig. 3.60 Possible outflow from MWC349A deduced from observations of RRLs. *Points and error bars* mark the central velocities of RRLs from MWC349A with respect to the local standard of rest plotted against distance from MWC349A. Notations identify the specific RRLs observed. Distances obtain from a model that presumes each RRL originates from a location within a circumstellar disk where its optical depth is unity. The *right ordinate* indicates the velocity with respect to the H6 α velocity. The *broken line* marks an approximate beta-law fit (Lamers and Cassinelli, 1999) to the observations. Note that the radial velocity of MWC349A with respect to the LSR appears to be 12 km s⁻¹. From Gordon (2003)

locations in the circumstellar disk where $\tau(\nu)_\ell = 1$ at their particular frequency. The analysis suggests a slow gas outflow from MWC349A that could be consistent with a wind driven by photoevaporation. The result also implies a radial velocity of 12 km s^{-1} with respect to the LSR for the MWC349A star itself.

The masering has been considered elsewhere. A series of papers by Ponomarev et al. (1994) and Strel'nitski et al. (1996; 1996) describe how the masering lines could form in the circumstellar disk of MWC349A as sketched in Fig. 3.61 – a geometry first proposed by Elmegreen and Morris (1979) for disk systems in general. In this model, the maser amplification occurs within a ring. The path lengths within the ring on either side of the star are sufficient to generate intense, masering RRLs seen as red- and blue-shifted components in the line profiles. The path length along the sight line through the star is insufficient to generate lines of comparable intensity and, moreover, could have a broader line profile depending upon the velocity dispersion through the front/back ring segments.

Calculations showed that, with this ring model, an unsaturated maser could match the observed line profiles. Furthermore, the model limits the range of physical parameters for the ring that can match the observations.

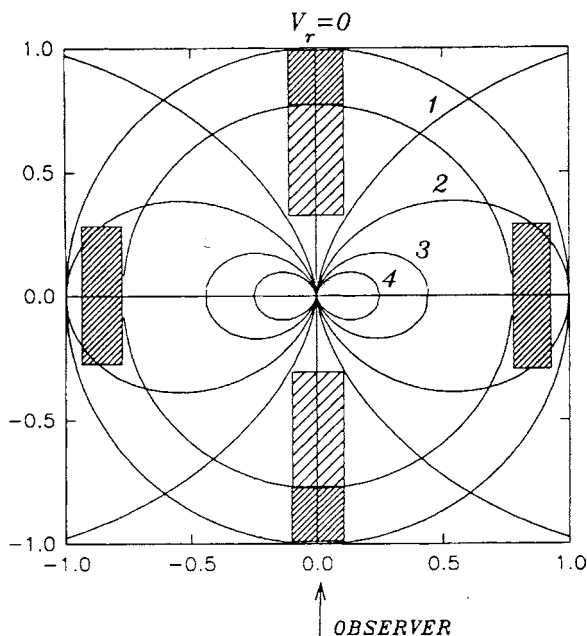


Fig. 3.61 Downward view of the differentially rotating, circumstellar disk of MWC349A. *Spider-leg curves* mark isochasms of the observed radial velocity in units of Doppler width. The maser amplification occurs within the two outer rings along sight lines marked by *crosshatching*. From Ponomarev et al. (1994)

What was not modeled was the asymmetrical conditions on either side of the ring that might account for the time variations of the red- and blue-shifted amplitudes and radial velocities. Presumably, these intensity variations are due to changes in local densities within the ring (Strelitski, Ponomarev and Smith, 1996) resulting from anisotropies in the stellar wind and, probably, to concurrent but not necessarily simultaneous variations in the Lyman photons incident upon the local masering regions.

Finally, recent observations have detected a magnetic field in MWC349A (Thum and Morris, 1999). The $H30\alpha$ lines have circular polarization characteristic of a strong magnetic field (22 mG) causing a Zeeman effect. Equipartition of the energy density between this field and the thermal environment suggests that the line emitting gas has a temperature of about 5,000 K, which is somewhat lower than expected for H II gas but an entirely reasonable value.

In conclusion, RRLs have revealed an unusual stellar system in our Galaxy, one that not only may represent an evolutionary stage of ultracompact H II regions but also provides the physical environment adequate to engender high-gain hydrogen masers. With regard to masering RRLs, the only other object like MWC349A may be the star η Carina from which narrow, asymmetrical RRLs have also been found at millimeter wavelengths (Cox, Martín-Pintado, Bachiller, Bronfman, Cernicharo, Lyman and Roelfsema, 1995). Like the lines from MWC349A, these RRLs sit atop a broad line component and have peculiar ratios of β/α intensities. Also like MWC349A, the star appears to be undergoing substantial mass loss. Unlike MWC349A, the continuum emission may be time varying.

3.5 RRLs from Extragalactic Objects

In the second half of the 1970s, RRL studies expanded toward extragalactic objects. Except for the Magellanic Clouds, the first target was the edge-on galaxy M82, located at a distance of 3.2 Mpc from the Sun. Shaver et al. (1977) soon detected the $H166\alpha$ (1.4 GHz) line with the Westerbork Synthesis Telescope in the Netherlands. They proposed that stimulated emission due to the nonthermal continuum made the principal contribution to the line intensity.

Other observers quickly confirmed this detection. Bell and Seaquist (1977) detected the $H102\alpha$ (6.1 GHz) recombination line from M82 with the 46-m Algonquin radio telescope in Canada, and Chaisson and Rodríguez (1977) detected the $H92\alpha$ (8.3 GHz) line. Subsequent observations showed the radial velocity and line width of the $H102\alpha$ and $H85\alpha$ (10.5 GHz) lines from M82 $\approx 175 \text{ km s}^{-1}$ and $\approx 150 \text{ km s}^{-1}$, respectively (Bell and Seaquist, 1978).

To investigate the circumstances of these RRLs further, Shaver et al. (1978) observed RRLs from M82 in three frequency ranges – 1.4, 4.9, and 14.7 GHz – with the 100-m Effelsberg telescope. The $H166\alpha$ (1.4 GHz) and

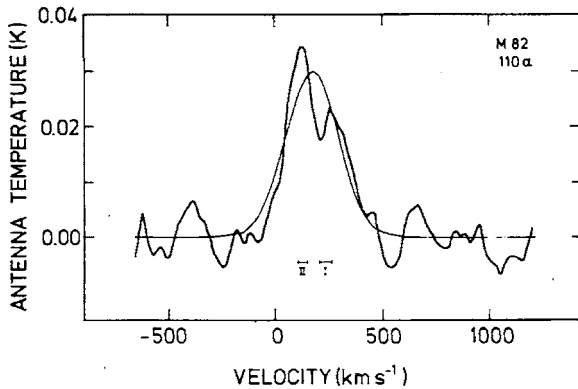


Fig. 3.62 The H110 α spectrum of M82 with a resolution of 19.2 km s^{-1} , made with the 100-m radio telescope at Effelsberg, Germany. The velocity scale is heliocentric relative to the RRL rest frequency. The *numbers I and II* mark the radial velocities of the two H II regions in M82 studied optically by Recillas-Cruz and Peimbert (1970). From Shaver et al. (1978)

the H110 α (4.9 GHz) lines were detected with good signal-to-noise ratios but the H76 α (14.7 GHz) was not seen. Figure 3.62 shows the spectrum of the H110 α line.

Comparison of all of these spectra from M82 confirmed that stimulated emission was a major factor in the line intensities. Its importance increased with wavelength – from 70% of the line intensity of the H102 α line to 90% of the H166 α line. At 14.7 GHz, the importance of stimulated emission had decreased so much that the H76 α line could not even be detected.

Nearly simultaneously with the M82 detections, Seaquist and Bell (1977) detected the H102 α line from the spiral galaxy NGC253, also located about 3 Mpc from the Sun. The radial velocity of the RRL was $132 \pm 25 \text{ km s}^{-1}$ and the line width was $309 \pm 65 \text{ km s}^{-1}$. Unlike the situation for M82, the NGC253 RRL seemed to be primarily spontaneous emission. Consequently, the authors proposed that the line originated from H II regions of $T_e \approx 5,000 \text{ K}$ embedded in the galaxy. In contrast, the M82 lines must have originated from gas in front of a background nonthermal source.

Based upon calculations, Shaver (1978) concluded that detections of RRLs with primarily spontaneous emission were possible only for galaxies with $D < 10 \text{ Mpc}$. On the other hand, if the RRL emission were primarily stimulated emission, then more distant galaxies could be detected. These galaxies would need to contain intense sources of nonthermal emission, such as quasars and radio galaxies. Because nonthermal emission increases with wavelength, Shaver suggested that the best window for searching for RRLs from such galaxies would be $1 \rightarrow 10 \text{ GHz}$. The line widths from such sources could be as large as 500 km s^{-1} .

Despite the large number of candidates, few galaxies were detected in RRL emission. New sources of extragalactic RRLs were detected only in the 1990s, more than 10 years after the detections of RRLs from M82 and NGC253. The first new detection was from NGC2146, a galaxy located about 13 Mpc from the Sun. This detection was the H53 α (42.9 GHz), made with the 45-m radio telescope of the Nobeyama Radio Observatory in Japan (Puxley, Brand, Moore, Mountain and Nakai, 1991). Later, Anantharamaiah et al. (1993) used the VLA to detect the H92 α (8.3 GHz) line in NGC3628, IC694, and NGC1365. The much greater angular resolution of the VLA at this frequency, about 3'', enabled localization of the emitting regions within these galaxies.

Figure 3.63 shows the continuum image of NGC3628 and the location of the H92 α RRL emission. This galaxy lies 11 Mpc from the Sun and is seen nearly edge on with an inclination angle of 89°. Its rotating central disk, with dimensions of about 5'', emits the recombination lines. Figure 3.64 shows observations (Zhao, Anantharamaiah, Goss and Viallefond, 1997) made with the still higher resolution of 1.5'', which resolve three distinct region of H92 α emission within the center: the nucleus, slightly SE, and slightly NW.

In the years following the initial detections, astronomers using the VLA detected H92 α line emission from five more galaxies: Arp 220, M83, NGC2146 (Zhao, Anantharamaiah, Goss and Viallefond, 1996), NGC3690 (a second component of the interacting system IC694 + NGC3690) (Zhao et al., 1997), and NGC660 (Phookun, Anantharamaiah and Goss, 1998). In general, the observations of RRLs from extragalactic objects began at decimeter and centimeter wavelengths and expanded into millimeter wavelengths.

Table 3.9 lists the detections. The first three columns list the galaxy, its type, and its distance from the Sun. The remaining columns give the

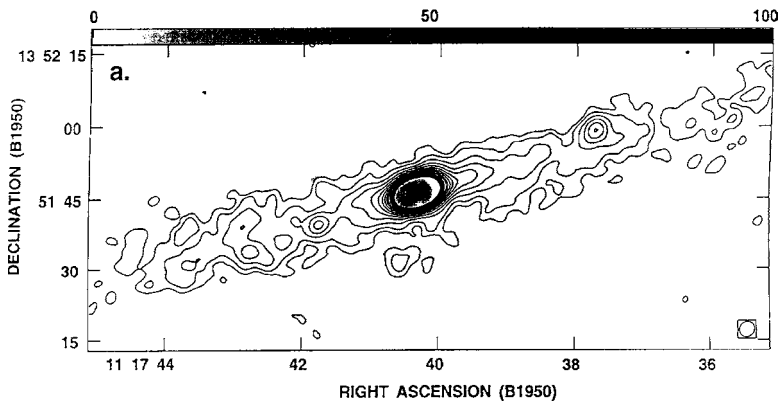


Fig. 3.63 An image of the continuum emission from NGC3628 obtained from VLA observations of the H92 α RRL. The beam size is 3.5'' \times 3.2'' and the contour levels are -0.1, 0.05, 0.1, 0.2, 0.35, 0.55, 0.8, 1.1, 1.5, 1.9, ... mJy beam⁻¹. Gray-scale densities at the top indicate the intensity of the RRL emission over the range 0 \rightarrow 100 mJy beam⁻¹. From Anantharamaiah et al. (1993)

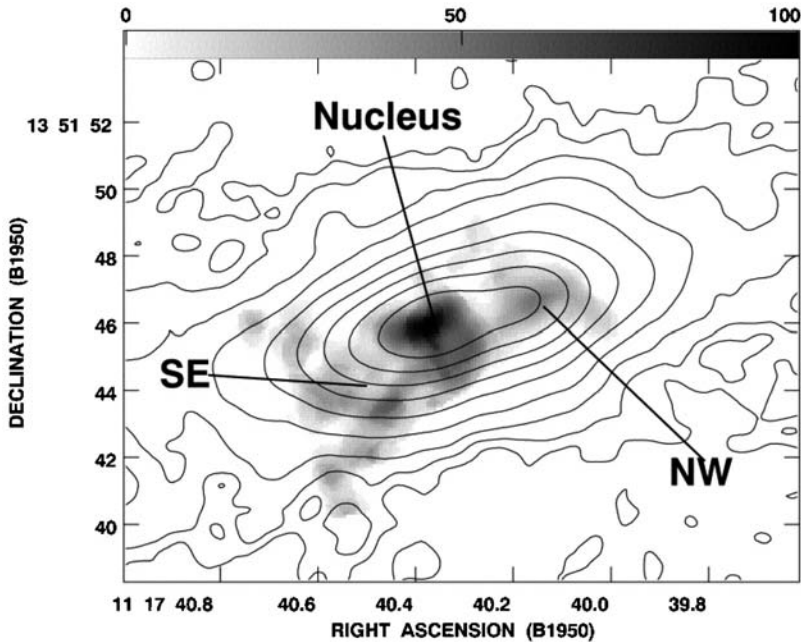


Fig. 3.64 The integrated H92 α line image of the nuclear region of NGC3628 superimposed on the continuum emission. The *gray scale at top* shows the line emission $0 \rightarrow 100 \text{ mJy beam}^{-1} \text{ km s}^{-1}$. The contour levels show the continuum intensity levels of 0.04, 0.08, 0.16, etc., to $10.3 \text{ mJy beam}^{-1}$. The beam size is $1.8'' \times 1.5''$. The three distinct RRL emission components are labeled: nucleus, SE, and NW. From Zhao et al. (1997)

transition, the line flux, the heliocentric velocity, the full line width at half-intensity, and the references. At present, RRLs have been detected from ten galaxies in addition to our own. The most distant is Arp 220. Its radial velocity $\approx 5,500 \text{ km s}^{-1}$, which corresponds to a distance of 73 Mpc for a Hubble constant of $75 \text{ km s}^{-1} \text{ kpc}^{-1}$. Attempts to detect RRLs from the more distant galaxy NGC6240, lying at 100 Mpc, have not yet been successful (Zhao et al., 1996).

These observations led to a revision of the accepted nature of RRL formation in galaxies. As mentioned earlier, theoretical calculations (Shaver, 1978) suggested that stimulated emission would be a major factor in RRLs emitted from galaxies. Therefore, observers had believed that the lines would be more intense at lower frequencies. But, the observations indicated otherwise. In many case, the RRLs were strongest at higher frequencies where stimulated emission from external sources or from the Galactic nuclei would have to be negligibly small. For example, the detections listed in Table 3.9 for M82 and Arp 220 show the line fluxes to be greater at millimeter wavelengths than at decimeter or centimeter wavelengths. The situation is similar for NGC2146. The flux of the H53 α line considerably exceeds that of the H92 α line. In turn, the intensity of the H92 α line in NGC253 can be explained if 75% of

Table 3.9 RRLs detected from extragalactic objects

| Galaxy | Type | Dist. (Mpc) | RRL | $I_L \Delta \nu_L$ ($10^{-19} \text{ W m}^{-2}$) | V_{Helio} (km s^{-1}) | ΔV_L (km s^{-1}) | References |
|----------------------|----------|----------------|---------------|---|---------------------------------------|--|--------------------------------|
| M82 | Ir | 3.25 | H27 α | 21.1 \pm 4.3 | | | Seaquist et al. (1996) |
| | | | H30 α | 5.12 \pm 1.0 | | | Seaquist et al. (1994) |
| | | | H41 α | 1.32 \pm 0.22 | | | Seaquist et al. (1996) |
| | | | H53 α | 0.22 \pm 0.06 | | | Puxley et al. (1989) |
| | | | H85 α | (4.6 \pm 0.7) $\times 10^{-3}$ | 180 \pm 10 | 149 \pm 25 | Bell and Seaquist (1978) |
| | | | H102 α | (8.5 \pm 1.4) $\times 10^{-3}$ | 173 \pm 10 | 174 \pm 20 | Bell and Seaquist (1978) |
| | | | H110 α | (1.05 \pm 0.1) $\times 10^{-2}$ | 163 \pm 10 | 293 \pm 20 | Shaver et al. (1978) |
| | | | H166 α | $> 1.7 \times 10^{-3}$ | 150 \pm 40 | 250 \pm 40 | Shaver et al. (1977; 1978) |
| | | | H166 α | (5.1 \pm 0.8) $\times 10^{-3}$ | 254 \pm 20 | 338 \pm 40 | Shaver et al. (1978) |
| NGC253 | Spiral | 3.4 | H92 α | (4.0 \pm 0.4) $\times 10^{-3}$ | 217 \pm 8 | 189 \pm 19 | Anantharamaiah and Goss (1990) |
| | | | H102 α | (1.3 \pm 0.3) $\times 10^{-2}$ | 132 \pm 25 | 309 \pm 65 | Seaquist and Bell (1977) |
| | | | H110 α | (2.9 \pm 0.6) $\times 10^{-3}$ | 209 \pm 13 | 185 \pm 32 | Anantharamaiah and Goss (1990) |
| | | | H166 α | (3.8 \pm 1.0) $\times 10^{-3}$ | 195 \pm 36 | 220 \pm 87 | Anantharamaiah and Goss (1990) |
| NGC2146 | Spiral | 13 | H53 α | (9.5 \pm 1.7) $\times 10^{-2}$ | \approx 880 | \approx 300 | Puxley et al. (1991) |
| | | | H92 α | 2.7×10^{-4} | 960 \pm 7 | 200 \pm 95 | Zhao et al. (1996) |
| NGC1365 | Seyf. II | 22.0 | H92 α | (12 \pm 2) $\times 10^{-4}$ | 1,670 \pm 80 | 310 \pm 110 | Anantharamaiah et al. (1993) |
| NGC3628 | S3 pec | 11.5 | H92 α | (8.6 \pm 1.5) $\times 10^{-4}$ | 864 \pm 56 | 170 \pm 70 | Anantharamaiah et al. (1993) |
| IC694 ^a | Sc | 40.3 | H92 α | (3.9 \pm 1.0) $\times 10^{-4}$ | 3,020 \pm 90 | 350 \pm 110 | Anantharamaiah et al. (1993) |
| NGC3690 ^a | Sc | | H92 α | (1.5 \pm 0.2) $\times 10^{-4}$ | 3,080 \pm 40 | 210 \pm 30 | Zhao et al. (1997) |
| Arp 220 | FIR | 73 | H31 α | $> 1.65 \pm 0.1$ | | | Anantharamaiah et al. (2000) |
| | | | H40 α | 0.39 \pm 0.05 | 5,513 | 179 | Anantharamaiah et al. (2000) |
| | | | H42 α | 0.22 \pm 0.02 | 5,424 | 210 | Anantharamaiah et al. (2000) |
| | | | H92 α | 3.5×10^{-4} | 5,560 \pm 70 | 320 \pm 120 | Zhao et al. (1996) |
| | | | H92 α | (8 \pm 1.5) 10^{-4} | 5,450 \pm 20 | 363 \pm 45 | Anantharamaiah et al. (2000) |
| M83 | SBc/b | 5 | H92 α | 2.8×10^{-4} | 500 \pm 30 | 95 \pm 30 | Zhao et al. (1996) |
| NGC660 | SBa pec | 11.3 | H92 α | 5.6×10^{-4} | 850 | 377 | Phookun et al. (1998) |

^aIC694 and NGC3690 are an interacting system

the line intensity is contributed by spontaneous emission (Anantharamaiah and Goss, 1990). The situation is similar for observations of the H92 α line in other galaxies.

To explain the observations of the H92 α line from the galaxies NGC3628, NGC1365, and IC694, Anantharamaiah et al. (1993) suggested a model of a collection of discrete H II regions. According to this model, the principal part of the line emission is formed in 100–200 small ($1 \rightarrow 5$ pc), dense ($N_e = 5 \times 10^3 \rightarrow 5 \times 10^4 \text{ cm}^{-3}$) H II regions with a temperature of 10^4 K. The volume filling factor of the H II regions is very small, $<10^{-5}$. In this model, only a small part of Galactic nuclear nonthermal radiation crosses the H II regions, and stimulated emission is not important. The RRL emission is primarily spontaneous, arising from the H II regions. Because the H II regions are optically thick even at centimeter wavelengths, the line emission arises from their outer layers. There, it can be amplified by stimulated emission due to the thermal continuum emission generated within the H II regions.

A similar model was developed for the NGC660 galaxy. Here, the best agreement with the H92 α line observations was obtained from a model of several thousand H II regions with $N_e = 5,000 \text{ cm}^{-3}$ and with diameters ≈ 1 pc (Phookun et al., 1998).

With this kind of model, it was possible to explain the observations of RRLs from M82 from centimeter wavelengths up to $\lambda \approx 1$ mm. The observed integrated line flux densities of the H53 α (42.9 GHz), the H41 α (92.03 GHz), and the H30 α (231.9 GHz) lines are proportional to the squares of their line frequencies. This is what would be expected from pure spontaneous emission from optically thin H II regions in LTE (Seaquist, Kerton and Bell, 1994). At the same time, millimeter wave observations of RRLs from the other detected galaxies have shown that the multi-H II region model with the same density does not work. For these galaxies, the longer wavelength observations were inconsistent with the fits to the shorter wavelength data and vice versa. Fitting these galaxies will require more complicated models.

The RRL observations of M82 have been interpreted by a two-component model of ionized gas with an electron temperature of $5,000 \rightarrow 10,000$ K. One component consists of a low-density layer with a high filling factor; the other consists of a layer of compact ($d \leq 1$ pc), dense ($N_e > 10^{4.5} \text{ cm}^{-3}$) H II regions with a small filling factor. The idea is that RRL emission from the compact H II regions contains emission stimulated by the internal continuum while that from the densest regions – up to 10^7 cm^{-3} – may be undergoing maser amplification. This model rests on the observation that the integrated line flux density of the shortest wavelength RRLs (H27 α at 316.4 GHz) from M82 was much larger than what would be expected from a square law fitted to the observed intensities of the low-frequency RRLs H30 α , H41 α , and H53 α (Seaquist, Carlstrom, Bryant and Bell, 1996). This intensity excess implies that significant stimulated emission is present. Maser amplification of RRLs is known to exist in at least one Galactic source (see Sect. 3.4.3) and very well might exist in other galaxies.

A two-component model has also been proposed for the galaxy NGC2146. In this model, the H53 α (43 GHz) lines are emitted by 100 compact H II regions with the very high electron densities of 10^5 cm^{-3} and individual sizes of about 0.2 pc. In contrast, the centimeter wavelength RRLs would come from a few hundred, less dense ($N_e = 5 \times 10^3 \rightarrow 10^4 \text{ cm}^{-3}$) H II regions with sizes ≈ 1 pc. The temperature of these H II regions would be 5,000–7,000 K (Zhao et al., 1996).

The galaxy Arp 220 can be fitted by a similar model. The observed lines H92 α (8.1 GHz), H42 α (84 GHz), H40 α (96 GHz), H31 α (207 GHz), and the upper limit for the H167 α (1.4 GHz) line fit a model containing two groups of H II regions with a temperature of 7,500 K. One group containing about 20,000 discrete H II regions with electron densities of about 10^3 cm^{-3} and a large filling factor (≈ 0.7) can explain the H92 α line emission and the upper limit to the H167 α line. The other group of about 1,000 H II regions with a density of about $2 \times 10^4 \text{ cm}^{-3}$, sizes of about 0.1 pc, and a low filling factor ($\approx 10^{-5}$) can account for the shorter wavelength lines H42 α , H40 α , and H31 α (Anantharamaiah, Viallefond, Mohan, Goss and Zhao, 2000).

In general, RRLs have been found in galaxies containing active star formation sites – the starburst galaxies – based upon an analysis of 15 galaxies where the H92 α line was either observed or an upper limit determined. There seems to be a correlation of RRL intensities with the flux density of IR emission and the radio continuum at 8.4 GHz, which are the traditional indicators of the star formation rate (Phookun et al., 1998).

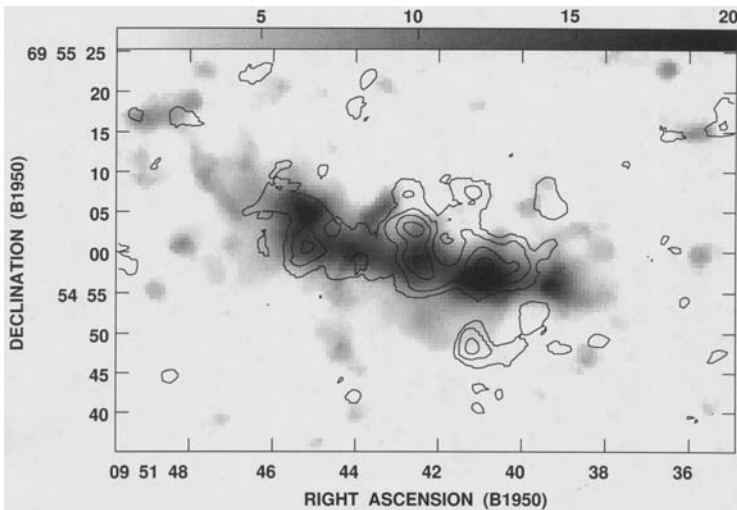


Fig. 3.65 Contours of the integrated H41 α emission from M82 overlaid on a *gray-scale map* of its HCO $^+(1 \rightarrow 0)$ emission. The angular resolution is approximately $4''$. From Seaquist et al. (1996)

In a number of galaxies, observations of the $\text{H}92\alpha$ line intensities correlate well with those of the HCN and HCO^+ lines (Zhao et al., 1996). Moreover, in M82, an excellent spatial correlation exists between the $\text{H}41\alpha$ line intensities and molecular line emission. Figure 3.65 shows an overlay of $\text{H}41\alpha$ emission upon HCO^+ emission. The emission centers are the same for both lines. There is similar agreement between the $\text{H}41\alpha$ and CO line emission (Seaquist et al., 1996). Molecular clouds are always associated with the formation of stars, and the correlation of the RRL emission with these clouds indicates that actual star formation is taking place within them.

To summarize, we note that, to date, detections of RRLs from extragalactic objects seem to come from starburst galaxies. These lines form in large complexes of H II regions located in the central parts of the galaxies, where stars are forming. The physical conditions of these regions are $T_e = 5,000 \rightarrow 10,000 \text{ K}$, $N_e = 10^3 \rightarrow 10^7 \text{ cm}^{-3}$, and the sizes of the H II regions are 0.2–5 pc – close to those of H II regions within our own Galaxy. The RRL emission is primarily spontaneous but also contains line emission stimulated by the continuum thermal emission of H II regions themselves. RRLs are most intense toward millimeter wavelengths. The densest H II regions may have masering RRLs. In contrast, RRL emission is weaker toward longer wavelengths.

The search for RRLs from quasars, carried out with the 100-m radio telescope at Effelsberg, was unsuccessful (Bell, Seaquist, Mebold, Reif and Shaver, 1984).

Appendixes

These appendixes give supplementary information. They include physical constants associated with radio recombination lines (RRLs); a table of line frequencies from 12 MHz to 29 THz for hydrogen, helium, carbon, and sulfur atoms; fine-structure components of lines associated with small principal quantum numbers; miscellaneous calculations relevant to line formation; a FORTRAN 77 computer code for calculating departure coefficients that index the relative population of atomic levels involved in RRLs; and a discussion of the relationship of the peculiar observational units used by astronomers to the more commonly used units of general physics.

Appendix A

Constants

A.1 Miscellaneous Constants

A.2 Rydberg Constants

A.2.1 *Reduced Mass*

The Bohr model used the center of the hydrogen mass as the centroid of the orbit. However, the reduced mass m_R of the nucleus M and the orbiting electron m gives better values. It results from

$$\frac{1}{m_R} = \frac{1}{m} + \frac{1}{M}, \quad \text{or} \quad (\text{A.1})$$

$$m_R = \frac{mM}{m + M}. \quad (\text{A.2})$$

Then, from (1.16) and (1.17), the correct value of R is

$$R = \frac{2\pi^2 m_R e^4}{ch^3} = R_\infty \frac{M}{M + m}. \quad (\text{A.3})$$

Substituting the modern physical constants listed in Table A.1 into (A.3) gives a value of $R_\infty = 109737.3 \text{ cm}^{-1}$.

Equation (A.3) enables a more general equation for the calculation of R for multielectron atoms and ions:

$$R = R_\infty \left[\frac{M_a - Zm}{M_a - (Z - 1)m} \right], \quad (\text{A.4})$$

where M_a is the mass of neutral atom and Z is the nuclear charge. It is assumed that these atoms are hydrogenic, i.e., only one electron is in an excited level and $Z - 1$ other electrons are in atomic core. For neutral atoms,

Table A.1 Recommended values of physical units

| Symbol | Identity | Value | CGS units | Basic units | Sources |
|------------|-------------------------|--------------------------------|---------------------|---|--|
| α | Fine-structure constant | $7.297352533 \times 10^{-3}$ | | | Mohr and Taylor (1999) IAU resolution 6, 1973 |
| c | Vacuum speed of light | $2.99792458 \times 10^{10}$ | cm s^{-1} | L T^{-1} | |
| e | Electronic charge | $-1.602176462 \times 10^{-19}$ | C | | |
| | | $-4.8032041 \times 10^{-10}$ | ESU | | Mohr and Taylor (1999) |
| h | Planck's constant | 6.626068×10^{-27} | erg s | MLL T^{-1} | |
| k | Boltzmann's constant | $1.3806503 \times 10^{-16}$ | erg K^{-1} | $\text{MLL T}^{-1} \text{ T}^{-1} \text{ K}^{-1}$ | Mohr and Taylor (1999) |
| m | Electronic mass | $9.10938188 \times 10^{-28}$ | g | M | Mohr and Taylor (1999) |
| M | Hydrogen mass | $1.67352499 \times 10^{-24}$ | g | M | Mohr and Taylor (1999) |
| R_∞ | Rydberg constant | 109,737.31568 | cm^{-1} | M^{-1} | Mohr and Taylor (1999) |

Table A.2 Masses and Rydberg constants

| Atom | Symbol | Mass (amu) | Rydberg constant (cm^{-1}) |
|--|-----------------|---------------|--|
| Hydrogen | H ¹ | 1.007825035 | 109,677.576 |
| Helium | He ⁴ | 4.00260324 | 109,722.27 |
| Carbon | C ¹² | 12.000000 | 109,732.30 |
| Nitrogen | N ¹⁴ | 14.003074 | 109,733.01 |
| Oxygen | O ¹⁶ | 15.994915 | 109,733.55 |
| Sulfur | S ³² | 31.97207070 | 109,735.43 |
| From Audi and Wapstra (1995) and (A.4) | | | |

the net negative charge of the inner electrons would screen the positive charge of the nucleus, so that the lone outer electron would see a single nuclear charge and $Z = 1$. For ions, $Z > 1$.

Therefore, from (1.17) and (A.4), the general expression for the frequency of an RRL becomes

$$\nu_{n_2 \rightarrow n_1} = R_\infty c Z^2 \left[\frac{M - (Z + 1)m}{M - Zm} \right] \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \tag{A.5}$$

or, in CGS units,

$$\begin{aligned}
\frac{\nu_{n_2 \rightarrow n_1}}{[\text{Hz}]} &= 3.28984196 \times 10^{15} Z^2 \\
&\times \left[\frac{M_u - 5.48579911 \times 10^{-4}(Z+1)}{M_u - 5.48579911 \times 10^{-4}Z} \right] \\
&\times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right), \tag{A.6}
\end{aligned}$$

where M_u is the mass of the neutral atom in atomic mass units (m_u) and $R_\infty = 109,737.315685 \text{ cm}^{-1}$.

A.2.2 Table of Rydberg Constants

Table A.2 lists Rydberg constants for the most abundant elements in the cosmos.

Appendix B

Tables of Line Frequencies

B.1 Frequencies Below 100 GHz

Table B.1 lists rest frequencies calculated for the $n\alpha$, $n\beta$, $n\gamma$, $n\delta$, and $n\epsilon$ transitions of atomic hydrogen for the range of principal quantum numbers $n = 40 \rightarrow 800$. The nomenclature is the usual. The term $n\alpha$ designates a transition from $n + 1 \rightarrow n$; $n\beta$, from $n + 2 \rightarrow n$; etc. The table also contains frequencies for the $n\alpha$ and $n\beta$ transitions of helium 4, and the $n\alpha$ transitions of carbon 12 and sulfur 32. The calculations use (A.5) and the constants listed in Tables A.1 and A.2. Below 100 GHz, frequency shifts due to the blending of fine-structure components are negligible. The shifts are less than 2 kHz for $n = 40$ and vary as n^{-5} (Lilley and Palmer, 1968).

Table B.1. H, ^4He , ^{12}C , and ^{32}S RRL rest frequencies for $n = 40 \rightarrow 800$

| n | Line transition (frequencies in MHz) | | | | | | | | |
|----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 40 | 99,022.96 | 191,057.4 | 276,745.8 | 356,658.4 | 431,303.0 | 99,063.31 | 191,135.3 | 99,072.36 | 99,075.19 |
| 41 | 92,034.44 | 177,722.8 | 257,635.5 | 332,280.1 | 402,109.6 | 92,071.94 | 177,795.3 | 92,080.35 | 92,082.98 |
| 42 | 85,688.39 | 165,601.1 | 240,245.6 | 310,075.2 | 375,495.1 | 85,723.31 | 165,668.5 | 85,731.14 | 85,733.59 |
| 43 | 79,912.66 | 154,557.2 | 224,386.8 | 289,806.7 | 351,180.7 | 79,945.22 | 154,620.2 | 79,952.52 | 79,954.81 |
| 44 | 74,644.57 | 144,474.1 | 209,894.1 | 271,268.0 | 328,922.8 | 74,674.98 | 144,533.0 | 74,681.81 | 74,683.94 |
| 45 | 69,829.56 | 135,249.5 | 196,623.4 | 254,278.3 | 308,508.5 | 69,858.01 | 135,304.6 | 69,864.39 | 69,866.39 |
| 46 | 65,419.94 | 126,793.9 | 184,448.7 | 238,679.0 | 289,750.6 | 65,446.60 | 126,845.5 | 65,452.58 | 65,454.45 |
| 47 | 61,373.94 | 119,028.8 | 173,259.0 | 224,330.6 | 272,484.2 | 61,398.95 | 119,077.3 | 61,404.56 | 61,406.31 |
| 48 | 57,654.83 | 111,885.1 | 162,956.7 | 211,110.3 | 256,564.0 | 57,678.32 | 111,930.7 | 57,683.59 | 57,685.24 |
| 49 | 54,230.25 | 105,301.9 | 153,455.5 | 198,909.2 | 241,861.2 | 54,252.35 | 105,344.8 | 54,257.30 | 54,258.85 |
| 50 | 51,071.61 | 99,225.21 | 144,678.9 | 187,630.9 | 228,261.4 | 51,092.43 | 99,265.65 | 51,097.09 | 51,098.55 |
| 51 | 48,153.60 | 93,607.32 | 136,559.3 | 177,189.8 | 215,663.2 | 48,173.22 | 93,645.47 | 48,177.62 | 48,179.00 |
| 52 | 45,453.72 | 88,405.69 | 129,036.2 | 167,509.6 | 203,975.8 | 45,472.24 | 88,441.72 | 45,476.40 | 45,477.70 |
| 53 | 42,951.97 | 83,582.47 | 122,055.8 | 158,522.1 | 193,118.5 | 42,969.47 | 83,616.53 | 42,973.40 | 42,974.63 |
| 54 | 40,630.50 | 79,103.86 | 115,570.1 | 150,166.5 | 183,018.7 | 40,647.06 | 79,136.09 | 40,650.77 | 40,651.93 |
| 55 | 38,473.36 | 74,939.62 | 109,536.0 | 142,388.2 | 173,611.5 | 38,489.04 | 74,970.16 | 38,492.55 | 38,493.65 |
| 56 | 36,466.26 | 71,062.65 | 103,914.8 | 135,138.2 | 164,838.5 | 36,481.12 | 71,091.60 | 36,484.45 | 36,485.50 |
| 57 | 34,596.38 | 67,448.58 | 98,671.90 | 128,372.3 | 156,647.1 | 34,610.48 | 67,476.07 | 34,613.64 | 34,614.63 |
| 58 | 32,852.20 | 64,075.51 | 93,775.88 | 122,050.7 | 148,989.9 | 32,865.58 | 64,101.62 | 32,868.59 | 32,869.53 |
| 59 | 31,223.32 | 60,923.68 | 89,198.55 | 116,137.7 | 141,824.0 | 31,236.04 | 60,948.50 | 31,238.89 | 31,239.78 |

(continued)

Table B.1. (continued)

| <i>n</i> | Line transition (frequencies in MHz) | | | | | | | | |
|----------|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 60 | 29,700.36 | 57,975.24 | 84,914.40 | 110,600.7 | 135,110.6 | 29,712.47 | 57,998.86 | 29,715.18 | 29,716.03 |
| 61 | 28,274.07 | 55,214.04 | 80,900.32 | 105,410.2 | 128,814.5 | 28,286.39 | 55,236.53 | 28,288.98 | 28,289.78 |
| 62 | 26,939.16 | 52,625.45 | 77,135.35 | 100,539.6 | 122,903.8 | 26,950.14 | 52,646.89 | 26,952.60 | 26,953.37 |
| 63 | 25,686.28 | 50,196.19 | 73,600.47 | 95,964.63 | 117,349.4 | 25,696.75 | 50,216.64 | 25,699.10 | 25,699.83 |
| 64 | 24,509.90 | 47,914.18 | 70,278.35 | 91,663.14 | 112,124.9 | 24,519.89 | 47,933.71 | 24,522.13 | 24,522.83 |
| 65 | 23,404.28 | 45,768.45 | 67,153.23 | 87,615.00 | 107,206.1 | 23,413.82 | 45,787.10 | 23,415.96 | 23,416.62 |
| 66 | 22,364.17 | 43,748.95 | 64,210.72 | 83,801.83 | 102,571.0 | 22,373.28 | 43,766.78 | 22,375.32 | 22,375.96 |
| 67 | 21,384.79 | 41,846.55 | 61,437.67 | 80,206.83 | 98,199.39 | 21,393.50 | 41,863.61 | 21,395.45 | 21,396.07 |
| 68 | 20,461.77 | 40,052.88 | 58,822.04 | 76,814.60 | 94,072.82 | 20,470.10 | 40,069.20 | 20,471.97 | 20,472.56 |
| 69 | 19,591.11 | 38,360.28 | 56,352.83 | 73,611.05 | 90,174.34 | 19,599.10 | 38,375.91 | 19,600.89 | 19,601.45 |
| 70 | 18,769.16 | 36,761.72 | 54,019.94 | 70,583.23 | 86,488.42 | 18,776.81 | 36,776.70 | 18,778.53 | 18,779.06 |
| 71 | 17,992.56 | 35,250.77 | 51,814.07 | 67,719.26 | 83,000.75 | 17,999.89 | 35,265.14 | 18,001.53 | 18,002.05 |
| 72 | 17,258.21 | 33,821.51 | 49,726.70 | 65,008.19 | 79,698.18 | 17,265.25 | 33,835.29 | 17,266.82 | 17,267.32 |
| 73 | 16,563.30 | 32,468.49 | 47,749.98 | 62,439.97 | 76,568.58 | 16,570.04 | 32,481.72 | 16,571.56 | 16,572.03 |
| 74 | 15,905.19 | 31,186.68 | 45,876.67 | 60,005.29 | 73,600.78 | 15,911.67 | 31,199.39 | 15,913.13 | 15,913.58 |
| 75 | 15,281.49 | 29,971.48 | 44,100.10 | 57,695.58 | 70,784.44 | 15,287.72 | 29,983.69 | 15,289.12 | 15,289.55 |
| 76 | 14,689.99 | 28,818.60 | 42,414.09 | 55,502.94 | 68,110.02 | 14,695.97 | 28,830.34 | 14,697.31 | 14,697.73 |
| 77 | 14,128.62 | 27,724.11 | 40,812.96 | 53,420.03 | 65,568.70 | 14,134.37 | 27,735.40 | 14,135.66 | 14,136.07 |
| 78 | 13,595.49 | 26,684.34 | 39,291.42 | 51,440.08 | 63,152.28 | 13,601.03 | 26,695.21 | 13,602.27 | 13,602.66 |
| 79 | 13,088.85 | 25,695.93 | 37,844.59 | 49,556.80 | 60,853.20 | 13,094.19 | 25,706.40 | 13,095.38 | 13,095.76 |
| 80 | 12,607.08 | 24,755.74 | 36,467.94 | 47,764.35 | 58,664.41 | 12,612.22 | 24,765.83 | 12,613.37 | 12,613.73 |
| 81 | 12,148.66 | 23,860.87 | 35,157.27 | 46,057.33 | 56,579.37 | 12,153.61 | 23,870.59 | 12,154.72 | 12,155.07 |
| 82 | 11,712.20 | 23,008.61 | 33,908.67 | 44,430.71 | 54,592.01 | 11,716.98 | 23,017.99 | 11,718.05 | 11,718.38 |
| 83 | 11,296.41 | 22,196.47 | 32,718.50 | 42,879.80 | 52,696.67 | 11,301.01 | 22,205.51 | 11,302.04 | 11,302.36 |
| 84 | 10,900.06 | 21,422.10 | 31,583.40 | 41,400.26 | 50,888.08 | 10,904.50 | 21,430.83 | 10,905.50 | 10,905.81 |
| 85 | 10,522.04 | 20,683.34 | 30,500.20 | 39,988.02 | 49,161.35 | 10,526.33 | 20,691.77 | 10,527.29 | 10,527.59 |
| 86 | 10,161.30 | 19,978.16 | 29,465.98 | 38,639.31 | 47,511.87 | 10,165.44 | 19,986.30 | 10,166.37 | 10,166.66 |
| 87 | 9,816.864 | 19,304.68 | 28,478.01 | 37,350.57 | 45,935.39 | 9,820.864 | 19,312.55 | 9,821.761 | 9,822.042 |
| 88 | 9,487.821 | 18,661.14 | 27,533.71 | 36,118.53 | 44,427.91 | 9,491.687 | 18,668.75 | 9,492.554 | 9,492.825 |
| 89 | 9,173.321 | 18,045.89 | 26,630.71 | 34,940.09 | 42,985.69 | 9,177.059 | 18,053.24 | 9,177.897 | 9,178.159 |
| 90 | 8,872.568 | 17,457.39 | 25,766.77 | 33,812.37 | 41,605.24 | 8,876.184 | 17,464.50 | 8,876.995 | 8,877.248 |
| 91 | 8,584.821 | 16,894.20 | 24,939.81 | 32,732.68 | 40,283.29 | 8,588.319 | 16,901.09 | 8,589.103 | 8,589.349 |
| 92 | 8,309.382 | 16,354.99 | 24,147.86 | 31,698.47 | 39,016.77 | 8,312.768 | 16,361.65 | 8,313.528 | 8,313.765 |
| 93 | 8,045.603 | 15,838.47 | 23,389.09 | 30,707.38 | 37,802.79 | 8,048.881 | 15,844.93 | 8,049.616 | 8,049.846 |
| 94 | 7,792.871 | 15,343.48 | 22,661.78 | 29,757.19 | 36,638.68 | 7,796.046 | 15,349.74 | 7,796.758 | 7,796.981 |
| 95 | 7,550.614 | 14,868.91 | 21,964.32 | 28,845.81 | 35,521.88 | 7,553.691 | 14,874.97 | 7,554.381 | 7,554.596 |
| 96 | 7,318.296 | 14,413.71 | 21,295.19 | 27,971.27 | 34,450.03 | 7,321.278 | 14,419.58 | 7,321.947 | 7,322.156 |
| 97 | 7,095.411 | 13,976.90 | 20,652.97 | 27,131.73 | 33,420.88 | 7,098.302 | 13,982.59 | 7,098.951 | 7,099.154 |
| 98 | 6,881.486 | 13,557.56 | 20,036.32 | 26,325.47 | 32,432.32 | 6,884.291 | 13,563.09 | 6,884.919 | 6,885.116 |
| 99 | 6,676.076 | 13,154.84 | 19,443.98 | 25,550.83 | 31,482.38 | 6,678.796 | 13,160.20 | 6,679.406 | 6,679.597 |
| 100 | 6,478.760 | 12,767.90 | 18,874.76 | 24,806.30 | 30,569.18 | 6,481.400 | 12,773.11 | 6,481.992 | 6,482.177 |
| 101 | 6,289.144 | 12,396.00 | 18,327.54 | 24,090.42 | 29,690.97 | 6,291.706 | 12,401.05 | 6,292.281 | 6,292.461 |
| 102 | 6,106.855 | 12,038.40 | 17,801.28 | 23,401.83 | 28,846.09 | 6,109.344 | 12,043.31 | 6,109.902 | 6,110.077 |
| 103 | 5,931.544 | 11,694.42 | 17,294.97 | 22,739.24 | 28,032.97 | 5,933.962 | 11,699.19 | 5,934.504 | 5,934.673 |
| 104 | 5,762.880 | 11,363.43 | 16,807.69 | 22,101.42 | 27,250.13 | 5,765.228 | 11,368.06 | 5,765.755 | 5,765.920 |
| 105 | 5,600.550 | 11,044.81 | 16,338.54 | 21,487.25 | 26,496.17 | 5,602.832 | 11,049.31 | 5,603.344 | 5,603.504 |
| 106 | 5,444.260 | 10,737.99 | 15,886.70 | 20,895.62 | 25,769.77 | 5,446.479 | 10,742.37 | 5,446.976 | 5,447.132 |
| 107 | 5,293.732 | 10,442.43 | 15,451.36 | 20,325.51 | 25,069.70 | 5,295.889 | 10,446.69 | 5,296.373 | 5,296.524 |
| 108 | 5,148.703 | 10,157.63 | 15,031.78 | 19,775.97 | 24,394.75 | 5,150.801 | 10,161.76 | 5,151.271 | 5,151.418 |
| 109 | 5,008.923 | 9,883.080 | 14,627.26 | 19,246.05 | 23,743.83 | 5,010.964 | 9,887.107 | 5,011.421 | 5,011.565 |
| 110 | 4,874.157 | 9,618.340 | 14,237.13 | 18,734.91 | 23,115.86 | 4,876.143 | 9,622.259 | 4,876.589 | 4,876.728 |
| 111 | 4,744.183 | 9,362.972 | 13,860.75 | 18,241.70 | 22,509.84 | 4,746.116 | 9,366.788 | 4,746.550 | 4,746.685 |
| 112 | 4,618.789 | 9,116.566 | 13,497.52 | 17,765.66 | 21,924.83 | 4,620.671 | 9,120.280 | 4,621.094 | 4,621.226 |
| 113 | 4,497.776 | 8,878.730 | 13,146.87 | 17,306.04 | 21,359.92 | 4,499.609 | 8,882.348 | 4,500.020 | 4,500.149 |
| 114 | 4,380.954 | 8,649.096 | 12,808.27 | 16,862.15 | 20,814.26 | 4,382.739 | 8,652.621 | 4,383.139 | 4,383.265 |
| 115 | 4,268.142 | 8,427.314 | 12,481.19 | 16,433.30 | 20,287.02 | 4,269.882 | 8,430.748 | 4,270.272 | 4,270.394 |
| 116 | 4,159.171 | 8,213.049 | 12,165.16 | 16,018.88 | 19,777.45 | 4,160.866 | 8,216.396 | 4,161.246 | 4,161.365 |
| 117 | 4,053.878 | 8,005.988 | 11,859.71 | 15,618.27 | 19,284.80 | 4,055.530 | 8,009.250 | 4,055.901 | 4,056.016 |
| 118 | 3,952.110 | 7,805.829 | 11,564.40 | 15,230.92 | 18,808.38 | 3,953.720 | 7,809.010 | 3,954.081 | 3,954.194 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 119 | 3,853.719 | 7,612.287 | 11,278.81 | 14,856.27 | 18,347.53 | 3,855.289 | 7,615.389 | 3,855.642 | 3,855.752 |
| 120 | 3,758.568 | 7,425.091 | 11,002.55 | 14,493.81 | 17,901.61 | 3,760.099 | 7,428.117 | 3,760.443 | 3,760.550 |
| 121 | 3,666.523 | 7,243.983 | 10,735.24 | 14,143.04 | 17,470.03 | 3,668.017 | 7,246.935 | 3,668.352 | 3,668.457 |
| 122 | 3,577.460 | 7,068.718 | 10,476.52 | 13,803.51 | 17,052.22 | 3,578.918 | 7,071.598 | 3,579.245 | 3,579.347 |
| 123 | 3,491.258 | 6,899.061 | 10,226.05 | 13,474.76 | 16,647.62 | 3,492.681 | 6,901.873 | 3,493.000 | 3,493.099 |
| 124 | 3,407.803 | 6,734.791 | 9,983.498 | 13,156.36 | 16,255.72 | 3,409.192 | 6,737.535 | 3,409.503 | 3,409.601 |
| 125 | 3,326.988 | 6,575.695 | 9,748.558 | 12,847.92 | 15,876.03 | 3,328.343 | 6,578.374 | 3,328.647 | 3,328.742 |
| 126 | 3,248.707 | 6,421.571 | 9,520.933 | 12,549.05 | 15,508.08 | 3,250.031 | 6,424.187 | 3,250.328 | 3,250.421 |
| 127 | 3,172.863 | 6,272.226 | 9,300.340 | 12,259.37 | 15,151.41 | 3,174.156 | 6,274.781 | 3,174.446 | 3,174.537 |
| 128 | 3,099.362 | 6,127.476 | 9,086.509 | 11,978.55 | 14,805.59 | 3,100.625 | 6,129.973 | 3,100.908 | 3,100.997 |
| 129 | 3,028.114 | 5,987.147 | 8,879.184 | 11,706.23 | 14,470.23 | 3,029.348 | 5,989.586 | 3,029.625 | 3,029.711 |
| 130 | 2,959.033 | 5,851.070 | 8,678.119 | 11,442.11 | 14,144.91 | 2,960.239 | 5,853.454 | 2,960.509 | 2,960.594 |
| 131 | 2,892.037 | 5,719.086 | 8,483.079 | 11,185.88 | 13,829.28 | 2,893.216 | 5,721.416 | 2,893.480 | 2,893.563 |
| 132 | 2,827.049 | 5,591.042 | 8,293.841 | 10,937.24 | 13,522.96 | 2,828.201 | 5,593.320 | 2,828.459 | 2,828.540 |
| 133 | 2,763.993 | 5,466.792 | 8,110.190 | 10,695.91 | 13,225.63 | 2,765.119 | 5,469.020 | 2,765.372 | 2,765.451 |
| 134 | 2,702.799 | 5,346.197 | 7,931.921 | 10,461.64 | 12,936.95 | 2,703.900 | 5,348.375 | 2,704.147 | 2,704.224 |
| 135 | 2,643.398 | 5,229.122 | 7,758.839 | 10,234.16 | 12,656.62 | 2,644.475 | 5,231.253 | 2,644.716 | 2,644.792 |
| 136 | 2,585.725 | 5,115.442 | 7,590.757 | 10,013.22 | 12,384.32 | 2,586.778 | 5,117.526 | 2,587.015 | 2,587.088 |
| 137 | 2,529.717 | 5,005.033 | 7,427.495 | 9,798.599 | 12,119.79 | 2,530.748 | 5,007.072 | 2,530.979 | 2,531.051 |
| 138 | 2,475.316 | 4,897.778 | 7,268.882 | 9,590.069 | 11,862.73 | 2,476.324 | 4,899.774 | 2,476.550 | 2,476.621 |
| 139 | 2,422.463 | 4,793.567 | 7,114.754 | 9,387.415 | 11,612.89 | 2,423.450 | 4,795.520 | 2,423.671 | 2,423.740 |
| 140 | 2,371.104 | 4,692.291 | 6,964.952 | 9,190.430 | 11,370.02 | 2,372.070 | 4,694.203 | 2,372.287 | 2,372.355 |
| 141 | 2,321.187 | 4,593.848 | 6,819.326 | 8,998.920 | 11,133.88 | 2,322.133 | 4,595.720 | 2,322.345 | 2,322.411 |
| 142 | 2,272.661 | 4,498.140 | 6,677.733 | 8,812.693 | 10,904.23 | 2,273.587 | 4,499.973 | 2,273.795 | 2,273.860 |
| 143 | 2,225.479 | 4,405.072 | 6,540.032 | 8,631.570 | 10,680.86 | 2,226.385 | 4,406.867 | 2,226.589 | 2,226.652 |
| 144 | 2,179.593 | 4,314.554 | 6,406.092 | 8,455.378 | 10,463.54 | 2,180.481 | 4,316.312 | 2,180.680 | 2,180.743 |
| 145 | 2,134.961 | 4,226.499 | 6,275.784 | 8,283.948 | 10,252.08 | 2,135.830 | 4,228.221 | 2,136.026 | 2,136.087 |
| 146 | 2,091.538 | 4,140.824 | 6,148.987 | 8,117.122 | 10,046.28 | 2,092.391 | 4,142.511 | 2,092.582 | 2,092.641 |
| 147 | 2,049.286 | 4,057.449 | 6,025.583 | 7,954.745 | 9,845.957 | 2,050.121 | 4,059.102 | 2,050.308 | 2,050.367 |
| 148 | 2,008.163 | 3,976.298 | 5,905.459 | 7,796.671 | 9,650.921 | 2,008.982 | 3,977.918 | 2,009.165 | 2,009.223 |
| 149 | 1,968.134 | 3,897.296 | 5,788.508 | 7,642.758 | 9,461.004 | 1,968.936 | 3,898.884 | 1,969.116 | 1,969.172 |
| 150 | 1,929.162 | 3,820.373 | 5,674.624 | 7,492.870 | 9,276.038 | 1,929.948 | 3,821.930 | 1,930.124 | 1,930.179 |
| 151 | 1,891.212 | 3,745.462 | 5,563.708 | 7,346.876 | 9,095.862 | 1,891.982 | 3,746.988 | 1,892.155 | 1,892.209 |
| 152 | 1,854.250 | 3,672.496 | 5,455.664 | 7,204.650 | 8,920.323 | 1,855.006 | 3,673.993 | 1,855.175 | 1,855.228 |
| 153 | 1,818.246 | 3,601.414 | 5,350.400 | 7,066.073 | 8,749.272 | 1,818.987 | 3,602.881 | 1,819.153 | 1,819.205 |
| 154 | 1,783.168 | 3,532.154 | 5,247.827 | 6,931.026 | 8,582.567 | 1,783.894 | 3,533.593 | 1,784.057 | 1,784.108 |
| 155 | 1,748.986 | 3,464.659 | 5,147.858 | 6,799.400 | 8,420.071 | 1,749.699 | 3,466.071 | 1,749.859 | 1,749.909 |
| 156 | 1,715.673 | 3,398.872 | 5,050.414 | 6,671.085 | 8,261.652 | 1,716.372 | 3,400.257 | 1,716.528 | 1,716.578 |
| 157 | 1,683.200 | 3,334.741 | 4,955.413 | 6,545.979 | 8,107.182 | 1,683.886 | 3,336.100 | 1,684.039 | 1,684.088 |
| 158 | 1,651.541 | 3,272.213 | 4,862.780 | 6,423.982 | 7,956.540 | 1,652.214 | 3,273.546 | 1,652.365 | 1,652.412 |
| 159 | 1,620.672 | 3,211.238 | 4,772.441 | 6,304.998 | 7,809.607 | 1,621.332 | 3,212.547 | 1,621.480 | 1,621.527 |
| 160 | 1,590.567 | 3,151.769 | 4,684.327 | 6,188.935 | 7,666.270 | 1,591.215 | 3,153.054 | 1,591.360 | 1,591.406 |
| 161 | 1,561.203 | 3,093.760 | 4,598.368 | 6,075.703 | 7,526.419 | 1,561.839 | 3,095.021 | 1,561.982 | 1,562.026 |
| 162 | 1,532.557 | 3,037.165 | 4,514.500 | 5,965.216 | 7,389.950 | 1,533.182 | 3,038.403 | 1,533.322 | 1,533.366 |
| 163 | 1,504.608 | 2,981.943 | 4,432.659 | 5,857.393 | 7,256.761 | 1,505.221 | 2,983.158 | 1,505.359 | 1,505.402 |
| 164 | 1,477.335 | 2,928.051 | 4,352.785 | 5,752.152 | 7,126.753 | 1,477.937 | 2,929.244 | 1,478.072 | 1,478.114 |
| 165 | 1,450.716 | 2,875.450 | 4,274.818 | 5,649.418 | 6,999.833 | 1,451.307 | 2,876.622 | 1,451.440 | 1,451.482 |
| 166 | 1,424.734 | 2,824.101 | 4,198.702 | 5,549.116 | 6,875.908 | 1,425.314 | 2,825.252 | 1,425.444 | 1,425.485 |
| 167 | 1,399.368 | 2,773.968 | 4,124.383 | 5,451.175 | 6,754.892 | 1,399.938 | 2,775.099 | 1,400.066 | 1,400.106 |
| 168 | 1,374.601 | 2,725.015 | 4,051.807 | 5,355.525 | 6,636.700 | 1,375.161 | 2,726.125 | 1,375.286 | 1,375.326 |
| 169 | 1,350.414 | 2,677.206 | 3,980.924 | 5,262.100 | 6,521.249 | 1,350.965 | 2,678.297 | 1,351.088 | 1,351.127 |
| 170 | 1,326.792 | 2,630.510 | 3,911.685 | 5,170.835 | 6,408.461 | 1,327.333 | 2,631.582 | 1,327.454 | 1,327.492 |
| 171 | 1,303.718 | 2,584.893 | 3,844.043 | 5,081.669 | 6,298.259 | 1,304.249 | 2,585.946 | 1,304.368 | 1,304.405 |
| 172 | 1,281.175 | 2,540.325 | 3,777.951 | 4,994.541 | 6,190.569 | 1,281.697 | 2,541.360 | 1,281.814 | 1,281.851 |
| 173 | 1,259.150 | 2,496.776 | 3,713.366 | 4,909.394 | 6,085.321 | 1,259.663 | 2,497.793 | 1,259.778 | 1,259.814 |
| 174 | 1,237.626 | 2,454.216 | 3,650.244 | 4,826.171 | 5,982.445 | 1,238.130 | 2,455.216 | 1,238.243 | 1,238.279 |
| 175 | 1,216.590 | 2,412.618 | 3,588.545 | 4,744.819 | 5,881.875 | 1,217.086 | 2,413.601 | 1,217.197 | 1,217.232 |
| 176 | 1,196.028 | 2,371.955 | 3,528.229 | 4,665.285 | 5,783.548 | 1,196.516 | 2,372.922 | 1,196.625 | 1,196.659 |
| 177 | 1,175.927 | 2,332.201 | 3,469.257 | 4,587.519 | 5,687.399 | 1,176.406 | 2,333.151 | 1,176.514 | 1,176.547 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Sn α |
| 178 | 1,156.274 | 2,293.330 | 3,411.592 | 4,511.472 | 5,593.371 | 1,156.745 | 2,294.265 | 1,156.884 |
| 179 | 1,137.056 | 2,255.318 | 3,355.198 | 4,437.097 | 5,501.403 | 1,137.520 | 2,256.237 | 1,137.656 |
| 180 | 1,118.262 | 2,218.142 | 3,300.040 | 4,364.347 | 5,411.442 | 1,118.718 | 2,219.046 | 1,118.852 |
| 181 | 1,099.880 | 2,181.778 | 3,246.085 | 4,293.179 | 5,323.431 | 1,100.328 | 2,182.667 | 1,100.460 |
| 182 | 1,081.898 | 2,146.205 | 3,193.300 | 4,223.551 | 5,237.318 | 1,082.339 | 2,147.080 | 1,082.469 |
| 183 | 1,064.307 | 2,111.401 | 3,141.652 | 4,155.420 | 5,153.053 | 1,064.740 | 2,112.262 | 1,064.838 |
| 184 | 1,047.094 | 2,077.346 | 3,091.113 | 4,088.746 | 5,070.586 | 1,047.521 | 2,078.192 | 1,047.617 |
| 185 | 1,030.251 | 2,044.019 | 3,041.652 | 4,023.492 | 4,989.870 | 1,030.671 | 2,044.851 | 1,030.765 |
| 186 | 1,013.767 | 2,011.401 | 2,993.240 | 3,959.618 | 4,910.857 | 1,014.180 | 2,012.220 | 1,014.273 |
| 187 | 997.6333 | 1,979.473 | 2,945.851 | 3,897.090 | 4,833.504 | 998.0398 | 1,980.280 | 998.1310 |
| 188 | 981.8398 | 1,948.218 | 2,899.457 | 3,835.871 | 4,757.768 | 982.2399 | 1,949.012 | 982.3296 |
| 189 | 966.3779 | 1,917.617 | 2,854.031 | 3,775.928 | 4,683.605 | 966.7717 | 1,918.398 | 966.8600 |
| 190 | 951.2389 | 1,887.653 | 2,809.550 | 3,717.227 | 4,610.976 | 951.6265 | 1,888.423 | 951.7135 |
| 191 | 936.4146 | 1,858.311 | 2,765.988 | 3,659.737 | 4,539.841 | 936.7961 | 1,859.068 | 936.8817 |
| 192 | 921.8966 | 1,829.574 | 2,723.323 | 3,603.427 | 4,470.162 | 922.2723 | 1,830.319 | 922.3565 |
| 193 | 907.6773 | 1,801.426 | 2,681.530 | 3,548.265 | 4,401.902 | 908.0471 | 1,802.160 | 908.1301 |
| 194 | 893.7488 | 1,773.853 | 2,640.588 | 3,494.224 | 4,335.024 | 894.1130 | 1,774.576 | 894.1947 |
| 195 | 880.1039 | 1,746.839 | 2,600.476 | 3,441.275 | 4,269.495 | 880.4626 | 1,747.551 | 880.5430 |
| 196 | 866.7354 | 1,720.372 | 2,561.171 | 3,389.391 | 4,205.279 | 867.0886 | 1,721.073 | 867.1678 |
| 197 | 853.6362 | 1,694.436 | 2,522.655 | 3,338.544 | 4,142.345 | 853.9841 | 1,695.126 | 854.0621 |
| 198 | 840.7997 | 1,669.019 | 2,484.908 | 3,288.709 | 4,080.661 | 841.1423 | 1,669.699 | 841.2192 |
| 199 | 828.2193 | 1,644.108 | 2,447.909 | 3,239.861 | 4,020.195 | 828.5567 | 1,644.778 | 828.6324 |
| 200 | 815.8886 | 1,619.690 | 2,411.642 | 3,191.976 | 3,960.918 | 816.2210 | 1,620.350 | 816.2956 |
| 201 | 803.8014 | 1,595.753 | 2,376.087 | 3,145.030 | 3,902.801 | 804.1290 | 1,596.404 | 804.2024 |
| 202 | 791.9519 | 1,572.286 | 2,341.228 | 3,099.000 | 3,845.816 | 792.2746 | 1,572.927 | 792.3469 |
| 203 | 780.3341 | 1,549.277 | 2,307.048 | 3,053.864 | 3,789.934 | 780.6521 | 1,549.908 | 780.7234 |
| 204 | 768.9424 | 1,526.714 | 2,273.530 | 3,009.600 | 3,735.130 | 769.2558 | 1,527.336 | 769.3260 |
| 205 | 757.7714 | 1,504.587 | 2,240.658 | 2,966.188 | 3,681.378 | 758.0802 | 1,505.200 | 758.1495 |
| 206 | 746.8158 | 1,482.886 | 2,208.416 | 2,923.606 | 3,628.652 | 747.1201 | 1,483.490 | 747.1884 |
| 207 | 736.0703 | 1,461.600 | 2,176.790 | 2,881.836 | 3,576.928 | 736.3703 | 1,462.196 | 736.4375 |
| 208 | 725.5300 | 1,440.720 | 2,145.766 | 2,840.858 | 3,526.183 | 725.8257 | 1,441.307 | 725.8920 |
| 209 | 715.1900 | 1,420.236 | 2,115.328 | 2,800.653 | 3,476.393 | 715.4814 | 1,420.814 | 715.5468 |
| 210 | 705.0455 | 1,400.138 | 2,085.463 | 2,761.203 | 3,427.536 | 705.3328 | 1,400.708 | 705.3973 |
| 211 | 695.0920 | 1,380.417 | 2,056.157 | 2,722.490 | 3,379.590 | 695.3753 | 1,380.980 | 695.4388 |
| 212 | 685.3250 | 1,361.065 | 2,027.398 | 2,684.498 | 3,332.535 | 685.6043 | 1,361.620 | 685.6669 |
| 213 | 675.7401 | 1,342.073 | 1,999.173 | 2,647.210 | 3,286.349 | 676.0155 | 1,342.620 | 676.0772 |
| 214 | 666.3331 | 1,323.433 | 1,971.470 | 2,610.609 | 3,241.012 | 666.6046 | 1,323.972 | 666.6655 |
| 215 | 657.0999 | 1,305.136 | 1,944.276 | 2,574.679 | 3,196.506 | 657.3677 | 1,305.668 | 657.4277 |
| 216 | 648.0365 | 1,287.176 | 1,917.579 | 2,539.406 | 3,152.811 | 648.3006 | 1,287.700 | 648.3598 |
| 217 | 639.1391 | 1,269.543 | 1,891.370 | 2,504.775 | 3,109.909 | 639.3995 | 1,270.060 | 639.4579 |
| 218 | 630.4038 | 1,252.231 | 1,865.636 | 2,470.770 | 3,067.782 | 630.6606 | 1,252.741 | 630.7183 |
| 219 | 621.8269 | 1,235.232 | 1,840.366 | 2,437.378 | 3,026.412 | 622.0803 | 1,235.735 | 622.1371 |
| 220 | 613.4049 | 1,218.539 | 1,815.551 | 2,404.585 | 2,985.783 | 613.6549 | 1,219.036 | 613.7109 |
| 221 | 605.1343 | 1,202.146 | 1,791.180 | 2,372.378 | 2,945.877 | 605.3809 | 1,202.636 | 605.4362 |
| 222 | 597.0118 | 1,186.046 | 1,767.243 | 2,340.743 | 2,906.680 | 597.2551 | 1,186.529 | 597.3096 |
| 223 | 589.0340 | 1,170.232 | 1,743.731 | 2,309.668 | 2,868.175 | 589.2740 | 1,170.708 | 589.3278 |
| 224 | 581.1976 | 1,154.697 | 1,720.634 | 2,279.141 | 2,830.348 | 581.4345 | 1,155.168 | 581.4876 |
| 225 | 573.4997 | 1,139.437 | 1,697.944 | 2,249.150 | 2,793.182 | 573.7334 | 1,139.901 | 573.7858 |
| 226 | 565.9371 | 1,124.444 | 1,675.650 | 2,219.682 | 2,756.665 | 566.1677 | 1,124.902 | 566.2195 |
| 227 | 558.5069 | 1,109.713 | 1,653.745 | 2,190.728 | 2,720.781 | 558.7345 | 1,110.165 | 558.7856 |
| 228 | 551.2062 | 1,095.238 | 1,632.221 | 2,162.274 | 2,685.517 | 551.4308 | 1,095.685 | 551.4812 |
| 229 | 544.0322 | 1,081.014 | 1,611.068 | 2,134.311 | 2,650.861 | 544.2539 | 1,081.455 | 544.3036 |
| 230 | 536.9822 | 1,067.036 | 1,590.279 | 2,106.828 | 2,616.798 | 537.2010 | 1,067.470 | 537.2501 |
| 231 | 530.0534 | 1,053.297 | 1,569.846 | 2,079.815 | 2,583.316 | 530.2694 | 1,053.726 | 530.3178 |
| 232 | 523.2433 | 1,039.793 | 1,549.762 | 2,053.262 | 2,550.403 | 523.4566 | 1,040.216 | 523.5044 |
| 233 | 516.5494 | 1,026.519 | 1,530.019 | 2,027.159 | 2,518.046 | 516.7599 | 1,026.937 | 516.8071 |
| 234 | 509.9692 | 1,013.470 | 1,510.610 | 2,001.497 | 2,486.235 | 510.1770 | 1,013.883 | 510.2236 |
| 235 | 503.5003 | 1,000.641 | 1,491.528 | 1,976.266 | 2,454.958 | 503.7055 | 1,001.048 | 503.7515 |
| 236 | 497.1404 | 988.0274 | 1,472.766 | 1,951.457 | 2,424.202 | 497.3429 | 988.4300 | 497.4026 |

(continued)

Table B.1. (continued)

| <i>n</i> | Line transition (frequencies in MHz) | | | | | | | | |
|----------|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 237 | 490.8871 | 975.6252 | 1,454.317 | 1,927.062 | 2,393.959 | 491.0871 | 976.0228 | 491.1319 | 491.1460 |
| 238 | 484.7382 | 963.4298 | 1,436.175 | 1,903.072 | 2,364.216 | 484.9357 | 963.8224 | 484.9800 | 484.9939 |
| 239 | 478.6916 | 951.4367 | 1,418.334 | 1,879.478 | 2,334.964 | 478.8867 | 951.8244 | 478.9304 | 478.9441 |
| 240 | 472.7452 | 939.6419 | 1,400.786 | 1,856.273 | 2,306.193 | 472.9378 | 940.0248 | 472.9810 | 472.9945 |
| 241 | 466.8968 | 928.0413 | 1,383.528 | 1,833.448 | 2,277.893 | 467.0870 | 928.4195 | 467.1297 | 467.1431 |
| 242 | 461.1445 | 916.6308 | 1,366.551 | 1,810.996 | 2,250.053 | 461.3324 | 917.0043 | 461.3746 | 461.3877 |
| 243 | 455.4863 | 905.4066 | 1,349.851 | 1,788.909 | 2,222.666 | 455.6719 | 905.7756 | 455.7135 | 455.7266 |
| 244 | 449.9203 | 894.3650 | 1,333.422 | 1,767.179 | 2,195.721 | 450.1037 | 894.7294 | 450.1448 | 450.1576 |
| 245 | 444.4447 | 883.5021 | 1,317.259 | 1,745.801 | 2,169.210 | 444.6258 | 883.8621 | 444.6664 | 444.6791 |
| 246 | 439.0575 | 872.8145 | 1,301.356 | 1,724.765 | 2,143.124 | 439.2364 | 873.1701 | 439.2765 | 439.2891 |
| 247 | 433.7570 | 862.2985 | 1,285.708 | 1,704.067 | 2,117.455 | 433.9338 | 862.6499 | 433.9734 | 433.9858 |
| 248 | 428.5415 | 851.9508 | 1,270.310 | 1,683.698 | 2,092.194 | 428.7161 | 852.2980 | 428.7553 | 428.7676 |
| 249 | 423.4093 | 841.7680 | 1,255.156 | 1,663.652 | 2,067.333 | 423.5818 | 842.1111 | 423.6205 | 423.6326 |
| 250 | 418.3587 | 831.7469 | 1,240.243 | 1,643.924 | 2,042.865 | 418.5292 | 832.0858 | 418.5674 | 418.5794 |
| 251 | 413.3882 | 821.8842 | 1,225.565 | 1,624.506 | 2,018.781 | 413.5566 | 822.2191 | 413.5944 | 413.6062 |
| 252 | 408.4960 | 812.1768 | 1,211.118 | 1,605.393 | 1,995.074 | 408.6625 | 812.5077 | 408.6998 | 408.7115 |
| 253 | 403.6808 | 802.6217 | 1,196.897 | 1,586.578 | 1,971.737 | 403.8453 | 802.9487 | 403.8822 | 403.8937 |
| 254 | 398.9409 | 793.2158 | 1,182.897 | 1,568.056 | 1,948.763 | 399.1035 | 793.5391 | 399.1399 | 399.1513 |
| 255 | 394.2749 | 783.9564 | 1,169.116 | 1,549.822 | 1,926.144 | 394.4356 | 784.2759 | 394.4716 | 394.4829 |
| 256 | 389.6815 | 774.8406 | 1,155.547 | 1,531.869 | 1,903.874 | 389.8403 | 775.1563 | 389.8759 | 389.8870 |
| 257 | 385.1591 | 765.8655 | 1,142.188 | 1,514.192 | 1,881.946 | 385.3160 | 766.1776 | 385.3512 | 385.3622 |
| 258 | 380.7064 | 757.0285 | 1,129.033 | 1,496.787 | 1,860.353 | 380.8615 | 757.3370 | 380.8963 | 380.9072 |
| 259 | 376.3221 | 748.3269 | 1,116.080 | 1,479.647 | 1,839.090 | 376.4754 | 748.6319 | 376.5098 | 376.5206 |
| 260 | 372.0048 | 739.7582 | 1,103.325 | 1,462.768 | 1,818.149 | 372.1564 | 740.0597 | 372.1904 | 372.2011 |
| 261 | 367.7534 | 731.3198 | 1,090.763 | 1,446.144 | 1,797.525 | 367.9032 | 731.6178 | 367.9368 | 367.9473 |
| 262 | 363.5664 | 723.0093 | 1,078.391 | 1,429.771 | 1,777.212 | 363.7146 | 723.3039 | 363.7478 | 363.7582 |
| 263 | 359.4428 | 714.8242 | 1,066.205 | 1,413.645 | 1,757.203 | 359.5893 | 715.1155 | 359.6222 | 359.6324 |
| 264 | 355.3814 | 706.7622 | 1,054.202 | 1,397.760 | 1,737.494 | 355.5262 | 707.0502 | 355.5587 | 355.5688 |
| 265 | 351.3808 | 698.8210 | 1,042.379 | 1,382.113 | 1,718.079 | 351.5240 | 699.1057 | 351.5561 | 351.5662 |
| 266 | 347.4401 | 690.9983 | 1,030.732 | 1,366.698 | 1,698.952 | 347.5817 | 691.2799 | 347.6135 | 347.6234 |
| 267 | 343.5581 | 683.2919 | 1,019.258 | 1,351.511 | 1,680.107 | 343.6981 | 683.5703 | 343.7295 | 343.7394 |
| 268 | 339.7338 | 675.6997 | 1,007.953 | 1,336.549 | 1,661.541 | 339.8722 | 675.9750 | 339.9033 | 339.9130 |
| 269 | 335.9659 | 668.2196 | 996.8154 | 1,321.807 | 1,643.247 | 336.1028 | 668.4919 | 336.1335 | 336.1431 |
| 270 | 332.2536 | 660.8494 | 985.8409 | 1,307.281 | 1,625.220 | 332.3890 | 661.1187 | 332.4194 | 332.4289 |
| 271 | 328.5958 | 653.5873 | 975.0270 | 1,292.966 | 1,607.456 | 328.7297 | 653.8536 | 328.7597 | 328.7691 |
| 272 | 324.9915 | 646.4312 | 964.3706 | 1,278.860 | 1,589.950 | 325.1239 | 646.6946 | 325.1536 | 325.1629 |
| 273 | 321.4397 | 639.3791 | 953.8690 | 1,264.959 | 1,572.698 | 321.5707 | 639.6397 | 321.6000 | 321.6092 |
| 274 | 317.9394 | 632.4293 | 943.5192 | 1,251.258 | 1,555.694 | 318.0690 | 632.6870 | 318.0981 | 318.1071 |
| 275 | 314.4898 | 625.5798 | 933.3187 | 1,237.755 | 1,538.934 | 314.6180 | 625.8347 | 314.6467 | 314.6557 |
| 276 | 311.0900 | 618.8289 | 923.2647 | 1,224.445 | 1,522.415 | 311.2167 | 619.0811 | 311.2452 | 311.2541 |
| 277 | 307.7389 | 612.1748 | 913.3546 | 1,211.325 | 1,506.131 | 307.8643 | 612.4242 | 307.8924 | 307.9012 |
| 278 | 304.4358 | 605.6157 | 903.5858 | 1,198.392 | 1,490.078 | 304.5599 | 605.8625 | 304.5877 | 304.5964 |
| 279 | 301.1799 | 599.1500 | 893.9558 | 1,185.642 | 1,474.253 | 301.3026 | 599.3941 | 301.3301 | 301.3387 |
| 280 | 297.9701 | 592.7760 | 884.4622 | 1,173.073 | 1,458.650 | 298.0916 | 593.0175 | 298.1188 | 298.1273 |
| 281 | 294.8059 | 586.4921 | 875.1026 | 1,160.680 | 1,443.268 | 294.9260 | 586.7311 | 294.9529 | 294.9614 |
| 282 | 291.6862 | 580.2967 | 865.8745 | 1,148.462 | 1,428.101 | 291.8051 | 580.5332 | 291.8318 | 291.8401 |
| 283 | 288.6105 | 574.1883 | 856.7757 | 1,136.414 | 1,413.145 | 288.7281 | 574.4222 | 288.7545 | 288.7627 |
| 284 | 285.5778 | 568.1653 | 847.8040 | 1,124.535 | 1,398.398 | 285.6942 | 568.3968 | 285.7203 | 285.7284 |
| 285 | 282.5875 | 562.2262 | 838.9571 | 1,112.820 | 1,383.855 | 282.7026 | 562.4553 | 282.7284 | 282.7365 |
| 286 | 279.6387 | 556.3696 | 830.2329 | 1,101.268 | 1,369.514 | 279.7527 | 556.5963 | 279.7782 | 279.7862 |
| 287 | 276.7309 | 550.5941 | 821.6292 | 1,089.875 | 1,355.369 | 276.8437 | 550.8185 | 276.8690 | 276.8769 |
| 288 | 273.8632 | 544.8983 | 813.1440 | 1,078.638 | 1,341.419 | 273.9748 | 545.1203 | 273.9999 | 274.0077 |
| 289 | 271.0350 | 539.2807 | 804.7752 | 1,067.556 | 1,327.660 | 271.1455 | 539.5005 | 271.1703 | 271.1780 |
| 290 | 268.2457 | 533.7401 | 796.5208 | 1,056.625 | 1,314.088 | 268.3550 | 533.9576 | 268.3795 | 268.3872 |
| 291 | 265.4945 | 528.2752 | 788.3790 | 1,045.842 | 1,300.700 | 265.6026 | 528.4904 | 265.6269 | 265.6345 |
| 292 | 262.7807 | 522.8846 | 780.3478 | 1,035.206 | 1,287.494 | 262.8878 | 523.0976 | 262.9118 | 262.9193 |
| 293 | 260.1038 | 517.5671 | 772.4252 | 1,024.713 | 1,274.466 | 260.2098 | 517.7780 | 260.2336 | 260.2410 |
| 294 | 257.4632 | 512.3214 | 764.6096 | 1,014.362 | 1,261.613 | 257.5681 | 512.5302 | 257.5917 | 257.5990 |
| 295 | 254.8582 | 507.1464 | 756.8991 | 1,004.150 | 1,248.933 | 254.9620 | 507.3531 | 254.9853 | 254.9926 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 296 | 252.2882 | 502.0409 | 749.2918 | 994.0744 | 1,236.421 | 252.3910 | 502.2454 | 252.4141 | 252.4213 |
| 297 | 249.7527 | 497.0036 | 741.7862 | 984.1331 | 1,224.077 | 249.8544 | 497.2061 | 249.8773 | 249.8844 |
| 298 | 247.2510 | 492.0335 | 734.3805 | 974.3240 | 1,211.896 | 247.3517 | 492.2340 | 247.3743 | 247.3814 |
| 299 | 244.7826 | 487.1295 | 727.0730 | 964.6448 | 1,199.876 | 244.8823 | 487.3280 | 244.9047 | 244.9117 |
| 300 | 242.3469 | 482.2905 | 719.8622 | 955.0934 | 1,188.015 | 242.4457 | 482.4870 | 242.4678 | 242.4748 |
| 301 | 239.9435 | 477.5153 | 712.7464 | 945.6676 | 1,176.309 | 240.0413 | 477.7098 | 240.0632 | 240.0701 |
| 302 | 237.5718 | 472.8029 | 705.7241 | 936.3655 | 1,164.757 | 237.6686 | 472.9956 | 237.6903 | 237.6971 |
| 303 | 235.2312 | 468.1524 | 698.7937 | 927.1850 | 1,153.355 | 235.3270 | 468.3431 | 235.3485 | 235.3552 |
| 304 | 232.9212 | 463.5626 | 691.9538 | 918.1241 | 1,142.102 | 233.0161 | 463.7515 | 233.0374 | 233.0441 |
| 305 | 230.6414 | 459.0326 | 685.2029 | 909.1809 | 1,130.995 | 230.7354 | 459.2197 | 230.7565 | 230.7630 |
| 306 | 228.3912 | 454.5615 | 678.5395 | 900.3535 | 1,120.031 | 228.4843 | 454.7467 | 228.5052 | 228.5117 |
| 307 | 226.1703 | 450.1483 | 671.9622 | 891.6399 | 1,109.209 | 226.2624 | 450.3317 | 226.2831 | 226.2896 |
| 308 | 223.9780 | 445.7920 | 665.4697 | 883.0385 | 1,098.525 | 224.0693 | 445.9736 | 224.0897 | 224.0961 |
| 309 | 221.8140 | 441.4917 | 659.0605 | 874.5473 | 1,087.979 | 221.9043 | 441.6716 | 221.9246 | 221.9310 |
| 310 | 219.6777 | 437.2465 | 652.7334 | 866.1647 | 1,077.567 | 219.7672 | 437.4247 | 219.7873 | 219.7936 |
| 311 | 217.5688 | 433.0556 | 646.4870 | 857.8888 | 1,067.287 | 217.6575 | 433.2321 | 217.6774 | 217.6836 |
| 312 | 215.4868 | 428.9181 | 640.3200 | 849.7181 | 1,057.138 | 215.5746 | 429.0929 | 215.5943 | 215.6005 |
| 313 | 213.4313 | 424.8332 | 634.2312 | 841.6508 | 1,047.117 | 213.5183 | 425.0063 | 213.5378 | 213.5439 |
| 314 | 211.4019 | 420.7999 | 628.2194 | 833.6852 | 1,037.222 | 211.4880 | 420.9714 | 211.5073 | 211.5134 |
| 315 | 209.3981 | 416.8176 | 622.2834 | 825.8199 | 1,027.451 | 209.4834 | 416.9874 | 209.5025 | 209.5085 |
| 316 | 207.4195 | 412.8853 | 616.4218 | 818.0532 | 1,017.803 | 207.5040 | 413.0535 | 207.5230 | 207.5289 |
| 317 | 205.4658 | 409.0023 | 610.6337 | 810.3836 | 1,008.275 | 205.5495 | 409.1690 | 205.5683 | 205.5742 |
| 318 | 203.5365 | 405.1679 | 604.9178 | 802.8096 | 998.8662 | 203.6195 | 405.3330 | 203.6381 | 203.6439 |
| 319 | 201.6314 | 401.3813 | 599.2731 | 795.3297 | 989.5737 | 201.7135 | 401.5448 | 201.7320 | 201.7377 |
| 320 | 199.7499 | 397.6417 | 593.6983 | 787.9424 | 980.3962 | 199.8313 | 397.8037 | 199.8496 | 199.8553 |
| 321 | 197.8918 | 393.9484 | 588.1925 | 780.6463 | 971.3318 | 197.9724 | 394.1089 | 197.9905 | 197.9962 |
| 322 | 196.0566 | 390.3007 | 582.7545 | 773.4400 | 962.3788 | 196.1365 | 390.4597 | 196.1544 | 196.1600 |
| 323 | 194.2441 | 386.6979 | 577.3834 | 766.3221 | 953.5354 | 194.3232 | 386.8555 | 194.3410 | 194.3465 |
| 324 | 192.4538 | 383.1393 | 572.0781 | 759.2914 | 944.8001 | 192.5322 | 383.2954 | 192.5498 | 192.5553 |
| 325 | 190.6855 | 379.6242 | 566.8375 | 752.3463 | 936.1712 | 190.7632 | 379.7789 | 190.7806 | 190.7861 |
| 326 | 188.9388 | 376.1520 | 561.6608 | 745.4857 | 927.6470 | 189.0157 | 376.3053 | 189.0330 | 189.0384 |
| 327 | 187.2133 | 372.7221 | 556.5470 | 738.7083 | 919.2260 | 187.2896 | 372.8739 | 187.3067 | 187.3120 |
| 328 | 185.5088 | 369.3337 | 551.4950 | 732.0128 | 910.9067 | 185.5844 | 369.4842 | 185.6013 | 185.6066 |
| 329 | 183.8249 | 365.9862 | 546.5040 | 725.3979 | 902.6874 | 183.8998 | 366.1353 | 183.9166 | 183.9218 |
| 330 | 182.1613 | 362.6791 | 541.5730 | 718.8625 | 894.5667 | 182.2555 | 362.8269 | 182.2522 | 182.2574 |
| 331 | 180.5178 | 359.4117 | 536.7012 | 712.4054 | 886.5431 | 180.5913 | 359.5581 | 180.6078 | 180.6130 |
| 332 | 178.8939 | 356.1834 | 531.8876 | 706.0254 | 878.6152 | 178.9668 | 356.3286 | 178.9832 | 178.9883 |
| 333 | 177.2895 | 352.9937 | 527.1314 | 699.7213 | 870.7816 | 177.3617 | 353.1375 | 177.3779 | 177.3830 |
| 334 | 175.7042 | 349.8419 | 522.4318 | 693.4921 | 863.0408 | 175.7758 | 349.9845 | 175.7919 | 175.7969 |
| 335 | 174.1377 | 346.7276 | 517.7879 | 687.3366 | 855.3914 | 174.2087 | 346.8689 | 174.2246 | 174.2296 |
| 336 | 172.5899 | 343.6501 | 513.1988 | 681.2537 | 847.8323 | 172.6602 | 343.7902 | 172.6760 | 172.6809 |
| 337 | 171.0603 | 340.6090 | 508.6638 | 675.2424 | 840.3619 | 171.1300 | 340.7478 | 171.1456 | 171.1505 |
| 338 | 169.5487 | 337.6036 | 504.1821 | 669.3016 | 832.9790 | 169.6178 | 337.7411 | 169.6333 | 169.6381 |
| 339 | 168.0549 | 334.6334 | 499.7529 | 663.4303 | 825.6823 | 168.1234 | 334.7698 | 168.1387 | 168.1435 |
| 340 | 166.5786 | 331.6980 | 495.3754 | 657.6275 | 818.4707 | 166.6464 | 331.8332 | 166.6617 | 166.6664 |
| 341 | 165.1195 | 328.7969 | 491.0489 | 651.8921 | 811.3428 | 165.1868 | 328.9308 | 165.2019 | 165.2066 |
| 342 | 163.6774 | 325.9294 | 486.7726 | 646.2233 | 804.2974 | 163.7441 | 326.0623 | 163.7590 | 163.7637 |
| 343 | 162.2520 | 323.0953 | 482.5459 | 640.6200 | 797.3333 | 162.3182 | 323.2269 | 162.3330 | 162.3376 |
| 344 | 160.8432 | 320.2938 | 478.3679 | 635.0812 | 790.4494 | 160.9087 | 320.4243 | 160.9234 | 160.9280 |
| 345 | 159.4506 | 317.5247 | 474.2380 | 629.6062 | 783.6445 | 159.5156 | 317.6541 | 159.5302 | 159.5347 |
| 346 | 158.0741 | 314.7874 | 470.1556 | 624.1939 | 776.9175 | 158.1385 | 314.9157 | 158.1529 | 158.1575 |
| 347 | 156.7133 | 312.0815 | 466.1198 | 618.8435 | 770.2673 | 156.7772 | 312.2087 | 156.7915 | 156.7960 |
| 348 | 155.3682 | 309.4065 | 462.1301 | 613.5540 | 763.6928 | 155.4315 | 309.5326 | 155.4457 | 155.4501 |
| 349 | 154.0383 | 306.7620 | 458.1858 | 608.3246 | 757.1929 | 154.1011 | 306.8870 | 154.1152 | 154.1196 |
| 350 | 152.7236 | 304.1475 | 454.2863 | 603.1546 | 750.7665 | 152.7859 | 304.2714 | 152.7998 | 152.8042 |
| 351 | 151.4239 | 301.5627 | 450.4309 | 598.0429 | 744.4127 | 151.4856 | 301.6856 | 151.4994 | 151.5037 |
| 352 | 150.1388 | 299.0070 | 446.6190 | 592.9888 | 738.1303 | 150.2000 | 299.1289 | 150.2137 | 150.2180 |
| 353 | 148.8682 | 296.4802 | 442.8500 | 587.9915 | 731.9185 | 148.9289 | 296.6010 | 148.9425 | 148.9468 |
| 354 | 147.6120 | 293.9818 | 439.1233 | 583.0502 | 725.7761 | 147.6721 | 294.1016 | 147.6856 | 147.6898 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 355 | 146.3698 | 291.5113 | 435.4383 | 578.1642 | 719.7023 | 146.4294 | 291.6301 | 146.4428 | 146.4470 |
| 356 | 145.1415 | 289.0685 | 431.7944 | 573.3325 | 713.6961 | 145.2007 | 289.1863 | 145.2139 | 145.2181 |
| 357 | 143.9270 | 286.6528 | 428.1910 | 568.5546 | 707.7565 | 143.9856 | 286.7697 | 143.9988 | 144.0029 |
| 358 | 142.7259 | 284.2641 | 424.6276 | 563.8296 | 701.8827 | 142.7841 | 284.3799 | 142.7971 | 142.8012 |
| 359 | 141.5382 | 281.9017 | 421.1037 | 559.1568 | 696.0737 | 141.5958 | 282.0166 | 141.6088 | 141.6128 |
| 360 | 140.3636 | 279.5655 | 417.6186 | 554.5355 | 690.3286 | 140.4208 | 279.6795 | 140.4336 | 140.4376 |
| 361 | 139.2020 | 277.2551 | 414.1719 | 549.9650 | 684.6465 | 139.2587 | 277.3680 | 139.2714 | 139.2754 |
| 362 | 138.0531 | 274.9700 | 410.7631 | 545.4446 | 679.0267 | 138.1094 | 275.0820 | 138.1220 | 138.1259 |
| 363 | 136.9169 | 272.7099 | 407.3915 | 540.9736 | 673.4682 | 136.9727 | 272.8211 | 136.9852 | 136.9891 |
| 364 | 135.7931 | 270.4746 | 404.0567 | 536.5513 | 667.9701 | 135.8484 | 270.5848 | 135.8608 | 135.8647 |
| 365 | 134.6815 | 268.2636 | 400.7582 | 532.1771 | 662.5318 | 134.7364 | 268.3729 | 134.7487 | 134.7526 |
| 366 | 133.5821 | 266.0767 | 397.4955 | 527.8503 | 657.1524 | 133.6365 | 266.1851 | 133.6487 | 133.6526 |
| 367 | 132.4946 | 263.9134 | 394.2682 | 523.5703 | 651.8310 | 132.5486 | 264.0210 | 132.5607 | 132.5645 |
| 368 | 131.4189 | 261.7736 | 391.0757 | 519.3364 | 646.5669 | 131.4724 | 261.8803 | 131.4844 | 131.4882 |
| 369 | 130.3547 | 259.6568 | 387.9175 | 515.1481 | 641.3594 | 130.4079 | 259.7626 | 130.4198 | 130.4235 |
| 370 | 129.3021 | 257.5628 | 384.7933 | 511.0046 | 636.2076 | 129.3548 | 257.6678 | 129.3666 | 129.3703 |
| 371 | 128.2607 | 255.4912 | 381.7026 | 506.9055 | 631.1109 | 128.3130 | 255.5954 | 128.3247 | 128.3284 |
| 372 | 127.2305 | 253.4418 | 378.6448 | 502.8502 | 626.0684 | 127.2824 | 253.5451 | 127.2940 | 127.2976 |
| 373 | 126.2113 | 251.4143 | 375.6196 | 498.8379 | 621.0796 | 126.2627 | 251.5167 | 126.2743 | 126.2779 |
| 374 | 125.2030 | 249.4083 | 372.6266 | 494.8683 | 616.1436 | 125.2540 | 249.5100 | 125.2654 | 125.2690 |
| 375 | 124.2053 | 247.4236 | 369.6653 | 490.9406 | 611.2598 | 124.2560 | 247.5245 | 124.2673 | 124.2709 |
| 376 | 123.2183 | 245.4599 | 366.7353 | 487.0544 | 606.4274 | 123.2685 | 245.5600 | 123.2798 | 123.2833 |
| 377 | 122.2417 | 243.5170 | 363.8361 | 483.2091 | 601.6459 | 122.2915 | 243.6162 | 122.3026 | 122.3061 |
| 378 | 121.2753 | 241.5945 | 360.9675 | 479.4042 | 596.9144 | 121.3247 | 241.6929 | 121.3358 | 121.3393 |
| 379 | 120.3191 | 239.6921 | 358.1289 | 475.6391 | 592.2325 | 120.3682 | 239.7898 | 120.3792 | 120.3826 |
| 380 | 119.3730 | 237.8097 | 355.3200 | 471.9134 | 587.5994 | 119.4216 | 237.9066 | 119.4325 | 119.4360 |
| 381 | 118.4367 | 235.9470 | 352.5404 | 468.2264 | 583.0145 | 118.4850 | 236.0431 | 118.4958 | 118.4992 |
| 382 | 117.5102 | 234.1036 | 349.7897 | 464.5778 | 578.4772 | 117.5581 | 234.1990 | 117.5689 | 117.5722 |
| 383 | 116.5934 | 232.2794 | 347.0676 | 460.9670 | 573.9869 | 116.6409 | 232.3741 | 116.6516 | 116.6549 |
| 384 | 115.6861 | 230.4742 | 344.3736 | 457.3935 | 569.5429 | 115.7332 | 230.5681 | 115.7438 | 115.7471 |
| 385 | 114.7881 | 228.6875 | 341.7074 | 453.8568 | 565.1446 | 114.8349 | 228.7807 | 114.8454 | 114.8486 |
| 386 | 113.8994 | 226.9193 | 339.0687 | 450.3565 | 560.7916 | 113.9458 | 227.0118 | 113.9562 | 113.9595 |
| 387 | 113.0199 | 225.1693 | 336.4571 | 446.8921 | 556.4831 | 113.0659 | 225.2610 | 113.0763 | 113.0795 |
| 388 | 112.1494 | 223.4372 | 333.8722 | 443.4632 | 552.2186 | 112.1951 | 223.5283 | 112.2053 | 112.2085 |
| 389 | 111.2878 | 221.7229 | 331.3138 | 440.0692 | 547.9976 | 111.3332 | 221.8132 | 111.3433 | 111.3465 |
| 390 | 110.4350 | 220.0260 | 328.7814 | 436.7098 | 543.8196 | 110.4800 | 220.1156 | 110.4901 | 110.4933 |
| 391 | 109.5909 | 218.3464 | 326.2748 | 433.3845 | 539.6839 | 109.6356 | 218.4354 | 109.6456 | 109.6488 |
| 392 | 108.7554 | 216.6838 | 323.7936 | 430.0929 | 535.5900 | 108.7998 | 216.7721 | 108.8097 | 108.8128 |
| 393 | 107.9284 | 215.0381 | 321.3375 | 426.8345 | 531.5374 | 107.9724 | 215.1258 | 107.9822 | 107.9853 |
| 394 | 107.1097 | 213.4091 | 318.9061 | 423.6090 | 527.5256 | 107.1534 | 213.4960 | 107.1632 | 107.1662 |
| 395 | 106.2993 | 211.7964 | 316.4992 | 420.4159 | 523.5541 | 106.3426 | 211.8827 | 106.3524 | 106.3554 |
| 396 | 105.4971 | 210.1999 | 314.1165 | 417.2547 | 519.6223 | 105.5401 | 210.2856 | 105.5497 | 105.5527 |
| 397 | 104.7029 | 208.6195 | 311.7577 | 414.1252 | 515.7298 | 104.7455 | 208.7045 | 104.7551 | 104.7581 |
| 398 | 103.9166 | 207.0548 | 309.4224 | 411.0270 | 511.8761 | 103.9590 | 207.1392 | 103.9684 | 103.9714 |
| 399 | 103.1382 | 205.5058 | 307.1103 | 407.9595 | 508.0607 | 103.1802 | 205.5895 | 103.1897 | 103.1926 |
| 400 | 102.3676 | 203.9721 | 304.8213 | 404.9225 | 504.2831 | 102.4093 | 204.0553 | 102.4186 | 102.4216 |
| 401 | 101.6046 | 202.4537 | 302.5549 | 401.9156 | 500.5429 | 101.6460 | 202.5362 | 101.6553 | 101.6582 |
| 402 | 100.8492 | 200.9504 | 300.3110 | 398.9383 | 496.8396 | 100.8902 | 201.0322 | 100.8995 | 100.9023 |
| 403 | 100.1012 | 199.4618 | 298.0892 | 395.9904 | 493.1727 | 100.1420 | 199.5431 | 100.1511 | 100.1540 |
| 404 | 99.36063 | 197.9880 | 295.8892 | 393.0715 | 489.5418 | 99.40111 | 198.0686 | 99.41019 | 99.41303 |
| 405 | 98.62734 | 196.5286 | 293.7109 | 390.1812 | 485.9465 | 98.66753 | 196.6087 | 98.67654 | 98.67936 |
| 406 | 97.90125 | 195.0835 | 291.5538 | 387.3191 | 482.3863 | 97.94114 | 195.1630 | 97.95009 | 97.95289 |
| 407 | 97.18227 | 193.6526 | 289.4179 | 384.4850 | 478.8607 | 97.22187 | 193.7315 | 97.23075 | 97.23353 |
| 408 | 96.47031 | 192.2356 | 287.3027 | 381.6785 | 475.3695 | 96.50962 | 192.3139 | 96.51844 | 96.52120 |
| 409 | 95.76529 | 190.8324 | 285.2082 | 378.8992 | 471.9121 | 95.80432 | 190.9102 | 95.81307 | 95.81581 |
| 410 | 95.06713 | 189.4429 | 283.1339 | 376.1468 | 468.4882 | 95.10587 | 189.5201 | 95.11455 | 95.11727 |
| 411 | 94.37573 | 188.0668 | 281.0797 | 373.4210 | 465.0973 | 94.41419 | 188.1434 | 94.42281 | 94.42551 |
| 412 | 93.69102 | 186.7039 | 279.0453 | 370.7215 | 461.7390 | 93.72920 | 186.7800 | 93.73776 | 93.74044 |
| 413 | 93.01292 | 185.3543 | 277.0305 | 368.0480 | 458.4130 | 93.05083 | 185.4298 | 93.05933 | 93.06198 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 414 | 92.34135 | 184.0176 | 275.0351 | 365.4001 | 455.1189 | 92.37898 | 184.0926 | 92.38742 | 92.39006 |
| 415 | 91.67623 | 182.6937 | 273.0587 | 362.7775 | 451.8562 | 91.71359 | 182.7682 | 91.72196 | 91.72458 |
| 416 | 91.01748 | 181.3825 | 271.1013 | 360.1800 | 448.6247 | 91.05457 | 181.4564 | 91.06289 | 91.06549 |
| 417 | 90.36503 | 180.0838 | 269.1625 | 357.6072 | 445.4239 | 90.40185 | 180.1572 | 90.41011 | 90.41269 |
| 418 | 89.71879 | 178.7975 | 267.2422 | 355.0589 | 442.2535 | 89.75535 | 178.8704 | 89.76355 | 89.76612 |
| 419 | 89.07871 | 177.5234 | 265.3401 | 352.5347 | 439.1131 | 89.11501 | 177.5957 | 89.12315 | 89.12569 |
| 420 | 88.44470 | 176.2614 | 263.4560 | 350.0344 | 436.0024 | 88.48074 | 176.3332 | 88.48882 | 88.49135 |
| 421 | 87.81669 | 175.0113 | 261.5897 | 347.5577 | 432.9209 | 87.85247 | 175.0826 | 87.86050 | 87.86301 |
| 422 | 87.19461 | 173.7730 | 259.7410 | 345.1043 | 429.8685 | 87.23014 | 173.8438 | 87.23811 | 87.24060 |
| 423 | 86.57840 | 172.5464 | 257.9096 | 342.6739 | 426.8447 | 86.61368 | 172.6167 | 86.62159 | 86.62406 |
| 424 | 85.96797 | 171.3313 | 256.0955 | 340.2663 | 423.8491 | 86.00300 | 171.4011 | 86.01086 | 86.01332 |
| 425 | 85.36328 | 170.1275 | 254.2983 | 337.8812 | 420.8816 | 85.39806 | 170.1968 | 85.40586 | 85.40830 |
| 426 | 84.76424 | 168.9350 | 252.5179 | 335.5183 | 417.9417 | 84.79878 | 169.0039 | 84.80652 | 84.80895 |
| 427 | 84.17079 | 167.7537 | 250.7541 | 333.1774 | 415.0291 | 84.20509 | 167.8220 | 84.21278 | 84.21519 |
| 428 | 83.58287 | 166.5833 | 249.0066 | 330.8583 | 412.1434 | 83.61693 | 166.6512 | 83.62457 | 83.62695 |
| 429 | 83.00041 | 165.4238 | 247.2754 | 328.5606 | 409.2845 | 83.03423 | 165.4912 | 83.04182 | 83.04419 |
| 430 | 82.42335 | 164.2750 | 245.5602 | 326.2841 | 406.4520 | 82.45694 | 164.3419 | 82.46447 | 82.46683 |
| 431 | 81.85163 | 163.1368 | 243.8608 | 324.0286 | 403.6455 | 81.88498 | 163.2033 | 81.89246 | 81.89480 |
| 432 | 81.28518 | 162.0091 | 242.1770 | 321.7939 | 400.8649 | 81.31831 | 162.0752 | 81.32574 | 81.32806 |
| 433 | 80.72395 | 160.8918 | 240.5087 | 319.5797 | 398.1097 | 80.75685 | 160.9574 | 80.76422 | 80.76653 |
| 434 | 80.16788 | 159.7848 | 238.8557 | 317.3857 | 395.3797 | 80.20054 | 159.8499 | 80.20787 | 80.21016 |
| 435 | 79.61690 | 158.6878 | 237.2178 | 315.2118 | 392.6746 | 79.64934 | 158.7525 | 79.65861 | 79.65889 |
| 436 | 79.07095 | 157.6009 | 235.5949 | 313.0577 | 389.9941 | 79.10317 | 157.6652 | 79.11040 | 79.11266 |
| 437 | 78.52999 | 156.5239 | 233.9867 | 310.9231 | 387.3379 | 78.56199 | 156.5877 | 78.56917 | 78.57141 |
| 438 | 77.99395 | 155.4567 | 232.3931 | 308.8080 | 384.7059 | 78.02573 | 155.5201 | 78.03286 | 78.03509 |
| 439 | 77.46278 | 154.3992 | 230.8140 | 306.7119 | 382.0976 | 77.49434 | 154.4621 | 77.50142 | 77.50363 |
| 440 | 76.93642 | 153.3512 | 229.2491 | 304.6348 | 379.5128 | 76.96777 | 153.4137 | 76.97480 | 76.97700 |
| 441 | 76.41481 | 152.3127 | 227.6984 | 302.5764 | 376.9513 | 76.44595 | 152.3748 | 76.45294 | 76.45512 |
| 442 | 75.89792 | 151.2836 | 226.1616 | 300.5365 | 374.4129 | 75.92884 | 151.3452 | 75.93578 | 75.93795 |
| 443 | 75.38567 | 150.2637 | 224.6386 | 298.5149 | 371.8971 | 75.41639 | 150.3249 | 75.42328 | 75.42543 |
| 444 | 74.87802 | 149.2529 | 223.1293 | 296.5114 | 369.4038 | 74.90854 | 149.3138 | 74.91538 | 74.91752 |
| 445 | 74.37492 | 148.2512 | 221.6334 | 294.5258 | 366.9328 | 74.40523 | 148.3117 | 74.41203 | 74.41415 |
| 446 | 73.87632 | 147.2585 | 220.1509 | 292.5579 | 364.4838 | 73.90643 | 147.3185 | 73.91318 | 73.91529 |
| 447 | 73.38217 | 146.2746 | 218.6816 | 290.6075 | 362.0565 | 73.41207 | 146.3342 | 73.41878 | 73.42087 |
| 448 | 72.89241 | 145.2994 | 217.2253 | 288.6743 | 359.6507 | 72.92211 | 145.3586 | 72.92877 | 72.93086 |
| 449 | 72.40700 | 144.3329 | 215.7819 | 286.7583 | 357.2662 | 72.43651 | 144.3917 | 72.44312 | 72.44519 |
| 450 | 71.92589 | 143.3749 | 214.3513 | 284.8592 | 354.9027 | 71.95520 | 143.4334 | 71.96177 | 71.96383 |
| 451 | 71.44904 | 142.4254 | 212.9333 | 282.9768 | 352.5600 | 71.47815 | 142.4835 | 71.48468 | 71.48672 |
| 452 | 70.97639 | 141.4843 | 211.5278 | 281.1110 | 350.2379 | 71.00531 | 141.5419 | 71.01179 | 71.01382 |
| 453 | 70.50790 | 140.5514 | 210.1346 | 279.2616 | 347.9362 | 70.53663 | 140.6087 | 70.54307 | 70.54508 |
| 454 | 70.04352 | 139.6267 | 208.7537 | 277.4283 | 345.6546 | 70.07206 | 139.6836 | 70.07846 | 70.08046 |
| 455 | 69.58321 | 138.7101 | 207.3848 | 275.6110 | 343.3928 | 69.61157 | 138.7667 | 69.61793 | 69.61991 |
| 456 | 69.12693 | 137.8016 | 206.0278 | 273.8096 | 341.1508 | 69.15510 | 137.8577 | 69.16142 | 69.16339 |
| 457 | 68.67463 | 136.9009 | 204.6827 | 272.0239 | 338.9282 | 68.70261 | 136.9567 | 68.70889 | 68.71085 |
| 458 | 68.22626 | 136.0081 | 203.3492 | 270.2536 | 336.7249 | 68.25406 | 136.0635 | 68.26030 | 68.26225 |
| 459 | 67.78179 | 135.1230 | 202.0273 | 268.4987 | 334.5407 | 67.80941 | 135.1780 | 67.81561 | 67.81754 |
| 460 | 67.34118 | 134.2455 | 200.7169 | 266.7589 | 332.3753 | 67.36862 | 134.3002 | 67.37477 | 67.37669 |
| 461 | 66.90437 | 133.3757 | 199.4177 | 265.0341 | 330.2286 | 66.93163 | 133.4300 | 66.93775 | 66.93966 |
| 462 | 66.47133 | 132.5134 | 198.1298 | 263.3242 | 328.1003 | 66.49842 | 132.5674 | 66.50449 | 66.50639 |
| 463 | 66.04202 | 131.6584 | 196.8529 | 261.6289 | 325.9902 | 66.06893 | 131.7121 | 66.07497 | 66.07686 |
| 464 | 65.61640 | 130.8108 | 195.5869 | 259.9482 | 323.8982 | 65.64314 | 130.8641 | 65.64914 | 65.65101 |
| 465 | 65.19443 | 129.9705 | 194.3318 | 258.2818 | 321.8241 | 65.22100 | 130.0235 | 65.22696 | 65.22882 |
| 466 | 64.77607 | 129.1374 | 193.0874 | 256.6297 | 319.7676 | 64.80247 | 129.1900 | 64.80839 | 64.81024 |
| 467 | 64.36129 | 128.3113 | 191.8536 | 254.9916 | 317.7287 | 64.38751 | 128.3636 | 64.39340 | 64.39523 |
| 468 | 63.95003 | 127.4923 | 190.6303 | 253.3674 | 315.7070 | 63.97609 | 127.5443 | 63.98194 | 63.98376 |
| 469 | 63.54228 | 126.6803 | 189.4174 | 251.7570 | 313.7024 | 63.56817 | 126.7319 | 63.57398 | 63.57579 |
| 470 | 63.13798 | 125.8751 | 188.2147 | 250.1602 | 311.7148 | 63.16371 | 125.9264 | 63.16948 | 63.17128 |
| 471 | 62.73710 | 125.0767 | 187.0222 | 248.5768 | 309.7440 | 62.76267 | 125.1277 | 62.76840 | 62.77019 |
| 472 | 62.33961 | 124.2851 | 185.8397 | 247.0069 | 307.7897 | 62.36502 | 124.3357 | 62.37071 | 62.37249 |

(continued)

Table B.1. (continued)

| <i>n</i> | Line transition (frequencies in MHz) | | | | | | | | |
|----------|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 473 | 61.94548 | 123.5001 | 184.6672 | 245.4501 | 305.8518 | 61.97072 | 123.5505 | 61.97638 | 61.97815 |
| 474 | 61.55465 | 122.7218 | 183.5046 | 243.9063 | 303.9301 | 61.57974 | 122.7718 | 61.58536 | 61.58712 |
| 475 | 61.16711 | 121.9499 | 182.3517 | 242.3755 | 302.0246 | 61.19204 | 121.9996 | 61.19763 | 61.19937 |
| 476 | 60.78282 | 121.1845 | 181.2084 | 240.8574 | 300.1349 | 60.80758 | 121.2339 | 60.81314 | 60.81488 |
| 477 | 60.40173 | 120.4256 | 180.0746 | 239.3521 | 298.2609 | 60.42635 | 120.4746 | 60.43187 | 60.43359 |
| 478 | 60.02383 | 119.6729 | 178.9503 | 237.8592 | 296.4025 | 60.04829 | 119.7217 | 60.05377 | 60.05549 |
| 479 | 59.64907 | 118.9265 | 177.8354 | 236.3787 | 294.5596 | 59.67338 | 118.9750 | 59.67883 | 59.68053 |
| 480 | 59.27743 | 118.1863 | 176.7296 | 234.9105 | 292.7318 | 59.30158 | 118.2344 | 59.30700 | 59.30869 |
| 481 | 58.90886 | 117.4522 | 175.6331 | 233.4544 | 290.9192 | 58.93287 | 117.5001 | 58.93825 | 58.93993 |
| 482 | 58.54335 | 116.7242 | 174.5455 | 232.0103 | 289.1215 | 58.56720 | 116.7718 | 58.57255 | 58.57423 |
| 483 | 58.18085 | 116.0022 | 173.4670 | 230.5781 | 287.3386 | 58.20456 | 116.0495 | 58.20988 | 58.21154 |
| 484 | 57.82134 | 115.2861 | 172.3973 | 229.1577 | 285.5703 | 57.84490 | 115.3331 | 57.85019 | 57.85184 |
| 485 | 57.46479 | 114.5759 | 171.3364 | 227.7489 | 283.8164 | 57.48820 | 114.6226 | 57.49345 | 57.49510 |
| 486 | 57.11116 | 113.8716 | 170.2841 | 226.3517 | 282.0770 | 57.13443 | 113.9180 | 57.13965 | 57.14128 |
| 487 | 56.76042 | 113.1730 | 169.2405 | 224.9658 | 280.3517 | 56.78355 | 113.2191 | 56.78874 | 56.79036 |
| 488 | 56.41256 | 112.4801 | 168.2054 | 223.5912 | 278.6404 | 56.43554 | 112.5259 | 56.44070 | 56.44231 |
| 489 | 56.06753 | 111.7928 | 167.1787 | 222.2279 | 276.9431 | 56.09037 | 111.8384 | 56.09550 | 56.09710 |
| 490 | 55.72530 | 111.1112 | 166.1603 | 220.8755 | 275.2595 | 55.74801 | 111.1564 | 55.75310 | 55.75470 |
| 491 | 55.38586 | 110.4350 | 165.1502 | 219.5342 | 273.5895 | 55.40843 | 110.4800 | 55.41349 | 55.41507 |
| 492 | 55.04917 | 109.7644 | 164.1483 | 218.2036 | 271.9330 | 55.07160 | 109.8091 | 55.07663 | 55.07820 |
| 493 | 54.71520 | 109.0991 | 163.1545 | 216.8838 | 270.2898 | 54.73750 | 109.1436 | 54.74250 | 54.74406 |
| 494 | 54.38393 | 108.4393 | 162.1686 | 215.5746 | 268.6599 | 54.40609 | 108.4834 | 54.41106 | 54.41261 |
| 495 | 54.05532 | 107.7847 | 161.1907 | 214.2760 | 267.0430 | 54.07735 | 107.8286 | 54.08229 | 54.08384 |
| 496 | 53.72936 | 107.1354 | 160.2206 | 212.9877 | 265.4391 | 53.75126 | 107.1790 | 53.75617 | 53.75770 |
| 497 | 53.40602 | 106.4913 | 159.2583 | 211.7098 | 263.8480 | 53.42778 | 106.5347 | 53.43266 | 53.43419 |
| 498 | 53.08526 | 105.8523 | 158.3037 | 210.4420 | 262.2696 | 53.10689 | 105.8955 | 53.11174 | 53.11326 |
| 499 | 52.76707 | 105.2185 | 157.3568 | 209.1844 | 260.7038 | 52.78857 | 105.2614 | 52.79339 | 52.79490 |
| 500 | 52.45141 | 104.5897 | 156.4173 | 207.9367 | 259.1504 | 52.47279 | 104.6323 | 52.47758 | 52.47908 |
| 501 | 52.13827 | 103.9659 | 155.4853 | 206.6990 | 257.6093 | 52.15952 | 104.0083 | 52.16428 | 52.16577 |
| 502 | 51.82762 | 103.3470 | 154.5607 | 205.4710 | 256.0804 | 51.84873 | 103.3892 | 51.85347 | 51.85495 |
| 503 | 51.51942 | 102.7331 | 153.6434 | 204.2528 | 254.5636 | 51.54042 | 102.7750 | 51.54513 | 51.54660 |
| 504 | 51.21367 | 102.1240 | 152.7334 | 203.0442 | 253.0588 | 51.23454 | 102.1656 | 51.23922 | 51.24068 |
| 505 | 50.91033 | 101.5197 | 151.8305 | 201.8451 | 251.5657 | 50.93108 | 101.5611 | 50.93573 | 50.93719 |
| 506 | 50.60939 | 100.9202 | 150.9348 | 200.6554 | 250.0845 | 50.63001 | 100.9613 | 50.63463 | 50.63608 |
| 507 | 50.31081 | 100.3254 | 150.0460 | 199.4751 | 248.6148 | 50.33131 | 100.3663 | 50.33590 | 50.33734 |
| 508 | 50.01457 | 99.73522 | 149.1643 | 198.3040 | 247.1566 | 50.03495 | 99.77587 | 50.03952 | 50.04095 |
| 509 | 49.72066 | 99.14969 | 148.2894 | 197.1420 | 245.7098 | 49.74092 | 99.19010 | 49.74546 | 49.74688 |
| 510 | 49.42904 | 98.56874 | 147.4213 | 195.9891 | 244.2742 | 49.44918 | 98.60890 | 49.45370 | 49.45511 |
| 511 | 49.13970 | 97.99231 | 146.5601 | 194.8452 | 242.8498 | 49.15972 | 98.03224 | 49.16421 | 49.16562 |
| 512 | 48.85261 | 97.42037 | 145.7055 | 193.7101 | 241.4365 | 48.87252 | 97.46007 | 48.87698 | 48.87838 |
| 513 | 48.56776 | 96.85287 | 144.8575 | 192.5839 | 240.0341 | 48.58755 | 96.89234 | 48.59199 | 48.59337 |
| 514 | 48.28511 | 96.28977 | 144.0161 | 191.4664 | 238.6426 | 48.30479 | 96.32901 | 48.30920 | 48.31058 |
| 515 | 48.00466 | 95.73103 | 143.1813 | 190.3575 | 237.2618 | 48.02422 | 95.77004 | 48.02861 | 48.02998 |
| 516 | 47.72637 | 95.17660 | 142.3528 | 189.2571 | 235.8916 | 47.74582 | 95.21538 | 47.75018 | 47.75154 |
| 517 | 47.45023 | 94.62644 | 141.5308 | 188.1652 | 234.5320 | 47.46956 | 94.66500 | 47.47390 | 47.47526 |
| 518 | 47.17622 | 94.08052 | 140.7150 | 187.0817 | 233.1827 | 47.19544 | 94.11886 | 47.19975 | 47.20110 |
| 519 | 46.90431 | 93.53879 | 139.9055 | 186.0065 | 231.8439 | 46.92342 | 93.57691 | 46.92771 | 46.92905 |
| 520 | 46.63448 | 93.00121 | 139.1022 | 184.9396 | 230.5152 | 46.65349 | 93.03911 | 46.65775 | 46.65908 |
| 521 | 46.36673 | 92.46774 | 138.3051 | 183.8807 | 229.1967 | 46.38562 | 92.50542 | 46.38986 | 46.39118 |
| 522 | 46.10101 | 91.93834 | 137.5140 | 182.8300 | 227.8882 | 46.11980 | 91.97581 | 46.12401 | 46.12533 |
| 523 | 45.83733 | 91.41298 | 136.7289 | 181.7872 | 226.5897 | 45.85601 | 91.45023 | 45.86020 | 45.86151 |
| 524 | 45.57565 | 90.89161 | 135.9499 | 180.7523 | 225.3010 | 45.59422 | 90.92865 | 45.59839 | 45.59969 |
| 525 | 45.31596 | 90.37420 | 135.1767 | 179.7253 | 224.0220 | 45.33443 | 90.41103 | 45.33857 | 45.33986 |
| 526 | 45.05824 | 89.86071 | 134.4093 | 178.7061 | 222.7527 | 45.07660 | 89.89733 | 45.08072 | 45.08201 |
| 527 | 44.80247 | 89.35110 | 133.6478 | 177.6945 | 221.4930 | 44.82073 | 89.38751 | 44.82482 | 44.82610 |
| 528 | 44.54863 | 88.84534 | 132.8920 | 176.6905 | 220.2428 | 44.56678 | 88.88154 | 44.57086 | 44.57213 |
| 529 | 44.29671 | 88.34339 | 132.1419 | 175.6942 | 219.0020 | 44.31476 | 88.37939 | 44.31881 | 44.32007 |
| 530 | 44.04668 | 87.84521 | 131.3974 | 174.7052 | 217.7704 | 44.06463 | 87.88101 | 44.06865 | 44.06991 |
| 531 | 43.79853 | 87.35077 | 130.6586 | 173.7237 | 216.5481 | 43.81638 | 87.38636 | 43.82038 | 43.82163 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 532 | 43.55224 | 86.86003 | 129.9252 | 172.7496 | 215.3349 | 43.56999 | 86.89543 | 43.57397 | 43.57521 |
| 533 | 43.30779 | 86.37297 | 129.1973 | 171.7827 | 214.1308 | 43.32544 | 86.40816 | 43.32940 | 43.33064 |
| 534 | 43.06517 | 85.88954 | 128.4749 | 170.8230 | 212.9356 | 43.08272 | 85.92453 | 43.08666 | 43.08789 |
| 535 | 42.82436 | 85.40970 | 127.7578 | 169.8704 | 211.7493 | 42.84181 | 85.44451 | 42.84573 | 42.84695 |
| 536 | 42.58534 | 84.93344 | 127.0461 | 168.9249 | 210.5718 | 42.60270 | 84.96805 | 42.60659 | 42.60780 |
| 537 | 42.34810 | 84.46071 | 126.3396 | 167.9864 | 209.4030 | 42.36535 | 84.49513 | 42.36923 | 42.37044 |
| 538 | 42.11261 | 83.99149 | 125.6383 | 167.0549 | 208.2428 | 42.12977 | 84.02571 | 42.13362 | 42.13483 |
| 539 | 41.87887 | 83.52573 | 124.9423 | 166.1302 | 207.0912 | 41.89594 | 83.55976 | 41.89976 | 41.90096 |
| 540 | 41.64686 | 83.06341 | 124.2513 | 165.2124 | 205.9481 | 41.66383 | 83.09725 | 41.66763 | 41.66882 |
| 541 | 41.41655 | 82.60449 | 123.5655 | 164.3012 | 204.8134 | 41.43343 | 82.63815 | 41.43721 | 41.43840 |
| 542 | 41.18794 | 82.14895 | 122.8847 | 163.3968 | 203.6870 | 41.20472 | 82.18243 | 41.20849 | 41.20967 |
| 543 | 40.96101 | 81.69675 | 122.2089 | 162.4990 | 202.5688 | 40.97770 | 81.73004 | 40.98144 | 40.98262 |
| 544 | 40.73574 | 81.24787 | 121.5380 | 161.6078 | 201.4588 | 40.75234 | 81.28098 | 40.75607 | 40.75723 |
| 545 | 40.51213 | 80.80227 | 120.8720 | 160.7231 | 200.3569 | 40.52863 | 80.83519 | 40.53234 | 40.53349 |
| 546 | 40.29014 | 80.35992 | 120.2109 | 159.8448 | 199.2630 | 40.30656 | 80.39266 | 40.31024 | 40.31139 |
| 547 | 40.06978 | 79.92079 | 119.5546 | 158.9729 | 198.1771 | 40.08610 | 79.95336 | 40.08977 | 40.09091 |
| 548 | 39.85102 | 79.48486 | 118.9031 | 158.1073 | 197.0990 | 39.86725 | 79.51725 | 39.87090 | 39.87204 |
| 549 | 39.63384 | 79.05209 | 118.2563 | 157.2480 | 196.0288 | 39.64999 | 79.08430 | 39.65362 | 39.65475 |
| 550 | 39.41825 | 78.62246 | 117.6142 | 156.3950 | 194.9663 | 39.43431 | 78.65450 | 39.43791 | 39.43904 |
| 551 | 39.20421 | 78.19594 | 116.9767 | 155.5480 | 193.9114 | 39.22019 | 78.22780 | 39.22377 | 39.22489 |
| 552 | 38.99172 | 77.77249 | 116.3438 | 154.7072 | 192.8642 | 39.00761 | 77.80418 | 39.01118 | 39.01229 |
| 553 | 38.78077 | 77.35210 | 115.7155 | 153.8725 | 191.8245 | 38.79657 | 77.38362 | 38.80012 | 38.80122 |
| 554 | 38.57133 | 76.93473 | 115.0917 | 153.0437 | 190.7922 | 38.58705 | 76.96608 | 38.59057 | 38.59168 |
| 555 | 38.36340 | 76.52036 | 114.4724 | 152.2209 | 189.7673 | 38.37903 | 76.55154 | 38.38254 | 38.38363 |
| 556 | 38.15696 | 76.10896 | 113.8575 | 151.4039 | 188.7498 | 38.17251 | 76.13997 | 38.17600 | 38.17709 |
| 557 | 37.95200 | 75.70050 | 113.2470 | 150.5928 | 187.7395 | 37.96746 | 75.73135 | 37.97093 | 37.97202 |
| 558 | 37.74850 | 75.29496 | 112.6408 | 149.7875 | 186.7364 | 37.76389 | 75.32564 | 37.76733 | 37.76841 |
| 559 | 37.54646 | 74.89231 | 112.0390 | 148.9879 | 185.7405 | 37.56176 | 74.92283 | 37.56519 | 37.56626 |
| 560 | 37.34586 | 74.49253 | 111.4414 | 148.1940 | 184.7516 | 37.36107 | 74.52289 | 37.36449 | 37.36555 |
| 561 | 37.14668 | 74.09559 | 110.8481 | 147.4057 | 183.7697 | 37.16181 | 74.12578 | 37.16521 | 37.16627 |
| 562 | 36.94891 | 73.70147 | 110.2590 | 146.6230 | 182.7948 | 36.96397 | 73.73150 | 36.96735 | 36.96840 |
| 563 | 36.75255 | 73.31013 | 109.6741 | 145.8459 | 181.8267 | 36.76753 | 73.34000 | 36.77089 | 36.77194 |
| 564 | 36.55758 | 72.92156 | 109.0933 | 145.0742 | 180.8655 | 36.57247 | 72.95127 | 36.57582 | 36.57686 |
| 565 | 36.36398 | 72.53573 | 108.5166 | 144.3079 | 179.9110 | 36.37880 | 72.56529 | 36.38212 | 36.38316 |
| 566 | 36.17175 | 72.15262 | 107.9439 | 143.5471 | 178.9633 | 36.18649 | 72.18202 | 36.18979 | 36.19083 |
| 567 | 35.98087 | 71.77220 | 107.3753 | 142.7915 | 178.0222 | 35.99553 | 71.80144 | 35.99882 | 35.99985 |
| 568 | 35.79133 | 71.39445 | 106.8107 | 142.0413 | 177.0877 | 35.80591 | 71.42354 | 35.80919 | 35.81021 |
| 569 | 35.60312 | 71.01935 | 106.2500 | 141.2963 | 176.1597 | 35.61763 | 71.04829 | 35.62088 | 35.62190 |
| 570 | 35.41623 | 70.64687 | 105.6932 | 140.5566 | 175.2382 | 35.43066 | 70.67565 | 35.43390 | 35.43491 |
| 571 | 35.23064 | 70.27699 | 105.1403 | 139.8219 | 174.3230 | 35.24500 | 70.30562 | 35.24822 | 35.24922 |
| 572 | 35.04635 | 69.90969 | 104.5913 | 139.0924 | 173.4143 | 35.06063 | 69.93817 | 35.06383 | 35.06483 |
| 573 | 34.86334 | 69.54494 | 104.0461 | 138.3680 | 172.5119 | 34.87754 | 69.57328 | 34.88073 | 34.88173 |
| 574 | 34.68160 | 69.18272 | 103.5046 | 137.6485 | 171.6157 | 34.69573 | 69.21092 | 34.69890 | 34.69989 |
| 575 | 34.50112 | 68.82302 | 102.9669 | 136.9341 | 170.7257 | 34.51518 | 68.85107 | 34.51834 | 34.51932 |
| 576 | 34.32190 | 68.46581 | 102.4330 | 136.2246 | 169.8418 | 34.33588 | 68.49371 | 34.33902 | 34.34000 |
| 577 | 34.14391 | 68.11106 | 101.9027 | 135.5200 | 168.9641 | 34.15782 | 68.13882 | 34.16094 | 34.16192 |
| 578 | 33.96715 | 67.75876 | 101.3760 | 134.8202 | 168.0924 | 33.98099 | 67.78637 | 33.98410 | 33.98507 |
| 579 | 33.79161 | 67.40889 | 100.8530 | 134.1252 | 167.2266 | 33.80538 | 67.43636 | 33.80847 | 33.80943 |
| 580 | 33.61728 | 67.06142 | 100.3336 | 133.4350 | 166.3668 | 33.63098 | 67.08875 | 33.63405 | 33.63501 |
| 581 | 33.44414 | 66.71633 | 99.81775 | 132.7496 | 165.5129 | 33.45777 | 66.74352 | 33.46083 | 33.46178 |
| 582 | 33.27219 | 66.37361 | 99.30543 | 132.0688 | 164.6649 | 33.28575 | 66.40066 | 33.28879 | 33.28974 |
| 583 | 33.10142 | 66.03323 | 98.79660 | 131.3927 | 163.8226 | 33.11491 | 66.06014 | 33.11793 | 33.11888 |
| 584 | 32.93181 | 65.69518 | 98.29124 | 130.7211 | 162.9860 | 32.94523 | 65.72195 | 32.94824 | 32.94918 |
| 585 | 32.76337 | 65.35943 | 97.78933 | 130.0542 | 162.1551 | 32.77672 | 65.38606 | 32.77971 | 32.78065 |
| 586 | 32.59606 | 65.02596 | 97.29082 | 129.3918 | 161.3299 | 32.60935 | 65.05246 | 32.61232 | 32.61326 |
| 587 | 32.42990 | 64.69476 | 96.79570 | 128.7338 | 160.5102 | 32.44311 | 64.72112 | 32.44608 | 32.44700 |
| 588 | 32.26486 | 64.36580 | 96.30393 | 128.0804 | 159.6961 | 32.27801 | 64.39203 | 32.28096 | 32.28188 |
| 589 | 32.10094 | 64.03907 | 95.81549 | 127.4313 | 158.8875 | 32.11402 | 64.06517 | 32.11696 | 32.11787 |
| 590 | 31.93813 | 63.71455 | 95.33034 | 126.7866 | 158.0844 | 31.95114 | 63.74051 | 31.95406 | 31.95498 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 591 | 31.77642 | 63.39221 | 94.84847 | 126.1463 | 157.2866 | 31.78937 | 63.41805 | 31.79227 | 31.79318 |
| 592 | 31.61580 | 63.07205 | 94.36984 | 125.5102 | 156.4942 | 31.62868 | 63.09775 | 31.63157 | 31.63247 |
| 593 | 31.45625 | 62.75404 | 93.89442 | 124.8784 | 155.7072 | 31.46907 | 62.77961 | 31.47195 | 31.47285 |
| 594 | 31.29779 | 62.43816 | 93.42219 | 124.2509 | 154.9254 | 31.31054 | 62.46361 | 31.31340 | 31.31429 |
| 595 | 31.14038 | 62.12440 | 92.95312 | 123.6276 | 154.1488 | 31.15307 | 62.14972 | 31.15591 | 31.15680 |
| 596 | 30.98403 | 61.81274 | 92.48719 | 123.0084 | 153.3774 | 30.99665 | 61.83793 | 30.99948 | 31.00037 |
| 597 | 30.82872 | 61.50316 | 92.02436 | 122.3933 | 152.6111 | 30.84128 | 61.52822 | 30.84410 | 30.84498 |
| 598 | 30.67444 | 61.19565 | 91.56462 | 121.7824 | 151.8499 | 30.68694 | 61.22058 | 30.68975 | 30.69062 |
| 599 | 30.52120 | 60.89018 | 91.10794 | 121.1755 | 151.0938 | 30.53364 | 60.91499 | 30.53643 | 30.53730 |
| 600 | 30.36898 | 60.58674 | 90.65428 | 120.5726 | 150.3427 | 30.38135 | 60.61142 | 30.38413 | 30.38499 |
| 601 | 30.21776 | 60.28531 | 90.20364 | 119.9737 | 149.5966 | 30.23007 | 60.30987 | 30.23284 | 30.23370 |
| 602 | 30.06755 | 59.98588 | 89.75597 | 119.3788 | 148.8554 | 30.07980 | 60.01032 | 30.08255 | 30.08341 |
| 603 | 29.91833 | 59.68843 | 89.31127 | 118.7878 | 148.1191 | 29.93052 | 59.71275 | 29.93326 | 29.93411 |
| 604 | 29.77010 | 59.39294 | 88.86950 | 118.2007 | 147.3876 | 29.78223 | 59.41714 | 29.78495 | 29.78580 |
| 605 | 29.62284 | 59.09940 | 88.43063 | 117.6175 | 146.6609 | 29.63491 | 59.12348 | 29.63762 | 29.63847 |
| 606 | 29.47656 | 58.80779 | 87.99465 | 117.0381 | 145.9390 | 29.48857 | 58.83175 | 29.49126 | 29.49210 |
| 607 | 29.33123 | 58.51809 | 87.56153 | 116.4625 | 145.2219 | 29.34318 | 58.54194 | 29.34587 | 29.34670 |
| 608 | 29.18686 | 58.23030 | 87.13125 | 115.8906 | 144.5094 | 29.19876 | 58.25403 | 29.20142 | 29.20226 |
| 609 | 29.04344 | 57.94439 | 86.70379 | 115.3226 | 143.8016 | 29.05527 | 57.96800 | 29.05793 | 29.05876 |
| 610 | 28.90095 | 57.66035 | 86.27911 | 114.7582 | 143.0984 | 28.91273 | 57.68384 | 28.91537 | 28.91620 |
| 611 | 28.75940 | 57.37816 | 85.85721 | 114.1974 | 142.3998 | 28.77112 | 57.40154 | 28.77374 | 28.77457 |
| 612 | 28.61876 | 57.09781 | 85.43805 | 113.6404 | 141.7057 | 28.63043 | 57.12108 | 28.63304 | 28.63386 |
| 613 | 28.47905 | 56.81928 | 85.02161 | 113.0869 | 141.0161 | 28.49065 | 56.84244 | 28.49325 | 28.49407 |
| 614 | 28.34024 | 56.54257 | 84.60788 | 112.5371 | 140.3310 | 28.35179 | 56.56561 | 28.35438 | 28.35519 |
| 615 | 28.20233 | 56.26764 | 84.19683 | 111.9908 | 139.6503 | 28.21382 | 56.29057 | 28.21640 | 28.21720 |
| 616 | 28.06531 | 55.99450 | 83.78843 | 111.4480 | 138.9740 | 28.07675 | 56.01731 | 28.07931 | 28.08012 |
| 617 | 27.92918 | 55.72312 | 83.38267 | 110.9087 | 138.3021 | 27.94057 | 55.74583 | 27.94312 | 27.94392 |
| 618 | 27.79393 | 55.45349 | 82.97953 | 110.3729 | 137.6345 | 27.80526 | 55.47609 | 27.80780 | 27.80859 |
| 619 | 27.65956 | 55.18560 | 82.57899 | 109.8406 | 136.9712 | 27.67083 | 55.20809 | 27.67335 | 27.67414 |
| 620 | 27.52604 | 54.91943 | 82.18101 | 109.3116 | 136.3121 | 27.53726 | 54.94181 | 27.53977 | 27.54056 |
| 621 | 27.39339 | 54.65497 | 81.78559 | 108.7861 | 135.6573 | 27.40455 | 54.67724 | 27.40705 | 27.40784 |
| 622 | 27.26158 | 54.39220 | 81.39270 | 108.2639 | 135.0067 | 27.27269 | 54.41437 | 27.27518 | 27.27596 |
| 623 | 27.13062 | 54.13112 | 81.00233 | 107.7451 | 134.3602 | 27.14168 | 54.15318 | 27.14416 | 27.14493 |
| 624 | 27.00050 | 53.87171 | 80.61445 | 107.2295 | 133.7178 | 27.01150 | 53.89366 | 27.01397 | 27.01474 |
| 625 | 26.87121 | 53.61395 | 80.22904 | 106.7173 | 133.0795 | 26.88216 | 53.63579 | 26.88461 | 26.88538 |
| 626 | 26.74274 | 53.35783 | 79.84608 | 106.2083 | 132.4453 | 26.75364 | 53.37957 | 26.75608 | 26.75685 |
| 627 | 26.61509 | 53.10334 | 79.46556 | 105.7025 | 131.8151 | 26.62593 | 53.12498 | 26.62837 | 26.62913 |
| 628 | 26.48825 | 52.85047 | 79.08745 | 105.2000 | 131.1889 | 26.49904 | 52.87200 | 26.50146 | 26.50222 |
| 629 | 26.36222 | 52.59920 | 78.71173 | 104.7006 | 130.5666 | 26.37296 | 52.62063 | 26.37537 | 26.37612 |
| 630 | 26.23698 | 52.34952 | 78.33839 | 104.2044 | 129.9483 | 26.24767 | 52.37085 | 26.25007 | 26.25082 |
| 631 | 26.11254 | 52.10141 | 77.96741 | 103.7113 | 129.3339 | 26.12318 | 52.12264 | 26.12556 | 26.12631 |
| 632 | 25.98888 | 51.85488 | 77.59877 | 103.2213 | 128.7233 | 25.99947 | 51.87601 | 26.00184 | 26.00259 |
| 633 | 25.86600 | 51.60989 | 77.23245 | 102.7344 | 128.1166 | 25.87654 | 51.63092 | 25.87890 | 25.87964 |
| 634 | 25.74389 | 51.36645 | 76.86843 | 102.2506 | 127.5137 | 25.75438 | 51.38738 | 25.75674 | 25.75747 |
| 635 | 25.62256 | 51.12453 | 76.50669 | 101.7698 | 126.9145 | 25.63300 | 51.14537 | 25.63534 | 25.63607 |
| 636 | 25.50198 | 50.88414 | 76.14722 | 101.2920 | 126.3191 | 25.51237 | 50.90487 | 25.51470 | 25.51543 |
| 637 | 25.38216 | 50.64524 | 75.79000 | 100.8172 | 125.7275 | 25.39250 | 50.66588 | 25.39482 | 25.39555 |
| 638 | 25.26309 | 50.40784 | 75.43501 | 100.3453 | 125.1395 | 25.27338 | 50.42838 | 25.27569 | 25.27641 |
| 639 | 25.14476 | 50.17192 | 75.08223 | 99.87641 | 124.5552 | 25.15500 | 50.19237 | 25.15730 | 25.15802 |
| 640 | 25.02717 | 49.93748 | 74.73165 | 99.41042 | 123.9745 | 25.03737 | 49.95783 | 25.03965 | 25.04037 |
| 641 | 24.91031 | 49.70449 | 74.38325 | 98.94732 | 123.3974 | 24.92046 | 49.72474 | 24.92274 | 24.92345 |
| 642 | 24.79418 | 49.47294 | 74.03701 | 98.48710 | 122.8239 | 24.80428 | 49.49310 | 24.80655 | 24.80725 |
| 643 | 24.67877 | 49.24284 | 73.69292 | 98.02972 | 122.2539 | 24.68882 | 49.26290 | 24.69108 | 24.69178 |
| 644 | 24.56407 | 49.01415 | 73.35096 | 97.57517 | 121.6875 | 24.57408 | 49.03413 | 24.57632 | 24.57703 |
| 645 | 24.45008 | 48.78689 | 73.01110 | 97.12343 | 121.1246 | 24.46005 | 48.80677 | 24.46228 | 24.46298 |
| 646 | 24.33680 | 48.56102 | 72.67335 | 96.67447 | 120.5651 | 24.34672 | 48.58081 | 24.34894 | 24.34964 |
| 647 | 24.22422 | 48.33655 | 72.33767 | 96.22828 | 120.0090 | 24.23409 | 48.35624 | 24.23630 | 24.23700 |
| 648 | 24.11233 | 48.11345 | 72.00406 | 95.78483 | 119.4564 | 24.12215 | 48.13306 | 24.12436 | 24.12505 |
| 649 | 24.00113 | 47.89173 | 71.67250 | 95.34409 | 118.9072 | 24.01091 | 47.91125 | 24.01310 | 24.01379 |

(continued)

Table B.1. (continued)

| <i>n</i> | Line transition (frequencies in MHz) | | | | | | | | |
|----------|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 650 | 23.89061 | 47.67137 | 71.34297 | 94.90606 | 118.3613 | 23.90034 | 47.69080 | 23.90253 | 23.90321 |
| 651 | 23.78077 | 47.45236 | 71.01455 | 94.47071 | 117.8188 | 23.79046 | 47.47170 | 23.79263 | 23.79331 |
| 652 | 23.67160 | 47.23469 | 70.68994 | 94.03801 | 117.2796 | 23.68124 | 47.25394 | 23.68340 | 23.68408 |
| 653 | 23.56309 | 47.01835 | 70.36642 | 93.60796 | 116.7436 | 23.57269 | 47.03751 | 23.57485 | 23.57552 |
| 654 | 23.45525 | 46.80332 | 70.04486 | 93.18052 | 116.2109 | 23.46481 | 46.82239 | 23.46695 | 23.46762 |
| 655 | 23.34807 | 46.58961 | 69.72526 | 92.75568 | 115.6815 | 23.35758 | 46.60859 | 23.35972 | 23.36038 |
| 656 | 23.24154 | 46.37719 | 69.40761 | 92.33342 | 115.1553 | 23.25101 | 46.39609 | 23.25313 | 23.25380 |
| 657 | 23.13566 | 46.16607 | 69.09188 | 91.91371 | 114.6322 | 23.14508 | 46.18488 | 23.14720 | 23.14786 |
| 658 | 23.03041 | 45.95622 | 68.77806 | 91.49655 | 114.1123 | 23.03980 | 45.97495 | 23.04190 | 23.04256 |
| 659 | 22.92581 | 45.74765 | 68.46614 | 91.08191 | 113.5956 | 22.93515 | 45.76629 | 22.93725 | 22.93790 |
| 660 | 22.82184 | 45.54033 | 68.15610 | 90.66977 | 113.0820 | 22.83114 | 45.55889 | 22.83322 | 22.83387 |
| 661 | 22.71849 | 45.33427 | 67.84793 | 90.26011 | 112.5714 | 22.72775 | 45.35274 | 22.72983 | 22.73048 |
| 662 | 22.61577 | 45.12944 | 67.54162 | 89.85292 | 112.0639 | 22.62499 | 45.14783 | 22.62705 | 22.62770 |
| 663 | 22.51367 | 44.92585 | 67.23715 | 89.44817 | 111.5595 | 22.52284 | 44.94416 | 22.52490 | 22.52554 |
| 664 | 22.41218 | 44.72348 | 66.93451 | 89.04585 | 111.0581 | 22.42131 | 44.74170 | 22.42336 | 22.42400 |
| 665 | 22.31130 | 44.52233 | 66.63367 | 88.64594 | 110.5597 | 22.32039 | 44.54047 | 22.32243 | 22.32307 |
| 666 | 22.21102 | 44.32237 | 66.33464 | 88.24842 | 110.0643 | 22.22008 | 44.34044 | 22.22211 | 22.22274 |
| 667 | 22.11135 | 44.12362 | 66.03740 | 87.85328 | 109.5718 | 22.12036 | 44.14160 | 22.12238 | 22.12301 |
| 668 | 22.01227 | 43.92605 | 65.74193 | 87.46049 | 109.0823 | 22.02124 | 43.94395 | 22.02325 | 22.02388 |
| 669 | 21.91378 | 43.72966 | 65.44822 | 87.07003 | 108.5957 | 21.92271 | 43.74748 | 21.92471 | 21.92534 |
| 670 | 21.81588 | 43.53444 | 65.15625 | 86.68190 | 108.1120 | 21.82747 | 43.55218 | 21.82676 | 21.82739 |
| 671 | 21.71856 | 43.34038 | 64.86062 | 86.29607 | 107.6311 | 21.72741 | 43.35804 | 21.72939 | 21.73001 |
| 672 | 21.62182 | 43.14747 | 64.57752 | 85.91253 | 107.1531 | 21.63063 | 43.16505 | 21.63260 | 21.63322 |
| 673 | 21.52565 | 42.95570 | 64.29072 | 85.53126 | 106.6779 | 21.53442 | 42.97320 | 21.53639 | 21.53700 |
| 674 | 21.43005 | 42.76507 | 64.00561 | 85.15224 | 106.2055 | 21.43878 | 42.78249 | 21.44074 | 21.44135 |
| 675 | 21.33502 | 42.57556 | 63.72219 | 84.77546 | 105.7359 | 21.34371 | 42.59291 | 21.34566 | 21.34627 |
| 676 | 21.24054 | 42.38717 | 63.44044 | 84.40089 | 105.2691 | 21.24920 | 42.40445 | 21.25114 | 21.25175 |
| 677 | 21.14663 | 42.19990 | 63.16035 | 84.02853 | 104.8050 | 21.15525 | 42.21709 | 21.15718 | 21.15778 |
| 678 | 21.05327 | 42.01372 | 62.88190 | 83.65836 | 104.3436 | 21.06185 | 42.03084 | 21.06377 | 21.06437 |
| 679 | 20.96045 | 41.82864 | 62.60509 | 83.29036 | 103.8850 | 20.96899 | 41.84568 | 20.97091 | 20.97151 |
| 680 | 20.86818 | 41.64464 | 62.32990 | 82.92451 | 103.4290 | 20.87669 | 41.66161 | 20.87859 | 20.87919 |
| 681 | 20.77646 | 41.46172 | 62.05633 | 82.56080 | 102.9757 | 20.78492 | 41.47861 | 20.78682 | 20.78741 |
| 682 | 20.68526 | 41.27987 | 61.78435 | 82.19922 | 102.5250 | 20.69369 | 41.29669 | 20.69558 | 20.69617 |
| 683 | 20.59461 | 41.09908 | 61.51395 | 81.83974 | 102.0770 | 20.60300 | 41.11583 | 20.60488 | 20.60547 |
| 684 | 20.50448 | 40.91935 | 61.24514 | 81.48236 | 101.6315 | 20.51283 | 40.93602 | 20.51471 | 20.51529 |
| 685 | 20.41487 | 40.74066 | 60.97788 | 81.12706 | 101.1887 | 20.42319 | 40.75726 | 20.42506 | 20.42564 |
| 686 | 20.32579 | 40.56301 | 60.71218 | 80.77381 | 100.7484 | 20.33407 | 40.57954 | 20.33593 | 20.33651 |
| 687 | 20.23722 | 40.38639 | 60.44802 | 80.42262 | 100.3107 | 20.24547 | 40.40285 | 20.24732 | 20.24790 |
| 688 | 20.14917 | 40.21080 | 60.18540 | 80.07346 | 99.87549 | 20.15738 | 40.22719 | 20.15922 | 20.15980 |
| 689 | 20.06163 | 40.03622 | 59.92429 | 79.72632 | 99.44281 | 20.06980 | 40.05254 | 20.07164 | 20.07221 |
| 690 | 19.97459 | 39.86266 | 59.66469 | 79.38118 | 99.01262 | 19.98273 | 39.87890 | 19.98456 | 19.98513 |
| 691 | 19.88806 | 39.69009 | 59.40658 | 79.03803 | 98.58491 | 19.89617 | 39.70626 | 19.89798 | 19.89855 |
| 692 | 19.80203 | 39.51852 | 59.14997 | 78.69685 | 98.15967 | 19.81010 | 39.53462 | 19.81191 | 19.81247 |
| 693 | 19.71649 | 39.34794 | 58.89482 | 78.35764 | 97.73686 | 19.72453 | 39.36397 | 19.72633 | 19.72689 |
| 694 | 19.63145 | 39.17833 | 58.64115 | 78.02037 | 97.31648 | 19.63944 | 39.19430 | 19.64124 | 19.64180 |
| 695 | 19.54689 | 39.00970 | 58.38893 | 77.68504 | 96.89851 | 19.55485 | 39.02560 | 19.55664 | 19.55720 |
| 696 | 19.46282 | 38.84204 | 58.13815 | 77.35162 | 96.48293 | 19.47075 | 38.85787 | 19.47252 | 19.47308 |
| 697 | 19.37922 | 38.67533 | 57.88881 | 77.02011 | 96.06972 | 19.38712 | 38.69109 | 19.38889 | 19.38945 |
| 698 | 19.29611 | 38.50958 | 57.64089 | 76.69049 | 95.65886 | 19.30397 | 38.52528 | 19.30574 | 19.30629 |
| 699 | 19.21347 | 38.34478 | 57.39438 | 76.36275 | 95.25035 | 19.22130 | 38.36040 | 19.22306 | 19.22361 |
| 700 | 19.13130 | 38.18091 | 57.14928 | 76.03688 | 94.84416 | 19.13910 | 38.19647 | 19.14085 | 19.14140 |
| 701 | 19.04961 | 38.01798 | 56.90557 | 75.71285 | 94.44027 | 19.05737 | 38.03347 | 19.05911 | 19.05965 |
| 702 | 18.96837 | 37.85597 | 56.66325 | 75.39067 | 94.03868 | 18.97610 | 37.87139 | 18.97783 | 18.97838 |
| 703 | 18.88760 | 37.69488 | 56.42230 | 75.07031 | 93.63936 | 18.89529 | 37.71024 | 18.89702 | 18.89756 |
| 704 | 18.80728 | 37.53470 | 56.18171 | 74.75176 | 93.24229 | 18.81494 | 37.55000 | 18.81666 | 18.81720 |
| 705 | 18.72742 | 37.37543 | 55.94448 | 74.43501 | 92.84747 | 18.73505 | 37.39066 | 18.73676 | 18.73730 |
| 706 | 18.64801 | 37.21706 | 55.70759 | 74.12005 | 92.45488 | 18.65561 | 37.23223 | 18.65731 | 18.65785 |
| 707 | 18.56905 | 37.05958 | 55.47204 | 73.80687 | 92.06449 | 18.57662 | 37.07468 | 18.57831 | 18.57884 |
| 708 | 18.49053 | 36.90299 | 55.23782 | 73.49544 | 91.67630 | 18.49807 | 36.91803 | 18.49976 | 18.50029 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 709 | 18.41246 | 36.74728 | 55.00491 | 73.18577 | 91.29029 | 18.41996 | 36.76226 | 18.42164 | 18.42217 |
| 710 | 18.33482 | 36.59245 | 54.77331 | 72.87783 | 90.90644 | 18.34230 | 36.60736 | 18.34397 | 18.34450 |
| 711 | 18.25763 | 36.43848 | 54.54301 | 72.57162 | 90.52474 | 18.26506 | 36.45333 | 18.26673 | 18.26726 |
| 712 | 18.18086 | 36.28538 | 54.31399 | 72.26712 | 90.14518 | 18.18827 | 36.30017 | 18.18993 | 18.19045 |
| 713 | 18.10452 | 36.13313 | 54.08626 | 71.96432 | 89.76773 | 18.11190 | 36.14786 | 18.11355 | 18.11407 |
| 714 | 18.02861 | 35.98174 | 53.85980 | 71.66321 | 89.39239 | 18.03596 | 35.99640 | 18.03761 | 18.03812 |
| 715 | 17.95313 | 35.83119 | 53.63460 | 71.36378 | 89.01914 | 17.96044 | 35.84579 | 17.96208 | 17.96260 |
| 716 | 17.87806 | 35.68147 | 53.41066 | 71.06602 | 88.64797 | 17.88535 | 35.69601 | 17.88698 | 17.88749 |
| 717 | 17.80341 | 35.53259 | 53.18796 | 70.76990 | 88.27885 | 17.81067 | 35.54707 | 17.81230 | 17.81280 |
| 718 | 17.72918 | 35.38454 | 52.96649 | 70.47544 | 87.91178 | 17.73641 | 35.39896 | 17.73803 | 17.73853 |
| 719 | 17.65536 | 35.23731 | 52.74626 | 70.18260 | 87.54674 | 17.66256 | 35.25167 | 17.66417 | 17.66467 |
| 720 | 17.58195 | 35.09089 | 52.52724 | 69.89138 | 87.18372 | 17.58911 | 35.10519 | 17.59072 | 17.59122 |
| 721 | 17.50895 | 34.94529 | 52.30943 | 69.60178 | 86.82271 | 17.51608 | 34.95953 | 17.51768 | 17.51818 |
| 722 | 17.43634 | 34.80049 | 52.09283 | 69.31377 | 86.46369 | 17.44345 | 34.81467 | 17.44504 | 17.44554 |
| 723 | 17.36414 | 34.65649 | 51.87742 | 69.02734 | 86.10664 | 17.37122 | 34.67061 | 17.37281 | 17.37330 |
| 724 | 17.29234 | 34.51328 | 51.66320 | 68.74250 | 85.75155 | 17.29939 | 34.52734 | 17.30097 | 17.30146 |
| 725 | 17.22094 | 34.37086 | 51.45015 | 68.45921 | 85.39842 | 17.22795 | 34.38486 | 17.22953 | 17.23002 |
| 726 | 17.14992 | 34.22922 | 51.23828 | 68.17749 | 85.04722 | 17.15691 | 34.24317 | 17.15848 | 17.15897 |
| 727 | 17.07930 | 34.08836 | 51.02756 | 67.89730 | 84.69795 | 17.08626 | 34.10225 | 17.08782 | 17.08831 |
| 728 | 17.00906 | 33.94827 | 50.81800 | 67.61865 | 84.35058 | 17.01599 | 33.96210 | 17.01755 | 17.01803 |
| 729 | 16.93921 | 33.80894 | 50.60959 | 67.34152 | 84.00511 | 16.94611 | 33.82272 | 16.94766 | 16.94814 |
| 730 | 16.86974 | 33.67038 | 50.40232 | 67.06591 | 83.66153 | 16.87661 | 33.68410 | 16.87815 | 16.87864 |
| 731 | 16.80065 | 33.53258 | 50.19617 | 66.79179 | 83.31982 | 16.80749 | 33.54624 | 16.80903 | 16.80951 |
| 732 | 16.73193 | 33.39552 | 49.99115 | 66.51917 | 82.97996 | 16.73875 | 33.40913 | 16.74028 | 16.74076 |
| 733 | 16.66359 | 33.25922 | 49.78724 | 66.24803 | 82.64195 | 16.67038 | 33.27277 | 16.67190 | 16.67238 |
| 734 | 16.59562 | 33.12365 | 49.58444 | 65.97836 | 82.30578 | 16.60239 | 33.13715 | 16.60390 | 16.60438 |
| 735 | 16.52802 | 32.98882 | 49.38274 | 65.71016 | 81.97142 | 16.53476 | 33.00226 | 16.53627 | 16.53674 |
| 736 | 16.46079 | 32.85471 | 49.18213 | 65.44340 | 81.63888 | 16.46750 | 32.86810 | 16.46900 | 16.46947 |
| 737 | 16.39392 | 32.72134 | 48.98261 | 65.17809 | 81.30813 | 16.40060 | 32.73467 | 16.40210 | 16.40257 |
| 738 | 16.32742 | 32.58869 | 48.78416 | 64.91421 | 80.97916 | 16.33407 | 32.60196 | 16.33556 | 16.33603 |
| 739 | 16.26127 | 32.45675 | 48.58679 | 64.65175 | 80.65197 | 16.26790 | 32.46997 | 16.26938 | 16.26985 |
| 740 | 16.19548 | 32.32552 | 48.39048 | 64.39070 | 80.32654 | 16.20208 | 32.33869 | 16.20356 | 16.20402 |
| 741 | 16.13004 | 32.19500 | 48.19522 | 64.13106 | 80.00285 | 16.13661 | 32.20812 | 16.13809 | 16.13855 |
| 742 | 16.06496 | 32.06518 | 48.00102 | 63.87281 | 79.68091 | 16.07150 | 32.07825 | 16.07297 | 16.07343 |
| 743 | 16.00022 | 31.93606 | 47.80785 | 63.61595 | 79.36068 | 16.00674 | 31.94907 | 16.00821 | 16.00866 |
| 744 | 15.93584 | 31.80763 | 47.61572 | 63.36046 | 79.04217 | 15.94233 | 31.82059 | 15.94379 | 15.94424 |
| 745 | 15.87179 | 31.67989 | 47.42462 | 63.10634 | 78.72537 | 15.87826 | 31.69280 | 15.87971 | 15.88017 |
| 746 | 15.80809 | 31.55283 | 47.23454 | 62.85357 | 78.41025 | 15.81454 | 31.56569 | 15.81598 | 15.81643 |
| 747 | 15.74474 | 31.42645 | 47.04548 | 62.60216 | 78.09682 | 15.75115 | 31.43926 | 15.75259 | 15.75304 |
| 748 | 15.68171 | 31.30074 | 46.85742 | 62.35208 | 77.78505 | 15.68810 | 31.31350 | 15.68954 | 15.68999 |
| 749 | 15.61903 | 31.17571 | 46.67037 | 62.10333 | 77.47494 | 15.62539 | 31.18841 | 15.62682 | 15.62727 |
| 750 | 15.55668 | 31.05134 | 46.48430 | 61.85591 | 77.16647 | 15.56302 | 31.06399 | 15.56444 | 15.56488 |
| 751 | 15.49466 | 30.92763 | 46.29923 | 61.60979 | 76.85964 | 15.50097 | 30.94023 | 15.50239 | 15.50283 |
| 752 | 15.43297 | 30.80457 | 46.11514 | 61.36499 | 76.55444 | 15.43926 | 30.81712 | 15.44067 | 15.44111 |
| 753 | 15.37160 | 30.68217 | 45.93202 | 61.12147 | 76.25085 | 15.37787 | 30.69467 | 15.37927 | 15.37971 |
| 754 | 15.31057 | 30.56041 | 45.74987 | 60.87924 | 75.94886 | 15.31680 | 30.57287 | 15.31820 | 15.31864 |
| 755 | 15.24985 | 30.43930 | 45.56868 | 60.63830 | 75.64847 | 15.25606 | 30.45171 | 15.25746 | 15.25789 |
| 756 | 15.18945 | 30.31883 | 45.38845 | 60.39862 | 75.34965 | 15.19564 | 30.33118 | 15.19703 | 15.19747 |
| 757 | 15.12938 | 30.19899 | 45.20916 | 60.16020 | 75.05241 | 15.13554 | 30.21130 | 15.13692 | 15.13736 |
| 758 | 15.06962 | 30.07979 | 45.03082 | 59.92303 | 74.75673 | 15.07576 | 30.09204 | 15.07713 | 15.07756 |
| 759 | 15.01017 | 29.96121 | 44.85342 | 59.68712 | 74.46260 | 15.01629 | 29.97341 | 15.01766 | 15.01809 |
| 760 | 14.95104 | 29.84325 | 44.67695 | 59.45243 | 74.17002 | 14.95713 | 29.85541 | 14.95849 | 14.95892 |
| 761 | 14.89221 | 29.72591 | 44.50140 | 59.21898 | 73.87896 | 14.89828 | 29.73802 | 14.89964 | 14.90007 |
| 762 | 14.83370 | 29.60918 | 44.32677 | 58.98675 | 73.58942 | 14.83974 | 29.62125 | 14.84110 | 14.84152 |
| 763 | 14.77549 | 29.49307 | 44.15305 | 58.75573 | 73.30140 | 14.78151 | 29.50509 | 14.78286 | 14.78328 |
| 764 | 14.71758 | 29.37756 | 43.98024 | 58.52591 | 73.01487 | 14.72358 | 29.38953 | 14.72492 | 14.72535 |
| 765 | 14.65998 | 29.26266 | 43.80833 | 58.29729 | 72.72984 | 14.66595 | 29.27458 | 14.66729 | 14.66771 |
| 766 | 14.60268 | 29.14835 | 43.63731 | 58.06986 | 72.44629 | 14.60863 | 29.16023 | 14.60996 | 14.61038 |
| 767 | 14.54567 | 29.03464 | 43.46719 | 57.84361 | 72.16421 | 14.55160 | 29.04647 | 14.55293 | 14.55334 |

(continued)

Table B.1. (continued)

| n | Line transition (frequencies in MHz) | | | | | | | | |
|-----|--------------------------------------|------------|-------------|-------------|---------------|--------------|-------------|-------------|-------------|
| | Hn α | Hn β | Hn γ | Hn δ | Hn ϵ | Hen α | Hen β | Cn α | Sn α |
| 768 | 14.48896 | 28.92151 | 43.29794 | 57.61854 | 71.88360 | 14.49487 | 28.93330 | 14.49619 | 14.49661 |
| 769 | 14.43255 | 28.80898 | 43.12958 | 57.39463 | 71.60443 | 14.43843 | 28.82072 | 14.43975 | 14.44016 |
| 770 | 14.37643 | 28.69703 | 42.96208 | 57.17188 | 71.32671 | 14.38229 | 28.70872 | 14.38360 | 14.38401 |
| 771 | 14.32060 | 28.58565 | 42.79545 | 56.95028 | 71.05043 | 14.32643 | 28.59730 | 14.32774 | 14.32815 |
| 772 | 14.26506 | 28.47486 | 42.62969 | 56.72983 | 70.77557 | 14.27087 | 28.48646 | 14.27217 | 14.27258 |
| 773 | 14.20980 | 28.36463 | 42.46477 | 56.51051 | 70.50212 | 14.21559 | 28.37619 | 14.21689 | 14.21730 |
| 774 | 14.15483 | 28.25497 | 42.30071 | 56.29232 | 70.23009 | 14.16060 | 28.26649 | 14.16189 | 14.16230 |
| 775 | 14.10014 | 28.14588 | 42.13749 | 56.07526 | 69.95945 | 14.10589 | 28.15735 | 14.10718 | 14.10758 |
| 776 | 14.04574 | 28.03735 | 41.97511 | 55.85930 | 69.69020 | 14.05146 | 28.04877 | 14.05274 | 14.05315 |
| 777 | 13.99161 | 27.92937 | 41.81357 | 55.64446 | 69.42232 | 13.99731 | 27.94076 | 13.99859 | 13.99899 |
| 778 | 13.93776 | 27.82195 | 41.65285 | 55.43071 | 69.15582 | 13.94344 | 27.83329 | 13.94472 | 13.94511 |
| 779 | 13.88419 | 27.71508 | 41.49295 | 55.21806 | 68.89069 | 13.88985 | 27.72638 | 13.89112 | 13.89151 |
| 780 | 13.83089 | 27.60876 | 41.33387 | 55.00650 | 68.62690 | 13.83653 | 27.62001 | 13.83779 | 13.83819 |
| 781 | 13.77787 | 27.50298 | 41.17560 | 54.79601 | 68.36446 | 13.78348 | 27.51419 | 13.78474 | 13.78513 |
| 782 | 13.72511 | 27.39774 | 41.01814 | 54.58660 | 68.10336 | 13.73070 | 27.40890 | 13.73196 | 13.73235 |
| 783 | 13.67263 | 27.29303 | 40.86149 | 54.37825 | 67.84359 | 13.67820 | 27.30415 | 13.67945 | 13.67984 |
| 784 | 13.62041 | 27.18886 | 40.70562 | 54.17096 | 67.58513 | 13.62596 | 27.19994 | 13.62720 | 13.62759 |
| 785 | 13.56845 | 27.08522 | 40.55056 | 53.96473 | 67.32799 | 13.57398 | 27.09625 | 13.57522 | 13.57561 |
| 786 | 13.51676 | 26.98210 | 40.39627 | 53.75954 | 67.07215 | 13.52227 | 26.99310 | 13.52351 | 13.52389 |
| 787 | 13.46534 | 26.87951 | 40.24277 | 53.55538 | 66.81760 | 13.47082 | 26.89046 | 13.47205 | 13.47244 |
| 788 | 13.41417 | 26.77743 | 40.09005 | 53.35226 | 66.56434 | 13.41964 | 26.78835 | 13.42086 | 13.42125 |
| 789 | 13.36326 | 26.67588 | 39.93809 | 53.15017 | 66.31236 | 13.36871 | 26.68675 | 13.36993 | 13.37031 |
| 790 | 13.31261 | 26.57483 | 39.78691 | 52.94910 | 66.06165 | 13.31804 | 26.58566 | 13.31925 | 13.31963 |
| 791 | 13.26222 | 26.47430 | 39.63648 | 52.74904 | 65.81220 | 13.26762 | 26.48508 | 13.26883 | 13.26921 |
| 792 | 13.21208 | 26.37427 | 39.48682 | 52.54998 | 65.56400 | 13.21746 | 26.38501 | 13.21867 | 13.21905 |
| 793 | 13.16219 | 26.27474 | 39.33790 | 52.35193 | 65.31706 | 13.16755 | 26.28545 | 13.16876 | 13.16913 |
| 794 | 13.11255 | 26.17572 | 39.18974 | 52.15487 | 65.07135 | 13.11789 | 26.18638 | 13.11909 | 13.11947 |
| 795 | 13.06316 | 26.07719 | 39.04232 | 51.95879 | 64.82687 | 13.06849 | 26.08781 | 13.06968 | 13.07005 |
| 796 | 13.01402 | 25.97915 | 38.89563 | 51.76370 | 64.58361 | 13.01933 | 25.98974 | 13.02052 | 13.02089 |
| 797 | 12.96513 | 25.88161 | 38.74968 | 51.56959 | 64.34157 | 12.97041 | 25.89215 | 12.97160 | 12.97197 |
| 798 | 12.91648 | 25.78455 | 38.60446 | 51.37644 | 64.10074 | 12.92174 | 25.79506 | 12.92292 | 12.92329 |
| 799 | 12.86807 | 25.68798 | 38.45996 | 51.18426 | 63.86111 | 12.87332 | 25.69845 | 12.87449 | 12.87486 |
| 800 | 12.81991 | 25.59189 | 38.31619 | 50.99303 | 63.62267 | 12.82513 | 25.60232 | 12.82630 | 12.82667 |

B.2 Frequencies Above 100 GHz

Although negligible at 100 GHz, the shifts in rest frequencies due to blending of the fine-structure lines become increasingly significant at higher frequencies. To explore these differences, Towle et al. (1996) have calculated rest frequencies of recombination lines from both the classical Rydberg equation (A.5) and the Dirac equation that describes the fine-structure lines. Their calculations include frequencies from 100 GHz to 29 THz (about $\lambda = 10 \mu\text{m}$) and estimate relative strengths for all of these lines.

Unlike the Rydberg equation, Dirac theory gives the energy E_{nj} of a discrete state of a hydrogenic atom as

$$E_{nj} = \frac{m_R c^2}{(1 + X_{nj}^2)} - \frac{mZ^4 \alpha^2 \hbar c R_\infty}{4Mn^4}, \quad (\text{B.1})$$

where

$$X_{nj} = \frac{Z\alpha}{n - k + (k^2 - Z^2\alpha^2)^{1/2}}.$$

(B.2)

The rightmost term of (B.1) is a correction for the energy shift (not splitting) of a principal quantum level n due to nuclear motion, described by (42.2), (42.5), and (42.7) of Bethe and Salpeter (1957). Here, n is the principal quantum number, the angular momentum quantum number $j = 1/2, 3/2, \dots, n - 1/2$, $k = j + 1/2$, and α is the fine-structure constant given in Table A.1. The parameter m_R is the reduced mass given by (A.2). The energy of each line component results from $E_{n'j'} - E_{nj} = h\nu_{nj \rightarrow n'j'}$. The line strengths are given as $g'A$ in units of 10^4 s^{-1} , where g' is the statistical weight of the upper quantum level and A is the spontaneous emission rate for that transition.

These calculations do not include relativistic effects arising from the high velocity of the electron around the nucleus, because they lead to frequency shifts that are much smaller than the fine-structure shifts and, hence, too small to be measured in the spectra of astronomical objects.

Table B.2 reproduces the entries of Table 1 of Towle et al. (1996). The column headings are frequency, intensity in the units described above, lower principal quantum number n , and change in principal quantum number Δn . The frequencies are calculated from the Rydberg equation (A.5) for H, ^4He , ^{12}C , and ^{32}S . For the carbon and sulfur, the quoted line intensities assume hydrogenic atoms.

Table B.2. Rydberg H, ^4He , ^{12}C , and ^{32}S RRL rest frequencies for 100 GHz to 29 THz

| ν/MHz | Int. | n | Δn | | ν/MHz | Int. | n | Δn | |
|------------------|------|-----|------------|----|------------------|------|-----|------------|----|
| 100,527.40 | 0.2 | 82 | 10 | H | 122,050.75 | 1.7 | 58 | 4 | H |
| 100,539.63 | 1.4 | 62 | 4 | H | 122,055.83 | 2.9 | 53 | 3 | H |
| 100,580.60 | 1.4 | 62 | 4 | He | 122,100.48 | 1.7 | 58 | 4 | He |
| 101,769.92 | 0.6 | 70 | 6 | H | 122,105.57 | 2.9 | 53 | 3 | He |
| 102,253.46 | 0.3 | 79 | 9 | H | 122,903.80 | 1.1 | 62 | 5 | H |
| 102,571.00 | 0.9 | 66 | 5 | H | 122,953.88 | 1.1 | 62 | 5 | He |
| 102,612.79 | 0.9 | 66 | 5 | He | 123,157.57 | 0.3 | 74 | 9 | H |
| 103,252.92 | 0.5 | 73 | 7 | H | 124,689.39 | 0.3 | 76 | 10 | H |
| 103,267.29 | 0.4 | 76 | 8 | H | 124,746.74 | 23.2 | 37 | 1 | H |
| 103,914.85 | 2.5 | 56 | 3 | H | 124,797.57 | 23.2 | 37 | 1 | He |
| 103,957.19 | 2.5 | 56 | 3 | He | 124,808.97 | 23.2 | 37 | 1 | C |
| 104,091.24 | 0.2 | 81 | 10 | H | 124,812.54 | 23.2 | 37 | 1 | S |
| 105,301.86 | 5.4 | 49 | 2 | H | 125,414.84 | 0.4 | 71 | 8 | H |
| 105,344.77 | 5.4 | 49 | 2 | He | 125,975.28 | 0.8 | 65 | 6 | H |
| 105,410.22 | 1.4 | 61 | 4 | H | 126,541.30 | 0.6 | 68 | 7 | H |
| 105,453.18 | 1.4 | 61 | 4 | He | 126,793.88 | 6.5 | 46 | 2 | H |
| 106,032.09 | 0.3 | 78 | 9 | H | 126,845.54 | 6.5 | 46 | 2 | He |
| 106,079.54 | 0.7 | 69 | 6 | H | 128,008.66 | 0.4 | 73 | 9 | H |
| 106,737.36 | 19.9 | 39 | 1 | H | 128,372.26 | 1.7 | 57 | 4 | H |
| 106,780.86 | 19.9 | 39 | 1 | He | 128,424.57 | 1.7 | 57 | 4 | He |
| 106,790.61 | 19.9 | 39 | 1 | C | 128,814.50 | 1.1 | 61 | 5 | H |
| 106,793.66 | 19.9 | 39 | 1 | S | 128,866.99 | 1.1 | 61 | 5 | He |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|------------|------|-----|------------|----|------------|------|-----|------------|----|
| 107,206.11 | 0.9 | 65 | 5 | H | 129,036.19 | 3.1 | 52 | 3 | H |
| 107,249.80 | 0.9 | 65 | 5 | He | 129,088.77 | 3.1 | 52 | 3 | He |
| 107,252.38 | 0.4 | 75 | 8 | H | 129,448.85 | 0.3 | 75 | 10 | H |
| 107,422.29 | 0.5 | 72 | 7 | H | 130,588.52 | 0.5 | 70 | 8 | H |
| 107,825.75 | 0.2 | 80 | 10 | H | 131,716.02 | 0.8 | 64 | 6 | H |
| 109,536.01 | 2.6 | 55 | 3 | H | 132,020.90 | 0.6 | 67 | 7 | H |
| 109,580.64 | 2.6 | 55 | 3 | He | 133,118.21 | 0.4 | 72 | 9 | H |
| 109,999.40 | 0.3 | 77 | 9 | H | 134,453.98 | 0.3 | 74 | 10 | H |
| 110,600.68 | 1.5 | 60 | 4 | H | 135,110.59 | 1.2 | 60 | 5 | H |
| 110,636.11 | 0.7 | 68 | 6 | H | 135,138.16 | 1.8 | 56 | 4 | H |
| 110,645.75 | 1.5 | 60 | 4 | He | 135,165.64 | 1.2 | 60 | 5 | He |
| 111,445.37 | 0.4 | 74 | 8 | H | 135,193.23 | 1.8 | 56 | 4 | He |
| 111,741.29 | 0.3 | 79 | 10 | H | 135,249.50 | 7.0 | 45 | 2 | H |
| 111,819.35 | 0.5 | 71 | 7 | H | 135,286.04 | 25.2 | 36 | 1 | H |
| 111,885.08 | 5.8 | 48 | 2 | H | 135,304.61 | 7.0 | 45 | 2 | He |
| 111,930.67 | 5.8 | 48 | 2 | He | 135,341.17 | 25.2 | 36 | 1 | He |
| 112,124.91 | 1.0 | 64 | 5 | H | 135,353.53 | 25.2 | 36 | 1 | C |
| 112,170.60 | 1.0 | 64 | 5 | He | 135,357.40 | 25.2 | 36 | 1 | S |
| 114,167.35 | 0.3 | 76 | 9 | H | 136,051.02 | 0.5 | 69 | 8 | H |
| 115,274.41 | 21.5 | 38 | 1 | H | 136,559.29 | 3.2 | 51 | 3 | H |
| 115,321.38 | 21.5 | 38 | 1 | He | 136,614.94 | 3.2 | 51 | 3 | He |
| 115,331.91 | 21.5 | 38 | 1 | C | 137,811.19 | 0.9 | 63 | 6 | H |
| 115,335.21 | 21.5 | 38 | 1 | S | 137,821.77 | 0.6 | 66 | 7 | H |
| 115,457.60 | 0.7 | 67 | 6 | H | 138,503.70 | 0.4 | 71 | 9 | H |
| 115,570.12 | 2.7 | 54 | 3 | H | 139,720.87 | 0.3 | 73 | 10 | H |
| 115,617.22 | 2.7 | 54 | 3 | He | 141,822.80 | 0.5 | 68 | 8 | H |
| 115,848.95 | 0.3 | 78 | 10 | H | 141,824.00 | 1.3 | 59 | 5 | H |
| 115,860.00 | 0.4 | 73 | 8 | H | 141,881.79 | 1.3 | 59 | 5 | He |
| 116,137.71 | 1.6 | 59 | 4 | H | 142,388.20 | 1.9 | 55 | 4 | H |
| 116,185.04 | 1.6 | 59 | 4 | He | 142,446.23 | 1.9 | 55 | 4 | He |
| 116,459.90 | 0.5 | 70 | 7 | H | 143,967.84 | 0.7 | 65 | 7 | H |
| 117,349.42 | 1.0 | 63 | 5 | H | 144,184.01 | 0.4 | 70 | 9 | H |
| 117,397.24 | 1.0 | 63 | 5 | He | 144,288.58 | 0.9 | 62 | 6 | H |
| 118,548.79 | 0.3 | 75 | 9 | H | 144,474.12 | 7.4 | 44 | 2 | H |
| 119,028.76 | 6.1 | 47 | 2 | H | 144,532.99 | 7.4 | 44 | 2 | He |
| 119,077.27 | 6.1 | 47 | 2 | He | 144,678.94 | 3.4 | 50 | 3 | H |
| 120,160.71 | 0.3 | 77 | 10 | H | 144,737.89 | 3.4 | 50 | 3 | He |
| 120,511.14 | 0.4 | 72 | 8 | H | 145,266.88 | 0.3 | 72 | 10 | H |
| 120,563.56 | 0.7 | 66 | 6 | H | 147,046.89 | 27.4 | 35 | 1 | H |
| 121,361.03 | 0.6 | 69 | 7 | H | 147,106.81 | 27.4 | 35 | 1 | He |
| | | | | | | | | | |
| 147,120.24 | 27.4 | 35 | 1 | C | 183,018.71 | 1.6 | 54 | 5 | H |
| 147,124.45 | 27.4 | 35 | 1 | S | 183,093.28 | 1.6 | 54 | 5 | He |
| 147,926.09 | 0.5 | 67 | 8 | H | 183,586.30 | 1.1 | 57 | 6 | H |
| 148,989.91 | 1.3 | 58 | 5 | H | 184,341.46 | 0.6 | 62 | 8 | H |
| 149,050.62 | 1.3 | 58 | 5 | He | 184,448.70 | 4.4 | 46 | 3 | H |
| 150,166.51 | 2.0 | 54 | 4 | H | 184,523.87 | 4.4 | 46 | 3 | He |
| 150,179.63 | 0.4 | 69 | 9 | H | 185,571.75 | 0.4 | 66 | 10 | H |
| 150,227.70 | 2.0 | 54 | 4 | He | 185,735.95 | 0.5 | 64 | 9 | H |
| 150,485.18 | 0.7 | 64 | 7 | H | 187,630.91 | 2.6 | 50 | 4 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|------------|------|-----|------------|----|------------|------|-----|------------|----|
| 151,110.77 | 0.3 | 71 | 10 | H | 187,707.36 | 2.6 | 50 | 4 | He |
| 151,178.67 | 0.9 | 61 | 6 | H | 189,738.18 | 0.9 | 59 | 7 | H |
| 153,455.46 | 3.6 | 49 | 3 | H | 191,057.40 | 9.8 | 40 | 2 | H |
| 153,517.99 | 3.6 | 49 | 3 | He | 191,135.25 | 9.8 | 40 | 2 | He |
| 154,385.07 | 0.5 | 66 | 8 | H | 191,656.74 | 35.7 | 32 | 1 | H |
| 154,557.22 | 7.9 | 43 | 2 | H | 191,734.84 | 35.7 | 32 | 1 | He |
| 154,620.20 | 7.9 | 43 | 2 | He | 191,752.35 | 35.7 | 32 | 1 | C |
| 156,512.78 | 0.4 | 68 | 9 | H | 191,757.83 | 35.7 | 32 | 1 | S |
| 156,647.13 | 1.4 | 57 | 5 | H | 193,025.22 | 0.7 | 61 | 8 | H |
| 156,710.96 | 1.4 | 57 | 5 | He | 193,113.39 | 1.2 | 56 | 6 | H |
| 157,272.86 | 0.4 | 70 | 10 | H | 193,118.48 | 1.7 | 53 | 5 | H |
| 157,402.30 | 0.7 | 63 | 7 | H | 193,197.17 | 1.7 | 53 | 5 | He |
| 158,514.87 | 1.0 | 60 | 6 | H | 193,694.54 | 0.4 | 65 | 10 | H |
| 158,522.09 | 2.2 | 53 | 4 | H | 194,164.02 | 0.5 | 63 | 9 | H |
| 158,586.69 | 2.2 | 53 | 4 | He | 196,623.43 | 4.7 | 45 | 3 | H |
| 160,211.52 | 29.9 | 34 | 1 | H | 196,703.55 | 4.7 | 45 | 3 | He |
| 160,276.80 | 29.9 | 34 | 1 | He | 198,909.18 | 2.7 | 49 | 4 | H |
| 160,291.45 | 29.9 | 34 | 1 | C | 198,990.24 | 2.7 | 49 | 4 | He |
| 160,296.02 | 29.9 | 34 | 1 | S | 199,186.10 | 0.9 | 58 | 7 | H |
| 161,226.05 | 0.6 | 65 | 8 | H | 202,263.82 | 0.7 | 60 | 8 | H |
| 162,956.69 | 3.9 | 48 | 3 | H | 202,299.25 | 0.5 | 64 | 10 | H |
| 163,023.09 | 3.9 | 48 | 3 | He | 203,110.63 | 0.6 | 62 | 9 | H |
| 163,207.58 | 0.5 | 67 | 9 | H | 203,311.88 | 1.3 | 55 | 6 | H |
| 163,775.12 | 0.4 | 69 | 10 | H | 203,975.82 | 1.8 | 52 | 5 | H |
| 164,750.35 | 0.8 | 62 | 7 | H | 204,058.93 | 1.8 | 52 | 5 | He |
| 164,838.52 | 1.5 | 56 | 5 | H | 205,760.32 | 10.6 | 39 | 2 | H |
| 164,905.69 | 1.5 | 56 | 5 | He | 205,844.17 | 10.6 | 39 | 2 | He |
| 165,601.05 | 8.5 | 42 | 2 | H | 209,272.58 | 1.0 | 57 | 7 | H |
| 165,668.53 | 8.5 | 42 | 2 | He | 209,894.06 | 5.0 | 44 | 3 | H |
| 166,333.90 | 1.0 | 59 | 6 | H | 209,979.59 | 5.0 | 44 | 3 | He |
| 167,509.55 | 2.3 | 52 | 4 | H | 210,501.78 | 39.2 | 31 | 1 | H |
| 167,577.81 | 2.3 | 52 | 4 | He | 210,587.56 | 39.2 | 31 | 1 | He |
| 168,477.74 | 0.6 | 64 | 8 | H | 210,606.80 | 39.2 | 31 | 1 | C |
| 170,290.26 | 0.5 | 66 | 9 | H | 210,612.81 | 39.2 | 31 | 1 | S |
| 170,641.40 | 0.4 | 68 | 10 | H | 211,110.29 | 2.9 | 48 | 4 | H |
| 172,563.46 | 0.8 | 61 | 7 | H | 211,196.32 | 2.9 | 48 | 4 | He |
| 173,259.01 | 4.1 | 47 | 3 | H | 211,422.24 | 0.5 | 63 | 10 | H |
| 173,329.61 | 4.1 | 47 | 3 | He | 212,102.35 | 0.7 | 59 | 8 | H |
| 173,611.52 | 1.5 | 55 | 5 | H | 212,616.34 | 0.6 | 61 | 9 | H |
| 173,682.26 | 1.5 | 55 | 5 | He | 214,242.02 | 1.3 | 54 | 6 | H |
| 174,676.19 | 1.1 | 58 | 6 | H | 215,663.15 | 1.9 | 51 | 5 | H |
| 174,995.82 | 32.6 | 33 | 1 | H | 215,751.03 | 1.9 | 51 | 5 | He |
| 175,067.12 | 32.6 | 33 | 1 | He | 220,052.56 | 1.0 | 56 | 7 | H |
| 175,083.12 | 32.6 | 33 | 1 | C | 221,103.19 | 0.5 | 62 | 10 | H |
| 175,088.12 | 32.6 | 33 | 1 | S | 222,011.77 | 11.4 | 38 | 2 | H |
| 176,171.46 | 0.6 | 63 | 8 | H | 222,102.24 | 11.4 | 38 | 2 | He |
| 177,189.79 | 2.4 | 51 | 4 | H | 222,590.38 | 0.8 | 58 | 8 | H |
| 177,261.99 | 2.4 | 51 | 4 | He | 222,725.59 | 0.6 | 60 | 9 | H |
| 177,722.83 | 9.1 | 41 | 2 | H | 224,330.63 | 3.1 | 47 | 4 | H |
| 177,789.35 | 0.5 | 65 | 9 | H | 224,386.78 | 5.3 | 43 | 3 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|------------|------|-----|------------|----|------------|------|-----|------------|----|
| 177,795.25 | 9.1 | 41 | 2 | He | 224,422.04 | 3.1 | 47 | 4 | He |
| 177,897.57 | 0.4 | 67 | 10 | H | 224,478.21 | 5.3 | 43 | 3 | He |
| 180,879.03 | 0.8 | 60 | 7 | H | 225,970.68 | 1.4 | 53 | 6 | H |
| 228,261.41 | 2.0 | 50 | 5 | H | 289,750.57 | 2.6 | 46 | 5 | H |
| 228,354.42 | 2.0 | 50 | 5 | He | 289,806.72 | 3.9 | 43 | 4 | H |
| 231,385.50 | 0.5 | 61 | 10 | H | 289,868.64 | 2.6 | 46 | 5 | He |
| 231,586.76 | 1.1 | 55 | 7 | H | 289,924.81 | 3.9 | 43 | 4 | He |
| 231,900.94 | 43.2 | 30 | 1 | H | 293,653.03 | 0.7 | 56 | 10 | H |
| 231,995.44 | 43.2 | 30 | 1 | He | 297,794.76 | 7.1 | 39 | 3 | H |
| 232,016.63 | 43.2 | 30 | 1 | C | 297,916.11 | 7.1 | 39 | 3 | He |
| 232,023.26 | 43.2 | 30 | 1 | S | 299,156.42 | 0.8 | 54 | 9 | H |
| 233,487.14 | 0.7 | 59 | 9 | H | 299,515.98 | 1.9 | 48 | 6 | H |
| 233,782.48 | 0.8 | 57 | 8 | H | 302,647.71 | 1.1 | 52 | 8 | H |
| 238,572.20 | 1.5 | 52 | 6 | H | 303,201.03 | 1.4 | 50 | 7 | H |
| 238,678.95 | 3.3 | 46 | 4 | H | 307,258.41 | 15.8 | 34 | 2 | H |
| 238,776.21 | 3.3 | 46 | 4 | He | 307,383.61 | 15.8 | 34 | 2 | He |
| 240,021.14 | 12.3 | 37 | 2 | H | 308,508.51 | 2.7 | 45 | 5 | H |
| 240,118.95 | 12.3 | 37 | 2 | He | 308,634.22 | 2.7 | 45 | 5 | He |
| 240,245.62 | 5.7 | 42 | 3 | H | 308,722.11 | 0.7 | 55 | 10 | H |
| 240,343.51 | 5.7 | 42 | 3 | He | 310,075.17 | 4.2 | 42 | 4 | H |
| 241,861.16 | 2.1 | 49 | 5 | H | 310,201.52 | 4.2 | 42 | 4 | He |
| 241,959.71 | 2.1 | 49 | 5 | He | 315,169.23 | 0.9 | 53 | 9 | H |
| 242,316.70 | 0.5 | 60 | 10 | H | 316,415.44 | 59.0 | 27 | 1 | H |
| 243,942.39 | 1.1 | 54 | 7 | H | 316,544.38 | 59.0 | 27 | 1 | He |
| 244,954.55 | 0.7 | 58 | 9 | H | 316,573.30 | 59.0 | 27 | 1 | C |
| 245,738.84 | 0.9 | 56 | 8 | H | 316,582.34 | 59.0 | 27 | 1 | S |
| 252,129.42 | 1.6 | 51 | 6 | H | 317,937.95 | 2.0 | 47 | 6 | H |
| 253,948.90 | 0.6 | 59 | 10 | H | 319,578.00 | 1.1 | 51 | 8 | H |
| 254,278.26 | 3.5 | 45 | 4 | H | 320,965.02 | 1.5 | 49 | 7 | H |
| 254,381.87 | 3.5 | 45 | 4 | He | 321,034.73 | 7.6 | 38 | 3 | H |
| 256,302.05 | 47.8 | 29 | 1 | H | 321,165.54 | 7.6 | 38 | 3 | He |
| 256,406.49 | 47.8 | 29 | 1 | He | 324,842.70 | 0.7 | 54 | 10 | H |
| 256,429.91 | 47.8 | 29 | 1 | C | 328,922.83 | 2.9 | 44 | 5 | H |
| 256,437.24 | 47.8 | 29 | 1 | S | 329,056.86 | 2.9 | 44 | 5 | He |
| 256,564.01 | 2.3 | 48 | 5 | H | 332,280.06 | 4.5 | 41 | 4 | H |
| 256,668.56 | 2.3 | 48 | 5 | He | 332,348.08 | 0.9 | 52 | 9 | H |
| 257,186.76 | 0.7 | 57 | 9 | H | 332,415.46 | 4.5 | 41 | 4 | He |
| 257,193.99 | 1.2 | 53 | 7 | H | 335,207.34 | 17.2 | 33 | 2 | H |
| 257,635.49 | 6.1 | 41 | 3 | H | 335,343.93 | 17.2 | 33 | 2 | He |
| 257,740.47 | 6.1 | 41 | 3 | He | 337,797.41 | 1.2 | 50 | 8 | H |
| 258,525.92 | 0.9 | 55 | 8 | H | 337,904.17 | 2.1 | 46 | 6 | H |
| 260,032.78 | 13.4 | 36 | 2 | H | 340,146.48 | 1.6 | 48 | 7 | H |
| 260,138.74 | 13.4 | 36 | 2 | He | 342,108.39 | 0.8 | 53 | 10 | H |
| 266,339.33 | 0.6 | 58 | 10 | H | 346,758.51 | 8.2 | 37 | 3 | H |
| 266,734.77 | 1.6 | 50 | 6 | H | 346,899.81 | 8.2 | 37 | 3 | He |
| 270,248.75 | 0.8 | 56 | 9 | H | 350,801.31 | 1.0 | 51 | 9 | H |
| 271,268.00 | 3.7 | 44 | 4 | H | 351,180.66 | 3.1 | 43 | 5 | H |
| 271,378.54 | 3.7 | 44 | 4 | He | 351,323.76 | 3.1 | 43 | 5 | He |
| 271,424.40 | 1.2 | 52 | 7 | H | 353,622.77 | 65.9 | 26 | 1 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|------------|------|-----|------------|----|------------|-------|-----|------------|----|
| 272,217.26 | 1.0 | 54 | 8 | H | 353,766.86 | 65.9 | 26 | 1 | He |
| 272,484.23 | 2.4 | 47 | 5 | H | 353,799.18 | 65.9 | 26 | 1 | C |
| 272,595.26 | 2.4 | 47 | 5 | He | 353,809.29 | 65.9 | 26 | 1 | S |
| 276,745.79 | 6.6 | 40 | 3 | H | 356,658.45 | 4.9 | 40 | 4 | H |
| 276,858.56 | 6.6 | 40 | 3 | He | 356,803.78 | 4.9 | 40 | 4 | He |
| 279,550.93 | 0.6 | 57 | 10 | H | 357,431.28 | 1.3 | 49 | 8 | H |
| 282,332.93 | 14.5 | 35 | 2 | H | 359,580.12 | 2.2 | 45 | 6 | H |
| 282,447.97 | 14.5 | 35 | 2 | He | 360,622.95 | 0.8 | 52 | 10 | H |
| 282,491.66 | 1.7 | 49 | 6 | H | 360,889.92 | 1.6 | 47 | 7 | H |
| 284,212.20 | 0.8 | 55 | 9 | H | 366,652.55 | 18.8 | 32 | 2 | H |
| 284,250.59 | 53.0 | 28 | 1 | H | 366,801.96 | 18.8 | 32 | 2 | He |
| 284,366.42 | 53.0 | 28 | 1 | He | 370,649.61 | 1.0 | 50 | 9 | H |
| 284,392.39 | 53.0 | 28 | 1 | C | 375,307.18 | 8.9 | 36 | 3 | H |
| 284,400.52 | 53.0 | 28 | 1 | S | 375,460.12 | 8.9 | 36 | 3 | He |
| 286,725.80 | 1.3 | 51 | 7 | H | 375,495.11 | 3.3 | 42 | 5 | H |
| 286,894.35 | 1.0 | 53 | 8 | H | 375,648.12 | 3.3 | 42 | 5 | He |
| | | | | | | | | | |
| 378,619.84 | 1.3 | 48 | 8 | H | 507,175.51 | 94.5 | 23 | 1 | H |
| 380,501.68 | 0.9 | 51 | 10 | H | 507,382.18 | 94.5 | 23 | 1 | He |
| 383,153.08 | 2.4 | 44 | 6 | H | 507,428.53 | 94.5 | 23 | 1 | C |
| 383,357.89 | 1.8 | 46 | 7 | H | 507,443.02 | 94.5 | 23 | 1 | S |
| 383,483.16 | 5.2 | 39 | 4 | H | 514,137.35 | 1.8 | 43 | 8 | H |
| 383,639.42 | 5.2 | 39 | 4 | He | 522,354.07 | 7.1 | 35 | 4 | H |
| 392,027.66 | 1.1 | 49 | 9 | H | 522,566.92 | 7.1 | 35 | 4 | He |
| 396,900.86 | 73.9 | 25 | 1 | H | 526,864.07 | 12.5 | 32 | 3 | H |
| 397,062.59 | 73.9 | 25 | 1 | He | 527,078.77 | 12.5 | 32 | 3 | He |
| 397,098.86 | 73.9 | 25 | 1 | C | 527,832.01 | 1.5 | 44 | 9 | H |
| 397,110.20 | 73.9 | 25 | 1 | S | 528,903.49 | 2.4 | 41 | 7 | H |
| 401,520.42 | 1.4 | 47 | 8 | H | 536,769.92 | 1.2 | 45 | 10 | H |
| 401,872.93 | 0.9 | 50 | 10 | H | 537,815.90 | 4.7 | 37 | 5 | H |
| 402,109.61 | 3.5 | 41 | 5 | H | 538,035.06 | 4.7 | 37 | 5 | He |
| 402,158.52 | 20.7 | 31 | 2 | H | 538,040.38 | 3.3 | 39 | 6 | H |
| 402,273.47 | 3.5 | 41 | 5 | He | 540,552.64 | 27.8 | 28 | 2 | H |
| 402,322.40 | 20.7 | 31 | 2 | He | 540,772.91 | 27.8 | 28 | 2 | He |
| 407,079.66 | 9.7 | 35 | 3 | H | 548,754.13 | 1.9 | 42 | 8 | H |
| 407,245.54 | 9.7 | 35 | 3 | He | 562,290.95 | 1.6 | 43 | 9 | H |
| 407,733.72 | 1.9 | 45 | 7 | H | 566,552.51 | 2.6 | 40 | 7 | H |
| 408,835.48 | 2.5 | 43 | 6 | H | 567,291.18 | 7.7 | 34 | 4 | H |
| 413,069.17 | 5.6 | 38 | 4 | H | 567,522.35 | 7.7 | 34 | 4 | He |
| 413,237.49 | 5.6 | 38 | 4 | He | 570,783.98 | 1.3 | 44 | 10 | H |
| 415,086.11 | 1.2 | 48 | 9 | H | 577,154.34 | 13.7 | 31 | 3 | H |
| 424,879.86 | 1.0 | 49 | 10 | H | 577,389.52 | 13.7 | 31 | 3 | He |
| 426,309.86 | 1.5 | 46 | 8 | H | 577,896.49 | 107.6 | 22 | 1 | H |
| 431,303.02 | 3.8 | 40 | 5 | H | 578,131.97 | 107.6 | 22 | 1 | He |
| 431,478.77 | 3.8 | 40 | 5 | He | 578,184.79 | 107.6 | 22 | 1 | C |
| 434,224.69 | 2.0 | 44 | 7 | H | 578,201.30 | 107.6 | 22 | 1 | S |
| 436,869.05 | 2.7 | 42 | 6 | H | 578,670.22 | 3.6 | 38 | 6 | H |
| 439,993.78 | 1.2 | 47 | 9 | H | 581,067.51 | 5.1 | 36 | 5 | H |
| 442,402.72 | 22.7 | 30 | 2 | H | 581,304.28 | 5.1 | 36 | 5 | He |
| 442,544.45 | 10.5 | 34 | 3 | H | 586,558.32 | 2.1 | 41 | 8 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|------------|-------|-----|------------|----|--------------|-------|-----|------------|----|
| 442,583.00 | 22.7 | 30 | 2 | He | 599,825.74 | 1.7 | 42 | 9 | H |
| 442,724.78 | 10.5 | 34 | 3 | He | 600,666.03 | 30.9 | 27 | 2 | H |
| 445,781.47 | 6.1 | 37 | 4 | H | 600,910.80 | 30.9 | 27 | 2 | He |
| 445,963.11 | 6.1 | 37 | 4 | He | 607,744.67 | 1.4 | 43 | 10 | H |
| 447,540.30 | 83.4 | 24 | 1 | H | 607,869.93 | 2.8 | 39 | 7 | H |
| 447,722.67 | 83.4 | 24 | 1 | He | 617,540.26 | 8.4 | 33 | 4 | H |
| 447,763.57 | 83.4 | 24 | 1 | C | 617,791.90 | 8.4 | 33 | 4 | He |
| 447,776.36 | 83.4 | 24 | 1 | S | 623,504.30 | 3.8 | 37 | 6 | H |
| 449,682.49 | 1.0 | 48 | 10 | H | 627,926.45 | 2.2 | 40 | 8 | H |
| 453,187.45 | 1.6 | 45 | 8 | H | 629,091.43 | 5.5 | 35 | 5 | H |
| 463,065.73 | 2.1 | 43 | 7 | H | 629,347.78 | 5.5 | 35 | 5 | He |
| 463,395.81 | 4.1 | 39 | 5 | H | 634,059.46 | 15.0 | 30 | 3 | H |
| 463,584.64 | 4.1 | 39 | 5 | He | 634,317.83 | 15.0 | 30 | 3 | He |
| 466,940.36 | 1.3 | 46 | 9 | H | 640,788.57 | 1.8 | 41 | 9 | H |
| 467,529.55 | 2.9 | 41 | 6 | H | 647,979.34 | 1.5 | 42 | 10 | H |
| 476,460.04 | 1.1 | 47 | 10 | H | 653,314.78 | 3.0 | 38 | 7 | H |
| 482,044.55 | 6.6 | 36 | 4 | H | 662,404.20 | 123.4 | 21 | 1 | H |
| 482,240.97 | 6.6 | 36 | 4 | He | 662,674.12 | 123.4 | 21 | 1 | He |
| 482,254.22 | 11.4 | 33 | 3 | H | 662,734.66 | 123.4 | 21 | 1 | C |
| 482,378.29 | 1.7 | 44 | 8 | H | 662,753.59 | 123.4 | 21 | 1 | S |
| 482,450.73 | 11.4 | 33 | 3 | He | 670,038.21 | 34.4 | 26 | 2 | H |
| 488,202.99 | 25.1 | 29 | 2 | H | 670,311.24 | 34.4 | 26 | 2 | He |
| 488,401.93 | 25.1 | 29 | 2 | He | 673,101.94 | 4.1 | 36 | 6 | H |
| 494,523.88 | 2.3 | 42 | 7 | H | 673,289.87 | 2.4 | 39 | 8 | H |
| 496,139.42 | 1.4 | 45 | 9 | H | 673,910.96 | 9.2 | 32 | 4 | H |
| 498,757.56 | 4.4 | 38 | 5 | H | 674,185.57 | 9.2 | 32 | 4 | He |
| 498,960.80 | 4.4 | 38 | 5 | He | 682,565.59 | 6.0 | 34 | 5 | H |
| 501,132.57 | 3.1 | 40 | 6 | H | 682,843.73 | 6.0 | 34 | 5 | He |
| 505,413.72 | 1.1 | 46 | 10 | H | 685,581.28 | 1.9 | 40 | 9 | H |
| | | | | | | | | | |
| 691,860.18 | 1.6 | 41 | 10 | H | 985,749.88 | 2.7 | 35 | 9 | H |
| 698,704.78 | 16.6 | 29 | 3 | H | 1,019,698.79 | 6.3 | 31 | 6 | H |
| 698,989.49 | 16.6 | 29 | 3 | He | 1,040,131.11 | 193.7 | 18 | 1 | H |
| 703,416.95 | 3.2 | 37 | 7 | H | 1,040,554.95 | 193.7 | 18 | 1 | He |
| 723,144.34 | 2.5 | 38 | 8 | H | 1,040,650.01 | 193.7 | 18 | 1 | C |
| 728,114.39 | 4.5 | 35 | 6 | H | 1,040,679.73 | 193.7 | 18 | 1 | S |
| 734,663.81 | 2.0 | 39 | 9 | H | 1,049,218.15 | 4.8 | 32 | 7 | H |
| 737,365.86 | 10.0 | 31 | 4 | H | 1,060,394.45 | 2.4 | 35 | 10 | H |
| 737,666.32 | 10.0 | 31 | 4 | He | 1,063,321.73 | 3.7 | 33 | 8 | H |
| 739,811.53 | 1.7 | 40 | 10 | H | 1,065,357.33 | 9.4 | 29 | 5 | H |
| 742,287.00 | 6.5 | 33 | 5 | H | 1,065,791.45 | 9.4 | 29 | 5 | He |
| 742,589.47 | 6.5 | 33 | 5 | He | 1,066,048.75 | 2.9 | 34 | 9 | H |
| 750,523.63 | 38.5 | 25 | 2 | H | 1,066,939.07 | 25.3 | 25 | 3 | H |
| 750,829.45 | 38.5 | 25 | 2 | He | 1,067,373.83 | 25.3 | 25 | 3 | He |
| 758,790.34 | 3.5 | 36 | 7 | H | 1,085,072.00 | 55.7 | 22 | 2 | H |
| 764,229.59 | 142.3 | 20 | 1 | H | 1,085,514.15 | 55.7 | 22 | 2 | He |
| 764,541.01 | 142.3 | 20 | 1 | He | 1,088,869.02 | 14.8 | 27 | 4 | H |
| 764,610.85 | 142.3 | 20 | 1 | C | 1,089,312.72 | 14.8 | 27 | 4 | He |
| 764,632.69 | 142.3 | 20 | 1 | S | 1,116,313.69 | 6.9 | 30 | 6 | H |
| 772,453.58 | 18.3 | 28 | 3 | H | 1,144,445.52 | 5.2 | 31 | 7 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|--------------|-------|-----|------------|----|--------------|-------|-----|------------|----|
| 772,768.34 | 18.3 | 28 | 3 | He | 1,145,961.40 | 2.6 | 34 | 10 | H |
| 778,061.52 | 2.7 | 37 | 8 | H | 1,155,356.17 | 3.2 | 33 | 9 | H |
| 788,564.28 | 2.2 | 38 | 9 | H | 1,155,955.51 | 4.0 | 32 | 8 | H |
| 789,302.95 | 4.9 | 34 | 6 | H | 1,174,612.10 | 10.3 | 28 | 5 | H |
| 792,318.64 | 1.8 | 39 | 10 | H | 1,175,090.74 | 10.3 | 28 | 5 | He |
| 809,055.28 | 11.0 | 30 | 4 | H | 1,198,063.93 | 28.3 | 24 | 3 | H |
| 809,197.00 | 7.1 | 32 | 5 | H | 1,198,552.13 | 28.3 | 24 | 3 | He |
| 809,384.96 | 11.0 | 30 | 4 | He | 1,210,590.85 | 16.4 | 26 | 4 | H |
| 809,526.74 | 7.1 | 32 | 5 | He | 1,211,084.15 | 16.4 | 26 | 4 | He |
| 820,148.83 | 3.7 | 35 | 7 | H | 1,225,568.85 | 7.5 | 29 | 6 | H |
| 838,702.99 | 2.9 | 36 | 8 | H | 1,229,033.63 | 228.8 | 17 | 1 | H |
| 844,441.16 | 43.4 | 24 | 2 | H | 1,229,534.44 | 228.8 | 17 | 1 | He |
| 844,785.26 | 43.4 | 24 | 2 | He | 1,229,646.76 | 228.8 | 17 | 1 | C |
| 847,891.08 | 2.3 | 37 | 9 | H | 1,229,681.88 | 228.8 | 17 | 1 | S |
| 849,938.22 | 1.9 | 38 | 10 | H | 1,240,300.69 | 63.6 | 21 | 2 | H |
| 856,968.08 | 20.3 | 27 | 3 | H | 1,240,806.09 | 63.6 | 21 | 2 | He |
| 857,317.29 | 20.3 | 27 | 3 | He | 1,241,044.56 | 2.8 | 33 | 10 | H |
| 857,561.41 | 5.3 | 33 | 6 | H | 1,251,599.73 | 5.7 | 30 | 7 | H |
| 884,412.74 | 7.8 | 31 | 5 | H | 1,254,978.47 | 3.5 | 32 | 9 | H |
| 884,773.13 | 7.8 | 31 | 5 | He | 1,259,719.93 | 4.4 | 31 | 8 | H |
| 888,047.07 | 165.4 | 19 | 1 | H | 1,299,370.81 | 11.4 | 27 | 5 | H |
| 888,325.91 | 4.0 | 34 | 7 | H | 1,299,900.28 | 11.4 | 27 | 5 | He |
| 888,408.94 | 165.4 | 19 | 1 | He | 1,347,012.91 | 3.0 | 32 | 10 | H |
| 888,490.10 | 165.4 | 19 | 1 | C | 1,349,607.92 | 8.3 | 28 | 6 | H |
| 888,515.47 | 165.4 | 19 | 1 | S | 1,351,189.66 | 18.3 | 25 | 4 | H |
| 890,361.52 | 12.1 | 29 | 4 | H | 1,351,616.68 | 32.0 | 23 | 3 | H |
| 890,724.32 | 12.1 | 29 | 4 | He | 1,351,740.25 | 18.3 | 25 | 4 | He |
| 905,837.23 | 3.2 | 35 | 8 | H | 1,352,167.44 | 32.0 | 23 | 3 | He |
| 913,311.02 | 2.1 | 37 | 10 | H | 1,366,457.29 | 3.8 | 31 | 9 | H |
| 913,347.56 | 2.5 | 36 | 9 | H | 1,372,615.74 | 6.2 | 29 | 7 | H |
| 933,943.74 | 5.7 | 32 | 6 | H | 1,376,346.46 | 4.8 | 30 | 8 | H |
| 954,288.80 | 22.6 | 26 | 3 | H | 1,426,633.79 | 73.2 | 20 | 2 | H |
| 954,677.66 | 22.6 | 26 | 3 | He | 1,427,215.13 | 73.2 | 20 | 2 | He |
| 954,715.82 | 49.0 | 23 | 2 | H | 1,442,491.79 | 12.7 | 26 | 5 | H |
| 955,104.85 | 49.0 | 23 | 2 | He | 1,443,079.59 | 12.7 | 26 | 5 | He |
| 964,298.77 | 4.4 | 33 | 7 | H | 1,465,480.25 | 3.3 | 31 | 10 | H |
| 969,266.80 | 8.5 | 30 | 5 | H | 1,466,610.22 | 273.0 | 16 | 1 | H |
| 969,661.76 | 8.5 | 30 | 5 | He | 1,467,207.84 | 273.0 | 16 | 1 | He |
| 980,360.35 | 3.4 | 34 | 8 | H | 1,467,341.88 | 273.0 | 16 | 1 | C |
| 982,955.36 | 13.3 | 28 | 4 | H | 1,467,383.78 | 273.0 | 16 | 1 | S |
| 983,177.12 | 2.2 | 36 | 10 | H | 1,491,027.55 | 9.1 | 27 | 6 | H |
| 983,355.91 | 13.3 | 28 | 4 | He | 1,491,620.87 | 4.1 | 30 | 9 | H |
| 1,507,901.78 | 5.2 | 29 | 8 | H | 2,497,434.74 | 8.6 | 24 | 8 | H |
| 1,509,819.44 | 6.8 | 28 | 7 | H | 2,511,705.79 | 33.9 | 20 | 4 | H |
| 1,514,479.38 | 20.5 | 24 | 4 | H | 2,512,729.28 | 33.9 | 20 | 4 | He |
| 1,515,096.50 | 20.5 | 24 | 4 | He | 2,562,207.53 | 11.5 | 23 | 7 | H |
| 1,532,612.30 | 36.2 | 22 | 3 | H | 2,576,758.51 | 5.7 | 25 | 10 | H |
| 1,533,236.82 | 36.2 | 22 | 3 | He | 2,591,917.36 | 22.6 | 21 | 5 | H |
| 1,598,358.23 | 3.6 | 30 | 10 | H | 2,592,973.54 | 22.6 | 21 | 5 | He |

(continued)

Table B.2. (continued)

| ν/MHz | Int. | n | Δn | | ν/MHz | Int. | n | Δn | |
|------------------|-------|-----|------------|----|------------------|-------|-----|------------|----|
| 1,607,491.71 | 14.1 | 25 | 5 | H | 2,599,551.38 | 15.9 | 22 | 6 | H |
| 1,608,146.74 | 14.1 | 25 | 5 | He | 2,680,152.85 | 498.5 | 13 | 1 | H |
| 1,632,648.51 | 4.5 | 29 | 9 | H | 2,681,244.98 | 498.5 | 13 | 1 | He |
| 1,652,276.67 | 84.7 | 19 | 2 | H | 2,681,489.92 | 498.5 | 13 | 1 | C |
| 1,652,949.95 | 84.7 | 19 | 2 | He | 2,681,566.50 | 498.5 | 13 | 1 | S |
| 1,652,993.58 | 10.1 | 26 | 6 | H | 2,689,091.48 | 7.3 | 24 | 9 | H |
| 1,656,866.32 | 5.8 | 28 | 8 | H | 2,692,407.78 | 63.5 | 18 | 3 | H |
| 1,666,023.36 | 7.5 | 27 | 7 | H | 2,693,504.90 | 63.5 | 18 | 3 | He |
| 1,705,239.44 | 23.1 | 23 | 4 | H | 2,695,643.85 | 138.1 | 16 | 2 | H |
| 1,705,934.31 | 23.1 | 23 | 4 | He | 2,696,742.28 | 138.1 | 16 | 2 | He |
| 1,747,476.20 | 41.3 | 21 | 3 | H | 2,794,108.47 | 9.6 | 23 | 8 | H |
| 1,747,922.92 | 3.9 | 29 | 10 | H | 2,864,087.29 | 6.3 | 24 | 10 | H |
| 1,748,188.27 | 41.3 | 21 | 3 | He | 2,883,801.96 | 13.0 | 22 | 7 | H |
| 1,769,610.90 | 329.3 | 15 | 1 | H | 2,892,577.36 | 39.0 | 19 | 4 | H |
| 1,770,331.99 | 329.3 | 15 | 1 | He | 2,893,756.04 | 39.0 | 19 | 4 | He |
| 1,770,493.72 | 329.3 | 15 | 1 | C | 2,945,540.13 | 18.0 | 21 | 6 | H |
| 1,770,544.28 | 329.3 | 15 | 1 | S | 2,959,246.10 | 25.8 | 20 | 5 | H |
| 1,792,152.36 | 4.9 | 28 | 9 | H | 2,960,451.95 | 25.8 | 20 | 5 | He |
| 1,798,729.96 | 15.8 | 24 | 5 | H | 3,004,610.25 | 8.2 | 23 | 9 | H |
| 1,799,462.92 | 15.8 | 24 | 5 | He | 3,140,104.01 | 10.8 | 22 | 8 | H |
| 1,826,234.88 | 6.3 | 27 | 8 | H | 3,157,211.81 | 74.4 | 17 | 3 | H |
| 1,839,392.65 | 11.3 | 25 | 6 | H | 3,158,498.33 | 74.4 | 17 | 3 | He |
| 1,844,650.31 | 8.3 | 26 | 7 | H | 3,196,266.99 | 7.0 | 23 | 10 | H |
| 1,916,899.10 | 4.3 | 28 | 10 | H | 3,236,221.12 | 165.7 | 15 | 2 | H |
| 1,928,178.19 | 98.9 | 18 | 2 | H | 3,237,539.83 | 165.7 | 15 | 2 | He |
| 1,928,963.89 | 98.9 | 18 | 2 | He | 3,261,955.58 | 14.6 | 21 | 7 | H |
| 1,929,513.16 | 26.1 | 22 | 4 | H | 3,354,811.98 | 45.2 | 18 | 4 | H |
| 1,930,299.41 | 26.1 | 22 | 4 | He | 3,356,146.96 | 20.4 | 20 | 6 | H |
| 1,973,281.77 | 5.4 | 27 | 9 | H | 3,356,179.02 | 45.2 | 18 | 4 | He |
| 2,004,530.28 | 47.3 | 20 | 3 | H | 3,372,004.96 | 9.2 | 22 | 9 | H |
| 2,005,347.10 | 47.3 | 20 | 3 | He | 3,377,764.65 | 628.0 | 12 | 1 | H |
| 2,019,646.13 | 7.0 | 26 | 8 | H | 3,379,141.04 | 628.0 | 12 | 1 | He |
| 2,021,654.89 | 17.7 | 23 | 5 | H | 3,379,449.74 | 628.0 | 12 | 1 | C |
| 2,022,478.69 | 17.7 | 23 | 5 | He | 3,379,546.25 | 628.0 | 12 | 1 | S |
| 2,049,894.43 | 9.3 | 25 | 7 | H | 3,399,752.87 | 29.6 | 19 | 5 | H |
| 2,055,032.01 | 12.6 | 24 | 6 | H | 3,401,138.22 | 29.6 | 19 | 5 | He |
| 2,108,567.81 | 4.7 | 27 | 10 | H | 3,546,206.17 | 12.2 | 21 | 8 | H |
| 2,162,210.55 | 402.3 | 14 | 1 | H | 3,582,506.74 | 7.9 | 22 | 10 | H |
| 2,163,091.63 | 402.3 | 14 | 1 | He | 3,709,769.73 | 16.6 | 20 | 7 | H |
| 2,163,289.23 | 402.3 | 14 | 1 | C | 3,735,774.96 | 87.9 | 16 | 3 | H |
| 2,163,351.01 | 402.3 | 14 | 1 | S | 3,737,297.23 | 87.9 | 16 | 3 | He |
| 2,179,857.65 | 6.0 | 26 | 9 | H | 3,802,508.21 | 10.3 | 21 | 9 | H |
| 2,195,016.51 | 29.7 | 21 | 4 | H | 3,847,293.17 | 23.4 | 19 | 6 | H |
| 2,195,910.95 | 29.7 | 21 | 4 | He | 3,921,441.40 | 52.8 | 17 | 4 | H |
| 2,241,551.17 | 7.8 | 25 | 8 | H | 3,923,039.34 | 52.8 | 17 | 4 | He |
| 2,269,164.74 | 116.3 | 17 | 2 | H | 3,931,821.45 | 201.1 | 14 | 2 | H |
| 2,270,089.39 | 116.3 | 17 | 2 | He | 3,932,708.47 | 34.2 | 18 | 5 | H |
| 2,283,135.93 | 20.0 | 22 | 5 | H | 3,933,423.62 | 201.1 | 14 | 2 | He |
| 2,284,066.28 | 20.0 | 22 | 5 | He | 3,934,310.99 | 34.2 | 18 | 5 | He |
| 2,286,932.96 | 10.3 | 24 | 7 | H | 4,026,185.17 | 13.8 | 20 | 8 | H |

(continued)

Table B.2. (continued)

| ν /MHz | Int. | n | Δn | | ν /MHz | Int. | n | Δn | |
|--------------|--------|-----|------------|----|---------------|--------|-----|------------|----|
| 2,305,905.48 | 14.1 | 23 | 6 | H | 4,034,409.16 | 8.8 | 21 | 10 | H |
| 2,314,680.87 | 54.6 | 19 | 3 | H | 4,244,194.03 | 19.0 | 19 | 7 | H |
| 2,315,624.07 | 54.6 | 19 | 3 | He | 4,310,435.76 | 11.6 | 20 | 9 | H |
| 2,326,904.54 | 5.2 | 26 | 10 | H | 4,340,288.00 | 806.5 | 11 | 1 | H |
| 2,416,546.99 | 6.6 | 25 | 9 | H | 4,342,056.61 | 806.5 | 11 | 1 | He |
| | | | | | | | | | |
| 4,342,453.27 | 806.5 | 11 | 1 | C | 8,555,643.48 | 51.2 | 14 | 6 | H |
| 4,342,577.28 | 806.5 | 11 | 1 | S | 8,905,138.72 | 23.5 | 15 | 9 | H |
| 4,439,883.98 | 26.9 | 18 | 6 | H | 9,307,618.15 | 79.9 | 13 | 5 | H |
| 4,465,254.75 | 105.0 | 15 | 3 | H | 9,311,410.88 | 79.9 | 13 | 5 | He |
| 4,467,074.28 | 105.0 | 15 | 3 | He | 9,319,873.08 | 40.9 | 14 | 7 | H |
| 4,566,737.81 | 10.0 | 20 | 10 | H | 9,352,679.03 | 19.9 | 15 | 10 | H |
| 4,583,845.61 | 39.8 | 17 | 5 | H | 9,982,277.28 | 33.3 | 14 | 8 | H |
| 4,585,713.46 | 39.8 | 17 | 5 | He | 9,989,738.95 | 133.0 | 12 | 4 | H |
| 4,597,816.80 | 15.7 | 19 | 8 | H | 9,993,809.64 | 133.0 | 12 | 4 | He |
| 4,623,822.03 | 62.2 | 16 | 4 | H | 10,046,823.18 | 511.2 | 10 | 2 | H |
| 4,625,706.18 | 62.2 | 16 | 4 | He | 10,050,917.12 | 511.2 | 10 | 2 | He |
| 4,842,363.40 | 247.5 | 13 | 2 | H | 10,347,749.26 | 61.6 | 13 | 6 | H |
| 4,844,336.60 | 247.5 | 13 | 2 | He | 10,398,205.50 | 242.6 | 11 | 3 | H |
| 4,887,424.29 | 21.8 | 18 | 7 | H | 10,402,442.63 | 242.6 | 11 | 3 | He |
| 4,914,232.24 | 13.2 | 19 | 9 | H | 10,560,173.77 | 27.7 | 14 | 9 | H |
| 5,161,742.09 | 31.2 | 17 | 6 | H | 10,782,575.38 | 1998.3 | 8 | 1 | H |
| 5,198,482.83 | 11.3 | 19 | 10 | H | 10,786,969.13 | 1998.3 | 8 | 1 | He |
| 5,284,325.14 | 18.0 | 18 | 8 | H | 10,787,954.56 | 1998.3 | 8 | 1 | C |
| 5,388,051.62 | 46.7 | 16 | 5 | H | 10,788,262.64 | 1998.3 | 8 | 1 | S |
| 5,390,247.18 | 46.7 | 16 | 5 | He | 11,067,349.28 | 23.3 | 14 | 10 | H |
| 5,398,431.67 | 126.8 | 14 | 3 | H | 11,235,796.33 | 49.0 | 13 | 7 | H |
| 5,400,631.46 | 126.8 | 14 | 3 | He | 11,456,349.17 | 97.9 | 12 | 5 | H |
| 5,505,385.86 | 73.9 | 15 | 4 | H | 11,461,017.48 | 97.9 | 12 | 5 | He |
| 5,507,629.23 | 73.9 | 15 | 4 | He | 12,000,025.93 | 39.8 | 13 | 8 | H |
| 5,637,947.91 | 15.1 | 18 | 9 | H | 12,560,416.05 | 166.6 | 11 | 4 | H |
| 5,668,917.61 | 25.2 | 17 | 7 | H | 12,565,534.25 | 166.6 | 11 | 4 | He |
| 5,706,535.18 | 1059.7 | 10 | 1 | H | 12,662,430.13 | 32.9 | 13 | 9 | H |
| 5,708,860.51 | 1059.7 | 10 | 1 | He | 12,685,382.80 | 75.1 | 12 | 6 | H |
| 5,709,382.04 | 1059.7 | 10 | 1 | C | 13,240,326.62 | 27.6 | 13 | 10 | H |
| 5,709,545.09 | 1059.7 | 10 | 1 | S | 13,419,247.92 | 681.1 | 9 | 2 | H |
| 5,954,363.35 | 12.9 | 18 | 10 | H | 13,424,587.83 | 312.2 | 10 | 3 | H |
| 6,050,455.82 | 36.5 | 16 | 6 | H | 13,424,716.08 | 681.1 | 9 | 2 | He |
| 6,057,917.50 | 309.2 | 12 | 2 | H | 13,430,058.17 | 312.2 | 10 | 3 | He |
| 6,060,386.02 | 309.2 | 12 | 2 | He | 13,725,513.91 | 59.4 | 12 | 7 | H |
| 6,116,457.91 | 20.7 | 17 | 8 | H | 14,330,026.95 | 121.8 | 11 | 5 | H |
| 6,393,432.93 | 55.3 | 15 | 5 | H | 14,335,866.25 | 121.8 | 11 | 5 | He |
| 6,396,038.17 | 55.3 | 15 | 5 | He | 14,613,560.98 | 48.1 | 12 | 8 | H |
| 6,513,358.77 | 17.4 | 17 | 9 | H | 15,377,790.58 | 39.6 | 12 | 9 | H |
| 6,611,974.30 | 155.0 | 13 | 3 | H | 15,727,285.82 | 2909.8 | 7 | 1 | H |
| 6,614,668.59 | 155.0 | 13 | 3 | He | 15,733,694.47 | 2909.8 | 7 | 1 | He |
| 6,627,465.30 | 88.8 | 14 | 4 | H | 15,735,131.80 | 2909.8 | 7 | 1 | C |
| 6,628,352.31 | 29.3 | 16 | 7 | H | 15,735,581.17 | 2909.8 | 7 | 1 | S |
| 6,630,165.90 | 88.8 | 14 | 4 | He | 15,796,637.17 | 92.9 | 11 | 6 | H |
| 6,866,981.54 | 14.8 | 17 | 10 | H | 16,040,194.78 | 33.1 | 12 | 10 | H |

(continued)

Table B.2. (continued)

| ν/MHz | Int. | n | Δn | | ν/MHz | Int. | n | Δn | |
|------------------|---------|-----|------------|----|------------------|-------|-----|------------|----|
| 7,135,527.82 | 24.1 | 16 | 8 | H | 16,104,740.68 | 212.5 | 10 | 4 | H |
| 7,157,662.52 | 43.0 | 15 | 6 | H | 16,111,303.14 | 212.5 | 10 | 4 | He |
| 7,583,068.13 | 0.1 | 16 | 9 | H | 17,025,670.80 | 73.1 | 11 | 7 | H |
| 7,667,596.41 | 6.1 | 14 | 5 | H | 17,759,535.92 | 411.2 | 9 | 3 | H |
| 7,670,720.85 | 66.1 | 14 | 5 | He | 17,766,772.69 | 411.2 | 9 | 3 | He |
| 7,712,712.74 | 1431.1 | 9 | 1 | H | 18,065,801.91 | 58.8 | 11 | 8 | H |
| 7,715,855.57 | 1431.1 | 9 | 1 | He | 18,266,951.23 | 154.2 | 10 | 5 | H |
| 7,716,560.44 | 1431.1 | 9 | 1 | C | 18,274,394.77 | 154.2 | 10 | 5 | He |
| 7,716,780.81 | 1431.1 | 9 | 1 | S | 18,495,288.12 | 935.7 | 8 | 2 | H |
| 7,718,052.65 | 393.4 | 11 | 2 | H | 18,502,824.70 | 935.7 | 8 | 2 | He |
| 7,721,197.65 | 393.4 | 11 | 2 | He | 18,953,848.98 | 48.2 | 11 | 9 | H |
| 7,820,066.72 | 34.5 | 15 | 7 | H | 19,718,078.57 | 40.2 | 11 | 10 | H |
| 7,979,968.99 | 17.1 | 16 | 10 | H | 20,036,562.13 | 116.8 | 10 | 6 | H |
| 8,078,584.52 | 107.9 | 13 | 4 | H | 21,137,300.57 | 277.0 | 9 | 4 | H |
| 8,081,876.43 | 107.9 | 13 | 4 | He | 21,145,913.73 | 277.0 | 9 | 4 | He |
| 8,220,128.05 | 192.3 | 12 | 3 | H | 21,503,172.35 | 91.3 | 10 | 7 | H |
| 8,223,477.64 | 192.3 | 12 | 3 | He | 22,732,205.98 | 73.1 | 10 | 8 | H |
| 8,397,963.21 | 28.2 | 15 | 8 | H | 23,772,337.09 | 59.6 | 10 | 9 | H |
| <hr/> | | | | | | | | | |
| 23,817,453.42 | 199.2 | 9 | 5 | H | | | | | |
| 23,827,158.71 | 199.2 | 9 | 5 | He | | | | | |
| 24,201,823.30 | 557.1 | 8 | 3 | H | | | | | |
| 24,211,685.22 | 557.1 | 8 | 3 | He | | | | | |
| 24,231,670.00 | 4472.0 | 6 | 1 | H | | | | | |
| 24,241,544.08 | 4472.0 | 6 | 1 | He | | | | | |
| 24,243,758.62 | 4472.0 | 6 | 1 | C | | | | | |
| 24,244,450.98 | 4472.0 | 6 | 1 | S | | | | | |
| 24,660,384.16 | 49.4 | 10 | 10 | H | | | | | |
| 25,979,663.97 | 149.7 | 9 | 6 | H | | | | | |
| 26,509,861.20 | 1,335.1 | 7 | 2 | H | | | | | |
| 26,520,663.61 | 1,335.1 | 7 | 2 | He | | | | | |
| 27,749,274.87 | 116.2 | 9 | 7 | H | | | | | |
| 28,542,111.30 | 370.9 | 8 | 4 | H | | | | | |
| 28,553,741.82 | 370.9 | 8 | 4 | He | | | | | |
| 29,215,885.09 | 92.4 | 9 | 8 | H | | | | | |

Table B.3 compares the rest frequencies of hydrogen RRLs calculated with the Dirac equation and with the Rydberg equation. It shows the content of Table 2 of Towle et al. (1996). From left to right, the columns are the lower principal quantum number n , the change in principal quantum number Δn , the angular momentum quantum number $j_{I_{max}}$ of the most intense fine-structure line, the frequency $\nu_{I_{max}}$ of that component, the line frequency ν_{Ryd} in MHz calculated by the Rydberg equation, the difference between $\nu_{I_{max}}$ and ν_{Ryd} in MHz, the intensity I_{max} of $\nu_{I_{max}}$ as $g'A$ in units of 10^4 s^{-1} , the frequency $\bar{\nu}$ in MHz of the intensity-weighted mean of all fine-structure components, the minimum frequency ν_{min} in MHz, and the minimum frequency ν_{min} in MHz of the $j' - j = +1$ fine-structure components – a measure of half-width of the highly symmetric RRLs due to structure lines that appear at small principal quantum numbers.

Table B.3. Dirac hydrogen RRL frequencies for 100 GHz to 29 THz

| n | Δn | $j_{I_{max}}$ | $\nu_{I_{max}}$ | ν_{Ryd} | ν_{diff} | I_{max} | $\bar{\nu}$ | ν_{min} |
|-----|------------|---------------|-----------------|-------------|--------------|-----------|-------------|-------------|
| 62 | 4 | 39 | 100,539.63 | 100,539.63 | 0.00 | 0.04 | 100,539.63 | 100,539.63 |
| 66 | 5 | 39 | 102,571.00 | 102,571.00 | 0.00 | 0.03 | 102,571.00 | 102,571.00 |
| 56 | 3 | 39 | 103,914.85 | 103,914.85 | 0.00 | 0.08 | 103,914.85 | 103,914.85 |
| 49 | 2 | 39 | 105,301.87 | 105,301.86 | 0.00 | 0.22 | 105,301.87 | 105,301.86 |
| 61 | 4 | 39 | 105,410.22 | 105,410.22 | 0.00 | 0.05 | 105,410.23 | 105,410.22 |
| 39 | 1 | 38 | 106,737.37 | 106,737.36 | 0.00 | 1.63 | 106,737.37 | 106,737.37 |
| 65 | 5 | 39 | 107,206.12 | 107,206.11 | 0.00 | 0.03 | 107,206.12 | 107,206.11 |
| 55 | 3 | 38 | 109,536.01 | 109,536.01 | 0.00 | 0.09 | 109,536.01 | 109,536.01 |
| 60 | 4 | 38 | 110,600.68 | 110,600.68 | 0.00 | 0.05 | 110,600.68 | 110,600.68 |
| 48 | 2 | 38 | 111,885.08 | 111,885.08 | 0.00 | 0.24 | 111,885.08 | 111,885.08 |
| 64 | 5 | 38 | 112,124.91 | 112,124.91 | 0.00 | 0.03 | 112,124.91 | 112,124.91 |
| 38 | 1 | 37 | 115,274.41 | 115,274.41 | 0.00 | 1.80 | 115,274.41 | 115,274.41 |
| 54 | 3 | 38 | 115,570.12 | 115,570.12 | 0.00 | 0.10 | 115,570.13 | 115,570.12 |
| 59 | 4 | 38 | 116,137.72 | 116,137.71 | 0.00 | 0.05 | 116,137.72 | 116,137.71 |
| 63 | 5 | 37 | 117,349.42 | 117,349.42 | 0.00 | 0.03 | 117,349.43 | 117,349.42 |
| 47 | 2 | 37 | 119,028.77 | 119,028.76 | 0.00 | 0.26 | 119,028.77 | 119,028.77 |
| 58 | 4 | 37 | 122,050.75 | 122,050.75 | 0.00 | 0.06 | 122,050.75 | 122,050.75 |
| 53 | 3 | 37 | 122,055.83 | 122,055.83 | 0.00 | 0.10 | 122,055.84 | 122,055.83 |
| 62 | 5 | 37 | 122,903.80 | 122,903.80 | 0.00 | 0.03 | 122,903.80 | 122,903.80 |
| 37 | 1 | 36 | 124,746.74 | 124,746.74 | 0.00 | 2.00 | 124,746.75 | 124,746.74 |
| 46 | 2 | 37 | 126,793.88 | 126,793.88 | 0.00 | 0.28 | 126,793.88 | 126,793.88 |
| 57 | 4 | 36 | 128,372.26 | 128,372.26 | 0.00 | 0.06 | 128,372.27 | 128,372.26 |
| 61 | 5 | 36 | 128,814.50 | 128,814.50 | 0.00 | 0.04 | 128,814.51 | 128,814.50 |
| 52 | 3 | 36 | 129,036.20 | 129,036.19 | 0.00 | 0.11 | 129,036.20 | 129,036.19 |
| 60 | 5 | 36 | 135,110.59 | 135,110.59 | 0.00 | 0.04 | 135,110.59 | 135,110.59 |
| 56 | 4 | 36 | 135,138.16 | 135,138.16 | 0.00 | 0.06 | 135,138.17 | 135,138.16 |
| 45 | 2 | 36 | 135,249.50 | 135,249.50 | 0.00 | 0.30 | 135,249.50 | 135,249.50 |
| 36 | 1 | 35 | 135,286.04 | 135,286.04 | 0.00 | 2.22 | 135,286.05 | 135,286.04 |
| 51 | 3 | 36 | 136,559.30 | 136,559.29 | 0.00 | 0.12 | 136,559.30 | 136,559.29 |
| 59 | 5 | 35 | 141,824.00 | 141,824.00 | 0.00 | 0.04 | 141,824.00 | 141,824.00 |
| 55 | 4 | 35 | 142,388.21 | 142,388.20 | 0.00 | 0.07 | 142,388.21 | 142,388.20 |
| 44 | 2 | 35 | 144,474.12 | 144,474.12 | 0.00 | 0.33 | 144,474.13 | 144,474.12 |
| 50 | 3 | 35 | 144,678.94 | 144,678.94 | 0.00 | 0.13 | 144,678.94 | 144,678.94 |
| 35 | 1 | 34 | 147,046.89 | 147,046.89 | 0.00 | 2.48 | 147,046.90 | 147,046.89 |
| 58 | 5 | 35 | 148,989.92 | 148,989.91 | 0.00 | 0.04 | 148,989.92 | 148,989.91 |
| 54 | 4 | 34 | 150,166.51 | 150,166.51 | 0.00 | 0.07 | 150,166.52 | 150,166.51 |
| 49 | 3 | 34 | 153,455.47 | 153,455.46 | 0.00 | 0.14 | 153,455.47 | 153,455.46 |
| 43 | 2 | 34 | 154,557.23 | 154,557.22 | 0.00 | 0.36 | 154,557.23 | 154,557.22 |
| 57 | 5 | 34 | 156,647.14 | 156,647.13 | 0.00 | 0.05 | 156,647.14 | 156,647.13 |
| 53 | 4 | 34 | 158,522.10 | 158,522.09 | 0.00 | 0.08 | 158,522.10 | 158,522.09 |
| 34 | 1 | 33 | 160,211.52 | 160,211.52 | 0.00 | 2.77 | 160,211.54 | 160,211.52 |
| 48 | 3 | 34 | 162,956.69 | 162,956.69 | 0.00 | 0.15 | 162,956.70 | 162,956.69 |
| 56 | 5 | 33 | 164,838.53 | 164,838.52 | 0.00 | 0.05 | 164,838.53 | 164,838.52 |
| 42 | 2 | 34 | 165,601.05 | 165,601.05 | 0.00 | 0.40 | 165,601.06 | 165,601.05 |
| 52 | 4 | 33 | 167,509.56 | 167,509.55 | 0.00 | 0.08 | 167,509.56 | 167,509.55 |
| 47 | 3 | 33 | 173,259.02 | 173,259.01 | 0.00 | 0.17 | 173,259.02 | 173,259.02 |
| 55 | 5 | 33 | 173,611.52 | 173,611.52 | 0.00 | 0.05 | 173,611.53 | 173,611.52 |
| 33 | 1 | 32 | 174,995.82 | 174,995.82 | 0.01 | 3.11 | 174,995.83 | 174,995.82 |
| 51 | 4 | 33 | 177,189.80 | 177,189.79 | 0.00 | 0.09 | 177,189.80 | 177,189.79 |
| 41 | 2 | 33 | 177,722.84 | 177,722.83 | 0.00 | 0.44 | 177,722.85 | 177,722.84 |

(continued)

Table B.3. (continued)

| n | Δn | $j_{I_{max}}$ | $\nu_{I_{max}}$ | ν_{Ryd} | ν_{diff} | I_{max} | $\bar{\nu}$ | ν_{min} |
|-----|------------|---------------|-----------------|-------------|--------------|-----------|-------------|-------------|
| 54 | 5 | 32 | 183,018.71 | 183,018.71 | 0.00 | 0.06 | 183,018.71 | 183,018.71 |
| 46 | 3 | 32 | 184,448.71 | 184,448.70 | 0.01 | 0.18 | 184,448.72 | 184,448.71 |
| 50 | 4 | 32 | 187,630.91 | 187,630.91 | 0.01 | 0.10 | 187,630.92 | 187,630.91 |
| 40 | 2 | 32 | 191,057.40 | 191,057.40 | 0.01 | 0.48 | 191,057.41 | 191,057.40 |
| 32 | 1 | 31 | 191,656.74 | 191,656.74 | 0.01 | 3.50 | 191,656.76 | 191,656.74 |
| 53 | 5 | 32 | 193,118.48 | 193,118.48 | 0.01 | 0.06 | 193,118.49 | 193,118.48 |
| 45 | 3 | 32 | 196,623.44 | 196,623.43 | 0.01 | 0.20 | 196,623.45 | 196,623.43 |
| 49 | 4 | 31 | 198,909.19 | 198,909.18 | 0.01 | 0.11 | 198,909.20 | 198,909.19 |
| 52 | 5 | 31 | 203,975.82 | 203,975.82 | 0.01 | 0.07 | 203,975.83 | 203,975.82 |
| 39 | 2 | 31 | 205,760.33 | 205,760.32 | 0.01 | 0.53 | 205,760.34 | 205,760.32 |
| 44 | 3 | 31 | 209,894.07 | 209,894.06 | 0.01 | 0.21 | 209,894.08 | 209,894.06 |
| 31 | 1 | 30 | 210,501.79 | 210,501.78 | 0.01 | 3.95 | 210,501.81 | 210,501.79 |
| 48 | 4 | 31 | 211,110.30 | 211,110.29 | 0.01 | 0.11 | 211,110.30 | 211,110.29 |
| 51 | 5 | 30 | 215,663.16 | 215,663.15 | 0.01 | 0.07 | 215,663.16 | 215,663.15 |
| 38 | 2 | 30 | 222,011.77 | 222,011.77 | 0.01 | 0.59 | 222,011.79 | 222,011.77 |
| 47 | 4 | 30 | 224,330.64 | 224,330.63 | 0.01 | 0.12 | 224,330.64 | 224,330.63 |
| 43 | 3 | 30 | 224,386.78 | 224,386.78 | 0.01 | 0.23 | 224,386.79 | 224,386.78 |
| 50 | 5 | 30 | 228,261.41 | 228,261.41 | 0.01 | 0.08 | 228,261.42 | 228,261.41 |
| 30 | 1 | 29 | 231,900.95 | 231,900.94 | 0.01 | 4.48 | 231,900.97 | 231,900.95 |
| 46 | 4 | 29 | 238,678.96 | 238,678.95 | 0.01 | 0.14 | 238,678.97 | 238,678.95 |
| 37 | 2 | 30 | 240,021.15 | 240,021.14 | 0.01 | 0.65 | 240,021.16 | 240,021.15 |
| 42 | 3 | 29 | 240,245.62 | 240,245.62 | 0.01 | 0.26 | 240,245.63 | 240,245.62 |
| 49 | 5 | 29 | 241,861.16 | 241,861.16 | 0.01 | 0.09 | 241,861.17 | 241,861.16 |
| 45 | 4 | 29 | 254,278.27 | 254,278.26 | 0.01 | 0.15 | 254,278.28 | 254,278.26 |
| 29 | 1 | 28 | 256,302.06 | 256,302.05 | 0.01 | 5.10 | 256,302.08 | 256,302.06 |
| 48 | 5 | 29 | 256,564.02 | 256,564.01 | 0.01 | 0.09 | 256,564.03 | 256,564.01 |
| 41 | 3 | 29 | 257,635.50 | 257,635.49 | 0.01 | 0.28 | 257,635.51 | 257,635.49 |
| 36 | 2 | 29 | 260,032.79 | 260,032.78 | 0.01 | 0.72 | 260,032.80 | 260,032.78 |
| 44 | 4 | 28 | 271,268.01 | 271,268.00 | 0.01 | 0.16 | 271,268.02 | 271,268.00 |
| 47 | 5 | 28 | 272,484.24 | 272,484.23 | 0.01 | 0.10 | 272,484.24 | 272,484.23 |
| 40 | 3 | 28 | 276,745.80 | 276,745.79 | 0.01 | 0.31 | 276,745.81 | 276,745.79 |
| 35 | 2 | 28 | 282,332.94 | 282,332.93 | 0.01 | 0.81 | 282,332.95 | 282,332.93 |
| 28 | 1 | 27 | 284,250.60 | 284,250.59 | 0.01 | 5.84 | 284,250.63 | 284,250.60 |
| 46 | 5 | 28 | 289,750.58 | 289,750.57 | 0.01 | 0.11 | 289,750.59 | 289,750.57 |
| 43 | 4 | 28 | 289,806.73 | 289,806.72 | 0.01 | 0.17 | 289,806.74 | 289,806.72 |
| 39 | 3 | 27 | 297,794.77 | 297,794.76 | 0.01 | 0.34 | 297,794.78 | 297,794.76 |
| 34 | 2 | 27 | 307,258.42 | 307,258.41 | 0.01 | 0.90 | 307,258.44 | 307,258.41 |
| 45 | 5 | 27 | 308,508.52 | 308,508.51 | 0.01 | 0.12 | 308,508.53 | 308,508.51 |
| 42 | 4 | 27 | 310,075.18 | 310,075.17 | 0.01 | 0.19 | 310,075.19 | 310,075.17 |
| 27 | 1 | 26 | 316,415.46 | 316,415.44 | 0.01 | 6.71 | 316,415.49 | 316,415.46 |
| 38 | 3 | 27 | 321,034.74 | 321,034.73 | 0.01 | 0.38 | 321,034.75 | 321,034.73 |
| 44 | 5 | 26 | 328,922.84 | 328,922.83 | 0.01 | 0.13 | 328,922.85 | 328,922.83 |
| 41 | 4 | 26 | 332,280.07 | 332,280.06 | 0.01 | 0.21 | 332,280.08 | 332,280.06 |
| 33 | 2 | 26 | 335,207.35 | 335,207.34 | 0.01 | 1.01 | 335,207.37 | 335,207.34 |
| 37 | 3 | 26 | 346,758.52 | 346,758.51 | 0.01 | 0.42 | 346,758.54 | 346,758.51 |
| 43 | 5 | 26 | 351,180.67 | 351,180.66 | 0.01 | 0.14 | 351,180.68 | 351,180.66 |
| 26 | 1 | 25 | 353,622.78 | 353,622.77 | 0.02 | 7.75 | 353,622.82 | 353,622.78 |
| 40 | 4 | 26 | 356,658.46 | 356,658.45 | 0.01 | 0.23 | 356,658.48 | 356,658.45 |
| 32 | 2 | 26 | 366,652.57 | 366,652.55 | 0.01 | 1.14 | 366,652.59 | 366,652.56 |
| 36 | 3 | 25 | 375,307.20 | 375,307.18 | 0.01 | 0.46 | 375,307.22 | 375,307.19 |

(continued)

Table B.3. (continued)

| n | Δn | $j_{I_{max}}$ | $\nu_{I_{max}}$ | ν_{Ryd} | ν_{diff} | I_{max} | $\bar{\nu}$ | ν_{min} |
|-----|------------|---------------|-----------------|-------------|--------------|-----------|-------------|-------------|
| 42 | 5 | 25 | 375,495.13 | 375,495.11 | 0.01 | 0.15 | 375,495.14 | 375,495.11 |
| 39 | 4 | 25 | 383,483.17 | 383,483.16 | 0.01 | 0.25 | 383,483.19 | 383,483.16 |
| 25 | 1 | 24 | 396,900.88 | 396,900.86 | 0.02 | 9.00 | 396,900.92 | 396,900.87 |
| 41 | 5 | 25 | 402,109.63 | 402,109.61 | 0.01 | 0.17 | 402,109.64 | 402,109.61 |
| 31 | 2 | 25 | 402,158.54 | 402,158.52 | 0.02 | 1.29 | 402,158.57 | 402,158.53 |
| 35 | 3 | 25 | 407,079.68 | 407,079.66 | 0.02 | 0.52 | 407,079.70 | 407,079.67 |
| 38 | 4 | 24 | 413,069.18 | 413,069.17 | 0.02 | 0.28 | 413,069.20 | 413,069.17 |
| 40 | 5 | 24 | 431,303.03 | 431,303.02 | 0.02 | 0.18 | 431,303.05 | 431,303.02 |
| 30 | 2 | 24 | 442,402.74 | 442,402.72 | 0.02 | 1.47 | 442,402.78 | 442,402.73 |
| 34 | 3 | 24 | 442,544.47 | 442,544.45 | 0.02 | 0.58 | 442,544.49 | 442,544.45 |
| 37 | 4 | 24 | 445,781.48 | 445,781.47 | 0.02 | 0.31 | 445,781.51 | 445,781.47 |
| 24 | 1 | 23 | 447,540.33 | 447,540.30 | 0.02 | 10.51 | 447,540.39 | 447,540.33 |
| 39 | 5 | 23 | 463,395.83 | 463,395.81 | 0.02 | 0.20 | 463,395.85 | 463,395.81 |
| 36 | 4 | 23 | 482,044.57 | 482,044.55 | 0.02 | 0.34 | 482,044.59 | 482,044.55 |
| 33 | 3 | 23 | 482,254.24 | 482,254.22 | 0.02 | 0.65 | 482,254.27 | 482,254.23 |
| 29 | 2 | 23 | 488,203.02 | 488,202.99 | 0.02 | 1.67 | 488,203.05 | 488,203.00 |
| 38 | 5 | 23 | 498,757.58 | 498,757.56 | 0.02 | 0.22 | 498,757.61 | 498,757.57 |
| 23 | 1 | 22 | 507,175.54 | 507,175.51 | 0.03 | 12.35 | 507,175.61 | 507,175.54 |
| 35 | 4 | 23 | 522,354.09 | 522,354.07 | 0.02 | 0.38 | 522,354.12 | 522,354.07 |
| 32 | 3 | 23 | 526,864.10 | 526,864.07 | 0.02 | 0.73 | 526,864.13 | 526,864.08 |
| 37 | 5 | 22 | 537,815.93 | 537,815.90 | 0.03 | 0.25 | 537,815.96 | 537,815.91 |
| 28 | 2 | 22 | 540,552.67 | 540,552.64 | 0.03 | 1.91 | 540,552.71 | 540,552.65 |
| 34 | 4 | 22 | 567,291.21 | 567,291.18 | 0.03 | 0.43 | 567,291.24 | 567,291.19 |
| 31 | 3 | 22 | 577,154.37 | 577,154.34 | 0.03 | 0.82 | 577,154.41 | 577,154.34 |
| 22 | 1 | 21 | 577,896.53 | 577,896.49 | 0.04 | 14.61 | 577,896.61 | 577,896.52 |
| 36 | 5 | 22 | 581,067.53 | 581,067.51 | 0.03 | 0.27 | 581,067.56 | 581,067.51 |
| 27 | 2 | 22 | 600,666.06 | 600,666.03 | 0.03 | 2.20 | 600,666.12 | 600,666.04 |
| 33 | 4 | 21 | 617,540.30 | 617,540.26 | 0.03 | 0.48 | 617,540.33 | 617,540.27 |
| 35 | 5 | 21 | 629,091.47 | 629,091.43 | 0.03 | 0.30 | 629,091.50 | 629,091.44 |
| 30 | 3 | 21 | 634,059.50 | 634,059.46 | 0.04 | 0.93 | 634,059.55 | 634,059.47 |
| 21 | 1 | 20 | 662,404.25 | 662,404.20 | 0.05 | 7.42 | 662,404.36 | 662,404.24 |
| 26 | 2 | 21 | 670,038.25 | 670,038.21 | 0.04 | 2.55 | 670,038.32 | 670,038.23 |
| 32 | 4 | 21 | 673,911.00 | 673,910.96 | 0.04 | 0.54 | 673,911.04 | 673,910.97 |
| 34 | 5 | 21 | 682,565.63 | 682,565.59 | 0.04 | 0.34 | 682,565.67 | 682,565.59 |
| 29 | 3 | 20 | 698,704.82 | 698,704.78 | 0.04 | 1.06 | 698,704.87 | 698,704.79 |
| 31 | 4 | 20 | 737,365.90 | 737,365.86 | 0.04 | 0.61 | 737,365.95 | 737,365.87 |
| 33 | 5 | 20 | 742,287.04 | 742,287.00 | 0.04 | 0.38 | 742,287.09 | 742,287.01 |
| 25 | 2 | 20 | 750,523.67 | 750,523.63 | 0.05 | 2.97 | 750,523.75 | 750,523.64 |
| 20 | 1 | 19 | 764,229.65 | 764,229.59 | 0.06 | 20.93 | 764,229.79 | 764,229.64 |
| 28 | 3 | 20 | 772,453.63 | 772,453.58 | 0.05 | 1.21 | 772,453.69 | 772,453.59 |
| 30 | 4 | 19 | 809,055.33 | 809,055.28 | 0.06 | 0.69 | 809,055.39 | 809,055.29 |
| 32 | 5 | 19 | 809,197.05 | 809,197.00 | 0.05 | 0.42 | 809,197.10 | 809,197.01 |
| 24 | 2 | 19 | 844,441.22 | 844,441.16 | 0.06 | 3.47 | 844,441.32 | 844,441.18 |
| 27 | 3 | 19 | 856,968.14 | 856,968.08 | 0.06 | 1.40 | 856,968.22 | 856,968.09 |
| 31 | 5 | 19 | 884,412.80 | 884,412.74 | 0.06 | 0.48 | 884,412.86 | 884,412.75 |
| 19 | 1 | 18 | 888,047.15 | 888,047.07 | 0.08 | 5.38 | 888,047.32 | 888,047.14 |
| 29 | 4 | 19 | 890,361.57 | 890,361.52 | 0.06 | 0.78 | 890,361.64 | 890,361.53 |
| 26 | 3 | 18 | 954,288.87 | 954,288.80 | 0.07 | 1.61 | 954,288.96 | 954,288.82 |
| 23 | 2 | 18 | 954,715.89 | 954,715.82 | 0.08 | 4.07 | 954,716.01 | 954,715.84 |
| 30 | 5 | 18 | 969,266.87 | 969,266.80 | 0.07 | 0.54 | 969,266.94 | 969,266.81 |

(continued)

Table B.3. (continued)

| n | Δn | $j_{I_{max}}$ | $\nu_{I_{max}}$ | ν_{Ryd} | ν_{diff} | I_{max} | $\bar{\nu}$ | ν_{min} |
|-----|------------|---------------|-----------------|--------------|--------------|-----------|--------------|--------------|
| 28 | 4 | 18 | 982,955.44 | 982,955.36 | 0.07 | 0.89 | 982,955.51 | 982,955.38 |
| 15 | 1 | 17 | 1,040,131.21 | 1,040,131.11 | 0.10 | 31.06 | 1,040,131.44 | 1,040,131.19 |
| 29 | 5 | 18 | 1,065,357.41 | 1,065,357.33 | 0.08 | 0.61 | 1,065,357.49 | 1,065,357.34 |
| 25 | 3 | 18 | 1,066,939.15 | 1,066,939.07 | 0.08 | 1.87 | 1,066,939.26 | 1,066,939.09 |
| 22 | 2 | 18 | 1,085,072.08 | 1,085,072.00 | 0.08 | 4.84 | 1,085,072.23 | 1,085,072.03 |
| 27 | 4 | 18 | 1,088,869.10 | 1,088,869.02 | 0.08 | 1.02 | 1,088,869.20 | 1,088,869.04 |
| 28 | 5 | 17 | 1,174,612.20 | 1,174,612.10 | 0.10 | 0.70 | 1,174,612.29 | 1,174,612.12 |
| 24 | 3 | 17 | 1,198,064.03 | 1,198,063.93 | 0.10 | 2.18 | 1,198,064.16 | 1,198,063.95 |
| 26 | 4 | 17 | 1,210,590.95 | 1,210,590.85 | 0.10 | 1.17 | 1,210,591.06 | 1,210,590.87 |
| 17 | 1 | 16 | 1,229,033.77 | 1,229,033.63 | 0.14 | 38.44 | 1,229,034.05 | 1,229,033.74 |
| 21 | 2 | 17 | 1,240,300.80 | 1,240,300.69 | 0.11 | 5.79 | 1,240,300.98 | 1,240,300.73 |
| 27 | 5 | 17 | 1,299,370.91 | 1,299,370.81 | 0.10 | 0.79 | 1,299,371.03 | 1,299,370.82 |
| 25 | 4 | 16 | 1,351,189.78 | 1,351,189.66 | 0.13 | 1.36 | 1,351,189.91 | 1,351,189.68 |
| 23 | 3 | 16 | 1,351,616.81 | 1,351,616.68 | 0.13 | 2.55 | 1,351,616.96 | 1,351,616.70 |
| 20 | 2 | 16 | 1,426,633.94 | 1,426,633.79 | 0.14 | 6.98 | 1,426,634.16 | 1,426,633.85 |
| 26 | 5 | 16 | 1,442,491.92 | 1,442,491.79 | 0.13 | 0.92 | 1,442,492.06 | 1,442,491.81 |
| 16 | 1 | 15 | 1,466,610.41 | 1,466,610.22 | 0.19 | 48.14 | 1,466,610.79 | 1,466,610.36 |
| 24 | 4 | 16 | 1,514,479.51 | 1,514,479.38 | 0.14 | 1.58 | 1,514,479.68 | 1,514,479.40 |
| 22 | 3 | 16 | 1,532,612.45 | 1,532,612.30 | 0.14 | 3.02 | 1,532,612.65 | 1,532,612.34 |
| 25 | 5 | 15 | 1,607,491.88 | 1,607,491.71 | 0.17 | 1.06 | 1,607,492.02 | 1,607,491.73 |
| 19 | 2 | 15 | 1,652,276.85 | 1,652,276.67 | 0.19 | 8.46 | 1,652,277.14 | 1,652,276.73 |
| 23 | 4 | 15 | 1,705,239.63 | 1,705,239.44 | 0.18 | 1.85 | 1,705,239.82 | 1,705,239.48 |
| 21 | 3 | 15 | 1,747,476.39 | 1,747,476.20 | 0.19 | 3.60 | 1,747,476.64 | 1,747,476.25 |
| 15 | 1 | 14 | 1,769,611.17 | 1,769,610.90 | 0.27 | 61.11 | 1,769,611.68 | 1,769,611.10 |
| 24 | 5 | 15 | 1,798,730.15 | 1,798,729.96 | 0.19 | 1.23 | 1,798,730.34 | 1,798,729.99 |
| 18 | 2 | 14 | 1,928,178.44 | 1,928,178.19 | 0.26 | 10.34 | 1,928,178.79 | 1,928,178.27 |
| 22 | 4 | 14 | 1,929,513.40 | 1,929,513.16 | 0.24 | 2.17 | 1,929,513.62 | 1,929,513.20 |
| 20 | 3 | 14 | 2,004,530.53 | 2,004,530.28 | 0.25 | 4.31 | 2,004,530.83 | 2,004,530.34 |
| 23 | 5 | 14 | 2,021,655.13 | 2,021,654.89 | 0.24 | 1.43 | 2,021,655.35 | 2,021,654.93 |
| 14 | 1 | 13 | 2,162,210.93 | 2,162,210.55 | 0.38 | 78.74 | 2,162,211.64 | 2,162,210.83 |
| 21 | 4 | 14 | 2,195,016.77 | 2,195,016.51 | 0.26 | 2.59 | 2,195,017.08 | 2,195,016.56 |
| 17 | 2 | 14 | 2,269,165.02 | 2,269,164.74 | 0.28 | 12.89 | 2,269,165.53 | 2,269,164.85 |
| 22 | 5 | 14 | 2,283,136.20 | 2,283,135.93 | 0.27 | 1.68 | 2,283,136.50 | 2,283,135.98 |
| 19 | 3 | 14 | 2,314,681.15 | 2,314,680.87 | 0.28 | 5.20 | 2,314,681.56 | 2,314,680.94 |
| 20 | 4 | 13 | 2,511,706.15 | 2,511,705.79 | 0.35 | 3.09 | 2,511,706.51 | 2,511,705.86 |
| 21 | 5 | 13 | 2,591,917.72 | 2,591,917.36 | 0.35 | 1.99 | 2,591,918.06 | 2,591,917.42 |
| 13 | 1 | 12 | 2,680,153.40 | 2,680,152.85 | 0.55 | 103.20 | 2,680,154.40 | 2,680,153.24 |
| 18 | 3 | 13 | 2,692,408.16 | 2,692,407.78 | 0.38 | 6.39 | 2,692,408.67 | 2,692,407.88 |
| 16 | 2 | 13 | 2,695,644.24 | 2,695,643.85 | 0.39 | 16.25 | 2,695,644.90 | 2,695,644.00 |
| 19 | 4 | 13 | 2,892,577.75 | 2,892,577.36 | 0.40 | 3.71 | 2,892,578.26 | 2,892,577.44 |
| 20 | 5 | 12 | 2,959,246.58 | 2,959,246.10 | 0.48 | 2.36 | 2,959,246.97 | 2,959,246.17 |
| 17 | 3 | 12 | 3,157,212.35 | 3,157,211.81 | 0.54 | 7.88 | 3,157,212.97 | 3,157,211.94 |
| 15 | 2 | 12 | 3,236,221.68 | 3,236,221.12 | 0.56 | 20.70 | 3,236,222.55 | 3,236,221.33 |
| 18 | 4 | 12 | 3,354,812.53 | 3,354,811.98 | 0.55 | 4.54 | 3,354,813.13 | 3,354,812.09 |
| 12 | 1 | 11 | 3,377,765.49 | 3,377,764.65 | 0.83 | 137.95 | 3,377,766.92 | 3,377,765.23 |
| 19 | 5 | 12 | 3,399,753.41 | 3,399,752.87 | 0.54 | 2.85 | 3,399,753.96 | 3,399,752.97 |
| 16 | 3 | 11 | 3,735,775.72 | 3,735,774.96 | 0.76 | 9.78 | 3,735,776.48 | 3,735,775.13 |
| 17 | 4 | 11 | 3,921,442.17 | 3,921,441.40 | 0.77 | 5.57 | 3,921,442.90 | 3,921,441.55 |
| 14 | 2 | 11 | 3,931,822.28 | 3,931,821.45 | 0.82 | 26.68 | 3,931,823.43 | 3,931,821.75 |
| 18 | 5 | 11 | 3,932,709.21 | 3,932,708.47 | 0.75 | 3.45 | 3,932,709.86 | 3,932,708.60 |

(continued)

Table B.3. (continued)

| n | Δn | $j_{I_{max}}$ | $\nu_{I_{max}}$ | ν_{Ryd} | ν_{diff} | I_{max} | $\bar{\nu}$ | ν_{min} |
|-----|------------|---------------|-----------------|---------------|--------------|-----------|---------------|---------------|
| 11 | 1 | 10 | 4,340,289.31 | 4,340,288.00 | 1.31 | 188.61 | 4,340,291.43 | 4,340,288.88 |
| 15 | 3 | 11 | 4,465,255.62 | 4,465,254.75 | 0.87 | 12.48 | 4,465,256.80 | 4,465,254.98 |
| 17 | 5 | 11 | 4,583,846.46 | 4,583,845.61 | 0.86 | 4.22 | 4,583,847.40 | 4,583,845.78 |
| 16 | 4 | 11 | 4,623,822.91 | 4,623,822.03 | 0.88 | 6.92 | 4,623,823.99 | 4,623,822.23 |
| 13 | 2 | 11 | 4,842,364.32 | 4,842,363.40 | 0.92 | 34.79 | 4,842,366.18 | 4,842,363.82 |
| 16 | 5 | 10 | 5,388,052.85 | 5,388,051.62 | 1.23 | 5.24 | 5,388,053.96 | 5,388,051.85 |
| 14 | 3 | 10 | 5,398,432.97 | 5,398,431.67 | 1.29 | 16.06 | 5,398,434.48 | 5,398,432.01 |
| 15 | 4 | 10 | 5,505,387.14 | 5,505,385.86 | 1.28 | 8.75 | 5,505,388.47 | 5,505,386.13 |
| 10 | 1 | 9 | 5,706,537.32 | 5,706,535.18 | 2.14 | 264.77 | 5,706,540.56 | 5,706,536.57 |
| 12 | 2 | 10 | 6,057,918.92 | 6,057,917.50 | 1.42 | 47.28 | 6,057,921.52 | 6,057,918.12 |
| 15 | 5 | 10 | 6,393,434.36 | 6,393,432.93 | 1.43 | 6.51 | 6,393,436.04 | 6,393,433.25 |
| 13 | 3 | 9 | 6,611,976.29 | 6,611,974.30 | 1.99 | 20.81 | 6,611,978.23 | 6,611,974.79 |
| 14 | 4 | 9 | 6,627,467.22 | 6,627,465.30 | 1.92 | 11.09 | 6,627,468.85 | 6,627,465.69 |
| 14 | 5 | 9 | 7,667,598.54 | 7,667,596.41 | 2.13 | 8.35 | 7,667,600.62 | 7,667,596.86 |
| 9 | 1 | 8 | 7,712,716.43 | 7,712,712.74 | 3.69 | 383.47 | 7,712,721.58 | 7,712,715.04 |
| 11 | 2 | 9 | 7,718,054.93 | 7,718,052.65 | 2.28 | 65.47 | 7,718,058.65 | 7,718,053.60 |
| 13 | 4 | 9 | 8,078,586.79 | 8,078,584.52 | 2.27 | 14.48 | 8,078,589.45 | 8,078,585.08 |
| 12 | 3 | 9 | 8,220,130.39 | 8,220,128.05 | 2.33 | 27.90 | 8,220,133.67 | 8,220,128.77 |
| 13 | 5 | 8 | 9,307,621.43 | 9,307,618.15 | 3.28 | 10.71 | 9,307,623.95 | 9,307,618.79 |
| 12 | 4 | 8 | 9,989,742.54 | 9,989,738.95 | 3.59 | 19.17 | 9,989,745.96 | 9,989,739.78 |
| 10 | 2 | 8 | 10,046,827.01 | 10,046,823.18 | 3.83 | 92.59 | 10,046,832.44 | 10,046,824.68 |
| 11 | 3 | 8 | 10,398,209.30 | 10,398,205.50 | 3.80 | 38.21 | 10,398,213.79 | 10,398,206.59 |
| 8 | 1 | 7 | 10,782,582.17 | 10,782,575.38 | 6.79 | 576.60 | 10,782,590.70 | 10,782,579.40 |
| 12 | 5 | 8 | 11,456,353.12 | 11,456,349.17 | 3.95 | 14.10 | 11,456,357.36 | 11,456,350.11 |
| 11 | 4 | 8 | 12,560,420.36 | 12,560,416.05 | 4.30 | 25.49 | 12,560,426.28 | 12,560,417.31 |
| 9 | 2 | 7 | 13,419,254.74 | 13,419,247.92 | 6.82 | 134.03 | 13,419,262.83 | 13,419,250.41 |
| 10 | 3 | 7 | 13,424,594.35 | 13,424,587.83 | 6.52 | 52.96 | 13,424,600.46 | 13,424,589.56 |
| 11 | 5 | 7 | 14,330,033.44 | 14,330,026.95 | 6.49 | 18.93 | 14,330,038.82 | 14,330,028.38 |
| 7 | 1 | 6 | 15,727,299.41 | 15,727,285.82 | 13.60 | 907.61 | 15,727,314.27 | 15,727,293.37 |
| 10 | 4 | 7 | 16,104,748.00 | 16,104,740.68 | 7.33 | 35.91 | 16,104,756.11 | 16,104,742.67 |
| 9 | 3 | 7 | 17,759,543.88 | 17,759,535.92 | 7.97 | 76.22 | 17,759,555.96 | 17,759,538.79 |
| 10 | 5 | 7 | 18,266,959.22 | 18,266,951.23 | 7.99 | 25.63 | 18,266,968.98 | 18,266,953.49 |
| 8 | 2 | 7 | 18,495,296.16 | 18,495,288.12 | 8.04 | 199.25 | 18,495,313.39 | 18,495,292.49 |
| 9 | 4 | 6 | 21,137,313.80 | 21,137,300.57 | 13.23 | 50.99 | 21,137,324.77 | 21,137,303.87 |
| 9 | 5 | 6 | 23,817,467.74 | 23,817,453.42 | 14.32 | 36.78 | 23,817,480.98 | 23,817,457.13 |
| 8 | 3 | 6 | 24,201,838.41 | 24,201,823.30 | 15.12 | 115.89 | 24,201,856.68 | 24,201,828.34 |
| 6 | 1 | 5 | 24,231,700.42 | 24,231,670.00 | 30.42 | 1,512.56 | 24,231,727.71 | 24,231,685.55 |
| 7 | 2 | 6 | 26,509,877.29 | 26,509,861.20 | 16.09 | 327.01 | 26,509,906.82 | 26,509,869.43 |
| 8 | 4 | 6 | 28,542,128.09 | 28,542,111.30 | 16.80 | 74.36 | 28,542,151.07 | 28,542,117.06 |

Figure B.1 illustrates the difference between the frequencies calculated by the Rydberg and Dirac equations for the H19 α line at 888 GHz ($\lambda = 338 \mu\text{m}$). All fine-structure components lying within this spectral window have been plotted. At left, a broken line marks the frequency calculated from the Rydberg equation (A.5). The vertical lines mark the fine-structure components calculated from the Dirac equation (B.1). The strongest components correspond to the case $j' - j = +1$ and the weakest correspond to $j' - j = +1$. The $j' - j = 0$ components – although plotted – generally lie within the line width of the X-axis and are too weak to be visible. The weighted-intensity

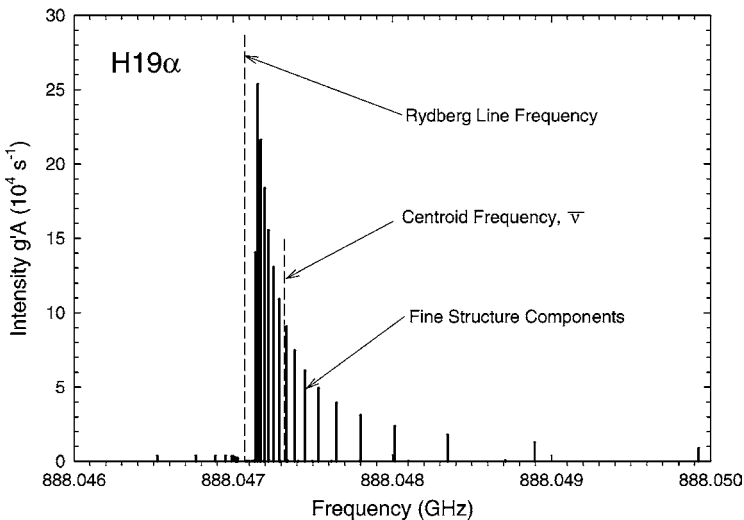


Fig. B.1 The fine-structure components of the H19 α recombination line plotted against frequency. *Broken lines* mark the frequency calculated by the classical Rydberg equation and the frequency–intensity weighted mean – the centroid – of *all* fine-structure components. Strong components with frequencies greater than the Rydberg frequency are $j' - j = +1$. The weak lower-frequency components seen on either side of the Rydberg frequency come principally from $j' - j = -1$. The $j' - j = 0$ components are too weak to be visible although this plot includes them. Data from Watson (2002)

mean of all of these is marked by another broken line labeled “centroid frequency.” The effect of the weighting is to shift the line to higher frequencies. The shift between the classical Rydberg frequency and the centroid of all the fine-structure components is about 250 kHz – a very small amount. Note also that the dispersion of the fine-structure components creates a line width. For the H19 α line, this full width at half-intensity is about 300 kHz. Of course, the composite line profile is highly asymmetrical and cannot be easily compared to a Gaussian.

Section B.3 lists the FORTRAN code used for these calculations.

B.3 FORTRAN Code for Fine-Structure Frequencies

To facilitate calculations of the frequencies of the fine-structure lines of hydrogen, we have included FORTRAN code written by Watson (2002). This code was not intended for publication but, with the inserted comments, can be read with clarity. It produces a listing of upper principal quantum number n' , upper j' value, lower principal quantum number n , lower j value, frequency in MHz (or wave number in cm^{-1}), and intensity gA in units of s^{-1} .

As discussed earlier in Sect. B.2, this code does not include corrections for relativistic effects.

```

PROGRAM HFINE

C  FORTRAN code to calculate Hydrogen fine-structure components
C  Relativistic effects on intensities are not included
C  Uses Dirac formula for frequencies but not for intensities
C  Written by James K.G. Watson
C  Steacie Institute for Molecular Sciences
C  National Research Council of Canada
C  Ottawa
C
      IMPLICIT DOUBLE PRECISION (A-H,P-Z), INTEGER(I-N), CHARACTER*2(0)
      PARAMETER(MAXSIZ=10000)
      DIMENSION NUP(MAXSIZ),NLO(MAXSIZ),FJUP(MAXSIZ),
+    FJLO(MAXSIZ),W(MAXSIZ),
+    SS(MAXSIZ),INDEX(MAXSIZ),GA(MAXSIZ)

      OPEN(UNIT=5,FILE='HFINE.IN',STATUS='OLD')

C
C  INPUTS ARE (1) ISPEC = 1 FOR NORMAL OUTPUT IN A TABLE, 2
C                    AND 3 FOR SPECIAL GRAPHICS OPTIONS
C                (2) IUNIT = 1 FOR WAVENUMBER IN 1/cm AND 2 FOR MHz
C                (3) NU = UPPER PRINCIPAL QUANTUM NUMBER
C                (4) NL = LOWER PRINCIPAL QUANTUM NUMBER
C
C  Read the run parameters from the input file
C
      READ(5,5001) ISPEC,IUNIT,NU,NL
5001 FORMAT(4I5)
C
C  Establish appropriate physical constants
C
      RINF=109737.31534D0
      IF (IUNIT.EQ.2) RINF=RINF*2.99792458D4
      ALF=7.29735308D-3
      FME=9.1093897D0/1.6605402D4
      FMH=1.007825035
      FAC=2.D0*RINF*(1.D0-FME/FMH)/(ALF*ALF)
      RH=RINF*(1.D0-FME/FMH)

C
C  Initialize variables
C
      LNCT=0
      FNL=DFLOAT(NL)
      SSUM=0.D0
      WSUM=0.D0
      SMAX=0.D0
      GAMAX=0.D0

C
C  Begin principal loop
C
      DO 99 J12=1,(2*NL-1),2
        FJL=0.5D0*J12
        FKL=FJL+0.5D0
        TEMP=FNL-FKL+DSQRT((FKL*FKL-ALF*ALF))
        TEMP=(ALF/TEMP)**2
        TEMPP=DSQRT((1+TEMP))
        ELO=-FAC*TEMP/(TEMPP*(1.D0+TEMPP))
        ELO0=-RINF*(1.D0-FME/FMH)/(FNL**2)
        ELO=ELO0+ELO*(FME/FMH)*(ALF/(2.D0*FNL))**2
        INTJ=INT(FJL)
      DO 99 IDJP2=1,3
        S=0.D0
        FJU=FJL+DFLOAT((IDJP2-2))
        IF (FJU.LT.0.D0) GO TO 99

```

```

FKU=FJU+0.5D0
DO 98 IDJL=1,2
  LL=INTJ+IDJL-1
  IF (LL.EQ.NL) GO TO 98
  FLL=DFLOAT(LL)
DO 97 IDLP2=1,3,2
  LU=LL+IDLP2-2
  FLU=DFLOAT(LU)

C
  FNU=DFLOAT(NU)
  IF (LU.LT.0) GO TO 97
  IF (LU.GT.(NU-1)) GO TO 97
  TEMP=FNU-FKU+DSQRT((FKU*FKU-ALF*ALF))
  TEMP=(ALF/TEMP)**2
  TEMPP=DSQRT((1+TEMP))
  EUP=-FAC*TEMP/(TEMPP*(1.DO+TEMPP))
  EUPO=-RINF*(1.DO-FME/FMH)/(FNU**2)
  EUP=EUP+EUPO*(FME/FMH)*(ALF/(2.DO*FNU))**2
  FREQ=EUP-EL0

C
C Calculate line strength
  S=S+0.5D0*(2.DO*FJU+1.DO)*(2.DO*FJL+1.DO)
  & *(FLU+FLL+1.DO)*(SIXJ(FJU,1.DO,FJL,FLL,0.5D0,FLU)
  & *GORD(NU,LU,NL,LL))**2
C Alternative output format
C WRITE(6,6001) NU,LU,FJU,(RH+EUP),NL,LL,FJL,(RH+EL0),FREQ,S
C
97 CONTINUE
98 CONTINUE
  IF (ISPEC.NE.1) GO TO 101
C Standard unsorted list output
C WRITE(6,6004) NU,FJU,(RH+EUP),NL,FJL,(RH+EL0),FREQ,S
101 SSUM=SSUM+S
  WSUM=WSUM+S*FREQ
  IF (IUNIT.EQ.1) A=S*2.0261273D-6*(FREQ**3)
  IF (IUNIT.EQ.2) A=S*2.0261273D-6*((FREQ/2.99792458D4)**3)
  IF (A.LT.1.D-6) GO TO 99
  LNCT=LNCT+1
  NUP(LNCT)=NU
  FJUP(LNCT)=FJU
  NLO(LNCT)=NL
  FJLO(LNCT)=FJL
  W(LNCT)=FREQ
  SS(LNCT)=S
  GA(LNCT)=A
  IF (A.GT.GAMAX) GAMAX=A
99 CONTINUE
  FNBAR=0.5D0*DFLOAT((NL+NU))
  IF (ISPEC.NE.1) GO TO 100
  WRITE(6,6003) NU,NL,(WSUM/SSUM),(EUPO-EL00)
6003 FORMAT(2I3,3F20.6)
  100 CONTINUE
6001 FORMAT(2I3,F4.1,1P,D15.6,3X,0P,2I3,F4.1,1P,D16.6,0P,2F20.6)
6004 FORMAT(I3,F5.1,1P,D15.6,3X,0P,I3,F5.1,1P,D15.6,0P,2F20.6)
C
C Sort output before printing
  CALL ASC2(LNCT,10000,INDEX,W)
C
C Output calculations (LNCT = line count)
C
DO 200 I=1,LNCT
  J=INDEX(I)
  IF (ISPEC.EQ.2) WRITE(6,6005) W(I),0.DO
  IF (ISPEC.EQ.2) WRITE(6,6005) W(I),SS(J)
  IF (ISPEC.EQ.3) WRITE(6,6006) W(I),SS(J)
  IF (ISPEC.EQ.2) WRITE(6,6005) W(I),0.DO
  IF ((ISPEC.NE.1).AND.(ISPEC.NE.4)) GO TO 200

```

```

C
C Optional filtering for threshold line strength GA
C IF (GA(J).LT.(0.1*GAMAX)) GO TO 200
  WRITE(6,6002) NUP(J),FJUP(J),NLO(J),FJLO(J),W(I),GA(J)
  200 CONTINUE
6002 FORMAT(I3,F5.1,3X,I3,F5.1,F15.3,F15.0)
6005 FORMAT(2F15.6)
6006 FORMAT(F11.3,F9.0)
C
  STOP
  END
C
C
  DOUBLE PRECISION FUNCTION gord(N1,L1,N2,L2)
  IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
C Calculate radial matrix integrals
  GORD=0.DO
  IF (IABS(L1-L2).NE.1) GO TO 200
C
  IF (L2.GT.L1) GO TO 10
  N=N1
  L=L1
  NP=N2
  GO TO 20
10 N=N2
  L=L2
  NP=N1
C
20 CONTINUE
  FN=DFLOAT(N)
  FNP=DFLOAT(NP)
  FL=DFLOAT(L)
  IF (N1.NE.N2) GO TO 30
  GORD=-1.5DO*FN*DSQRT((FN*FN-FL*FL))
  GO TO 200
30 X=- (FN-FNP)**2/(4.DO*FN*FNP)
C
  MU=MIN(N-L-1,NP-L)
  M=N+NP-2*L-1-MU
  MUP=MIN(N-L+1,NP-L)
  MP=N+NP-2*L+1-MUP
  X1=1.DO
  IF ((M-MU-1).NE.0) X1=(FN-FNP)**(M-MU-1)
  X2=1.DO
  IF (MU.NE.0) X2=(-4.DO*FN*FNP)**MU
  X3=1.DO
  IF ((MP-MUP-1).NE.0) X3=(FN-FNP)**(MP-MUP-1)
  X4=1.DO
  IF (MUP.NE.0) X4=(-4.DO*FN*FNP)**MUP
  TERM1=X2*X1*FAC(M)
&      *HYPERG(MU,(2*L+MU-1),(M-MU+1),X)
&      /(FAC((2*L-1+MU))*FAC((M-MU)))
  TERM2=X4*X3*FAC(MP)
&      *HYPERG(MUP,(2*L+MUP-1),(MP-MUP+1),X)
&      /(FAC((2*L-1+MUP))*FAC((MP-MUP)))*(N+NP)**2)
C
  X5=1.DO
  IF (NP.NE.L) X5=(-1.DO)**(NP-L)
  TERM1=X5*(4.DO*FN*FNP)**(L+1)/(4.DO*(FN+FNP)**(N+NP))
&      *DSQRT((FAC((N+L))*FAC((NP+L-1))/(FAC((N-L-1))*FAC((NP-L))))))
&      *TERM1
  TERM2=X5*(4.DO*FN*FNP)**(L+1)/(4.DO*(FN+FNP)**(N+NP))
&      *DSQRT((FAC((N+L))*FAC((NP+L-1))/(FAC((N-L-1))*FAC((NP-L))))))
&      *TERM2
  GORD=TERM1-TERM2
C Print statement for radial matrix integrals
C WRITE(6,6265) N1,L1,N2,L2,TERM1,TERM2,GORD

```



```

6265 FORMAT(4I3,1P,5D12.3)
200 RETURN
C
C   The hyperg(m,n,k,x) function is  ${}_2F_1(-m,-n;k;x)$ 
C
      DOUBLE PRECISION FUNCTION hyperg(m,n,k,x)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
C
      TERM=1.DO
      IF (IM.EQ.0) GO TO 200
      DO 100 I=1,IM
        A1=DFLOAT((M+1-I)*(N+1-I))
        A2=DFLOAT((K-1+I)*I)
        TERM=TERM*X*A1/A2
        HYPERG=HYPERG+TERM
100  CONTINUE
200  RETURN
      END
C
C   Program to calculate N!
C
      DOUBLE PRECISION FUNCTION FAC(N)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
      FAC=1.DO
      IF (N.EQ.0) GO TO 9000
      IF (N.GT.0) GO TO 8000
      FAC=0.DO
      GO TO 9000
8000  X=0.DO
      DO 8001 N=N,1,N
        X=X+1.DO
8001  FAC=FAC*X
9000  RETURN
      END
C
C   Evaluate 6J array See Richard Zare's book on angular momentum
C
      DOUBLE PRECISION FUNCTION SIXJ(X1,X2,X3,X4,X5,X6)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
      DIMENSION FL(400)
      COMMON/FACLOG/FL
C
      SIXJ=0.DO
C
      AA1=(-X1+X2+X3)*(X1-X2+X3)*(X1+X2-X3)
      IF (AA1.LT.0.DO) GO TO 1000
      AA2=(-X1+X5+X6)*(X1-X5+X6)*(X1+X5-X6)
      IF (AA2.LT.0.DO) GO TO 1000
      AA3=(-X4+X2+X6)*(X4-X2+X6)*(X4+X2-X6)
      IF (AA3.LT.0.DO) GO TO 1000
      AA4=(-X4+X5+X3)*(X4-X5+X3)*(X4+X5-X3)
      IF (AA4.LT.0.DO) GO TO 1000
C
      NL=400
      FL(1)=0.DO
      X=1.DO
      DO 100 I=2,NL
        X=X+1.DO
100  FL(I)=FL(I-1)+DLOG(X)
C
      I1=IDINT((X1+X2+X3))
      FRAC=X1+X2+X3-DFLOAT(I1)
      IF (DABS(FRAC).GT.1.D-9) GO TO 900
      I2=IDINT((X1+X5+X6))
      FRAC=X1+X5+X6-DFLOAT(I2)
      IF (DABS(FRAC).GT.1.D-9) GO TO 900

```

```

      I3=IDINT((X4+X2+X6))
      FRAC=X4+X2+X6-DFLOAT(I3)
      IF (DABS(FRAC).GT.1.D-9) GO TO 900
      I4=IDINT((X4+X5+X3))
      FRAC=X4+X5+X3-DFLOAT(I4)
      IF (DABS(FRAC).GT.1.D-9) GO TO 900
C
      K1=IDINT((X2+X3+X5+X6))
      K2=IDINT((X1+X3+X4+X6))
      K3=IDINT((X1+X2+X4+X5))
C
      KMIN=MAX0(I1,I2,I3,I4)
      KMAX=MIN0(K1,K2,K3)
C
      IF (KMIN.GT.KMAX) GO TO 1000
C
      I2X1=IDINT((2.D0*X1+0.1D0))
      I2X2=IDINT((2.D0*X2+0.1D0))
      I2X3=IDINT((2.D0*X3+0.1D0))
      I2X4=IDINT((2.D0*X4+0.1D0))
      I2X5=IDINT((2.D0*X5+0.1D0))
      I2X6=IDINT((2.D0*X6+0.1D0))
C
C   WRITE(6,6211) X1,X2,X3,X4,X5,X6
C6211 FORMAT(6F5.1)
C   WRITE(6,6212) I2X1,I2X2,I2X3,I2X4,I2X5,I2X6
C6212 FORMAT(6I5)
C
      EXPON=0.5D0*(FACL(I1-I2X1)+FACL(I1-I2X2)+FACL(I1-I2X3)
+      +FACL(I2-I2X1)+FACL(I2-I2X5)+FACL(I2-I2X6)
+      +FACL(I3-I2X4)+FACL(I3-I2X2)+FACL(I3-I2X6)
+      +FACL(I4-I2X4)+FACL(I4-I2X5)+FACL(I4-I2X3)
+      -FACL(I1+1)-FACL(I2+1)-FACL(I3+1)-FACL(I4+1))
+      +FACL(KMIN+1)
+      -FACL(KMIN-I1)-FACL(KMIN-I2)-FACL(KMIN-I3)-FACL(KMIN-I4)
+      -FACL(K1-KMIN)-FACL(K2-KMIN)-FACL(K3-KMIN)
      TERM=DFLOAT((-1)**KMIN)*DEXP(EXPON)
C
C   WRITE(6,6222) KMIN,TERM
C6222 FORMAT(I5,F20.9)
C
      SIXJ=TERM
C
      IF (KMAX.EQ.KMIN) GO TO 1000
C
      SUM=1.D0
      TERM=1.D0
      DO 200 K=(KMIN+1),KMAX
      XK=DFLOAT((K+1)*(K1-K+1)*(K2-K+1)*(K3-K+1))
      YK=DFLOAT((K-I1)*(K-I2)*(K-I3)*(K-I4))
      TERM=-TERM*XK/YK
      SUM=SUM+TERM
200 CONTINUE
C
      SIXJ=SIXJ*SUM
      GO TO 1000
C
C 900 CONTINUE
900 WRITE(6,6111)
6111 FORMAT('      Sum of J values in a triangle of 6j noninteger ')
      WRITE(6,6112) X1,X2,X3,X4,X5,X6
6112 FORMAT('      J values: ', 6F10.3)
C
1000 RETURN
      END
C
C

```

```

C      DOUBLE PRECISION FUNCTION FACL(I)
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
C      DIMENSION FL(400)
C      COMMON/FACLOG/FL
C
C      IF (I.LE.0) FACL=0.D0
C      IF (I.GE.1) FACL=FL(I)
C
C      RETURN
C      END
C
C      SUBROUTINE ASC2(N,NDIM,INDEX,WAV)
C
C      Subroutine to arrange calculated spectrum in order of wave number.
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z), INTEGER(I-N)
C      DIMENSION INDEX(NDIM),WAV(NDIM)
C      DO 50 I=1,NDIM
50    INDEX(I)=I
C      IF (N.EQ.1) RETURN
C      ILIM=N-1
C      DO 300 I=1,ILIM
C      JLIM=I+1
C      DO 300 J=JLIM,N
C      IF (WAV(J).GE.WAV(I)) GO TO 300
C      TEMP=WAV(I)
C      WAV(I)=WAV(J)
C      WAV(J)=TEMP
C      ITEMP=INDEX(I)
C      INDEX(I)=INDEX(J)
C      INDEX(J)=ITEMP
C      300 CONTINUE
9800 RETURN
C      END

```

Appendix C

Supplemental Calculations

C.1 Early Estimates of Stark Broadening

It is interesting to revisit the calculations of the effects of line broadening with regard to the detectability of RRLs as considered by van de Hulst. We try to reproduce his calculations below following the suggestions of Sullivan (1982).

The full width of a Gaussian line at half-intensity, $\Delta\nu_T$, due to thermal broadening is given by

$$\Delta\nu_T = \nu_0 \left(4 \ln 2 \frac{2kT}{Mc^2} \right)^{1/2}, \quad (\text{C.1})$$

where ν_0 is the rest frequency of the line and M is the mass of the radiating atom or molecule. Working from the van de Hulst (1945) and the Inglis and Teller (1939) papers, we derived the simplified expression for the Stark width, $\Delta\nu_{vdh}$, calculated by van de Hulst to be

$$\Delta\nu_{vdh} \approx \nu_0 \left(\frac{3 \times 10^6}{\nu_0} \right)^{3/5}, \quad (\text{C.2})$$

where we have used the inverted exponent suggested by Sullivan. In this case, the correct, simplified expression for the Stark width would be

$$\Delta\nu_S \approx \nu_0 \left(\frac{3 \times 10^6}{\nu_0} \right)^{5/3}. \quad (\text{C.3})$$

The generalized expression for the line-to-continuum ratio used by van de Hulst to estimate the detectability of RRLs was

$$\frac{I_L}{I_C} = \frac{\nu_0}{\Delta\nu} \frac{h\nu_0}{kT} g, \quad (\text{C.4})$$

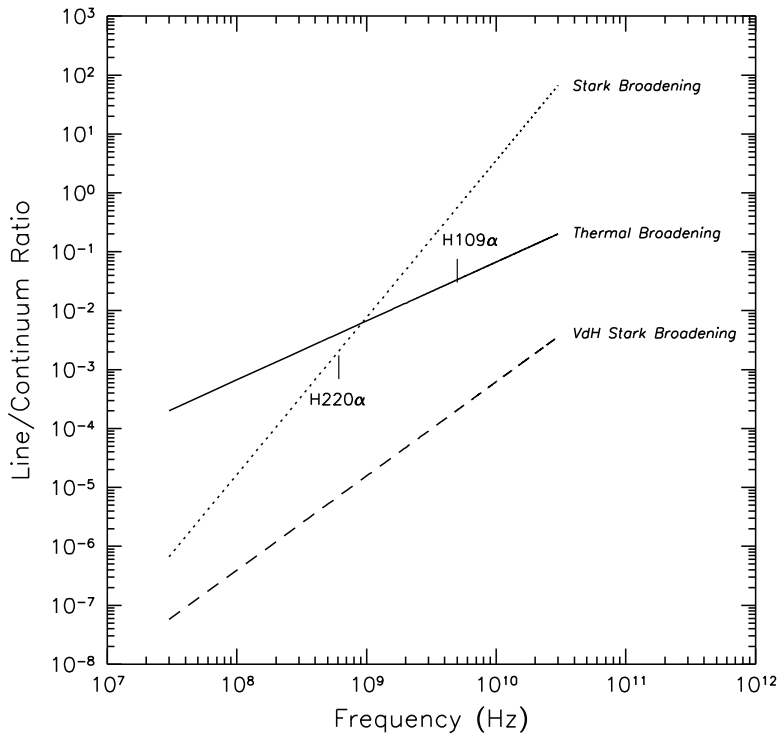


Fig. C.1 The line/continuum ratio of $Hn\alpha$ RRLs plotted against frequency. Calculations are based upon approximations used by van de Hulst (1945). *Solid line* assumes that line broadening is only thermal for a 10^4 K environment. *Dashed line* assumes Stark broadening estimated by the alleged “incorrect” formula with the inverted exponent of $3/5$. *Dotted line*: same Stark broadening formula but with the “correct” exponent, i.e., $5/3$. Also shown are observations of the H220 α and H109 α RRLs from the Orion nebula with appropriate *error bars*

where g is the ratio of the appropriate Gaunt factors and was taken to be ≈ 0.1 . Substituting the above expressions for $\Delta\nu$ and plotting the results gives Fig. C.1.

Inspection shows that the I_L/I_C ratio is indeed low for the calculation of Stark broadening using the $3/5$ exponent, in fact, lower than the thermal case by at least two orders of magnitude in the radio wavelength regime from, say, 10^7 to 10^{11} Hz. Furthermore, the VdH values of I_L/I_C are so low to constitute unrealistic detection prospects for equipment available in 1945 when the paper appeared. From the presumed calculations shown here, we can easily understand why van de Hulst rejected the possibility of detecting RRLs.

On the other hand, his approximate calculations make sense if we “correct” them by inverting the exponent to $5/3$. In these calculations, Fig. C.1 shows that thermal broadening dominates the line widths at frequencies above

900 MHz. Actual observations of the H220 α and H109 α lines from the Orion nebula fall in the appropriate positions. The former falls below the thermal line and the latter falls on that line. Stark broadening diminishes the I_L/I_C ratio for the H220 α line but has virtually no effect on the H109 α line.

We conclude that Sullivan's claim may be correct. An accidental inversion of an exponent would have changed van de Hulst's conclusion of whether or not RRLs would be detectable in radio astronomy.

C.2 Refinements to the Bohr Model

While the equations derived in Sect. 1.3 describe the salient features of recombination spectra, they need additional refinements for generality. For example, the electron orbits need not be circular in a classical sense; elliptical orbits are also possible. von Sommerfeld (1916a; 1916b) extended Bohr's work by generalizing the quantization of angular momentum to radial and to azimuthal coordinates:

$$L_r = n_r \frac{h}{2\pi}; \quad n_r = 0, 1, \dots \quad (\text{C.5})$$

and

$$L_\phi = k \frac{h}{2\pi}; \quad k = 0, 1, \dots, \quad (\text{C.6})$$

where n_r and k are called the radial and azimuthal quantum numbers, respectively. Accordingly, the diameters of the major and minor axes of an elliptical electron orbit can be written, respectively,

$$2a = \frac{2h^2}{4\pi^2 m_R e^2} \frac{n^2}{Z} \quad (\text{C.7})$$

and

$$2b = \frac{2h^2}{4\pi^2 m_R e^2} \frac{nk}{Z}, \quad (\text{C.8})$$

where $n = n_r + k$. Dividing (C.7) by (C.8) shows the axial ratio $2a/2b = n/k$.

While, classically, the energy of the orbiting electron does not depend upon the ellipticity and is still given by (1.14) after the substitution of the reduced mass m_R for m , the application of relativity (von Sommerfeld, 1916a) leads to a slight modification:

$$\begin{aligned} E_{n,k} &= -\frac{2\pi^2 m_R e^4}{h^2} \frac{Z^2}{n^2} \left[1 + \frac{\alpha^2 Z^2}{n} \left(\frac{1}{k} - \frac{3}{4n} \right) + \dots \right] \\ &\approx -2.17987 \times 10^{-11} \left(1 - \frac{5.48580 \times 10^{-4}}{M_A} \right) \frac{Z^2}{n^2} \end{aligned} \quad (\text{C.9})$$

$$\times \left[1 + \frac{5.32514 \times 10^{-5} Z^2}{n} \left(\frac{1}{k} - \frac{3}{4n} \right) \right] \quad \text{ergs}, \quad (\text{C.10})$$

where M_A is the mass of the atom in amu and the calculated dimensionless “fine-structure constant” is

$$\alpha = \frac{2\pi e^2}{hc} = 7.29735337 \times 10^{-3}, \quad (\text{C.11})$$

compared with the currently accepted value of $7.297352533 \times 10^{-3}$ (Audi and Wapstra, 1995).

The spectral line frequencies would then result in the usual way from

$$\nu = \frac{E_{n_2} k_2 - E_{n_1} k_1}{h}. \quad (\text{C.12})$$

The higher-order terms of α^2 in (C.9) are usually very small and can be neglected.

Appendix D

Hydrogen Oscillator Strengths

D.1 Population of Atomic Sublevels

Excited states of the hydrogen atom can be characterized by three quantum numbers: the principal quantum number n , the orbital quantum number ℓ , and the magnetic quantum number m . Because spectral lines can originate between any of these sublevels, the oscillator strengths of the sublevels involve these quantum numbers.

The simple oscillator strengths given by (2.111) and (2.112) involve only the principal quantum numbers n . They presume an LTE distribution of atoms in the sublevels ℓ and m . It is important to know whether these distributions are adequate for cosmic RRLs, i.e., whether our simple oscillator strengths characterized only by n are sufficiently accurate to analyze these RRLs.

The number of sublevels in each principal quantum state is

$$N(n) = \frac{n^2}{2\ell + 1} N(n, \ell), \quad (\text{D.1})$$

where $\ell = 0, 1, \dots, n - 1$ and $m = -\ell, -\ell + 1, \dots, +\ell$. If the population of these states is determined by collisions, each of these states will be populated, and we can characterize them as being in LTE and confidently assume that our simple oscillator strengths are sufficient.

On the other hand, if radiative processes dominate the population of the sublevels, they may not be fully populated, the LTE approximation will not hold, and our simple oscillator strengths will be incorrect.

Appropriate calculations can resolve this situation. Pengelly and Seaton (1964) have compared the rate of collisional transitions (protons, or H^+ s) between the sublevels of H I atoms (e.g., $n_2, \ell_2 \rightarrow n_1, \ell_1$) with the rate of radiation transitions ($n_2, \ell_2 \rightarrow n_1, \ell_2 \pm 1$) for a planetary nebula (an H II region) at 10^4 K. The critical density is where the rates are equal. Above this density, the sublevels will be fully populated and our simple approximation will be correct.

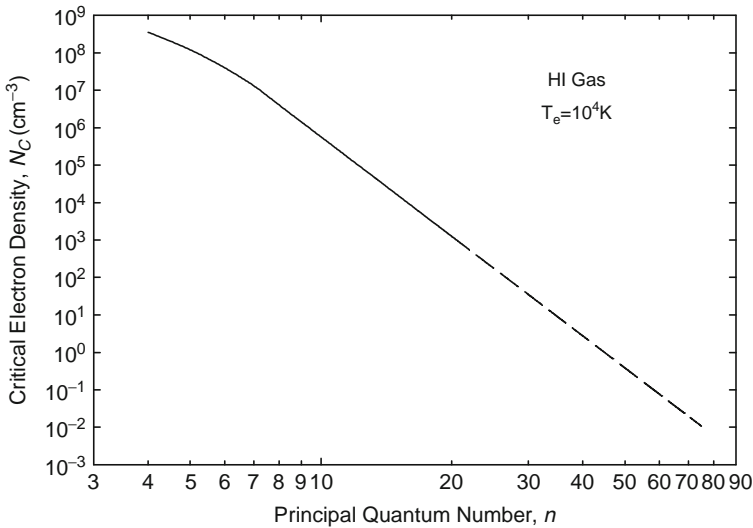


Fig. D.1 The minimum electron density required to thermalize the quantum sublevels ℓ and m in a hydrogen gas at 10^4 K. The *dashed part of the curve* indicates the extrapolation from the actual calculations. After Pengelly and Seaton (1964)

Figure D.1 shows the results for hydrogen. The situation is similar for singly ionized helium. The critical density N_c required to populate the quantum sublevels ℓ and m fully in a typical H II region has a large range depending upon the principal quantum number. For $n = 75$ corresponding to an H α RRL with a frequency of 15.2 GHz, a critical density of only 0.01 cm^{-3} is sufficient to thermalize the sublevels. In contrast, a density of $2,000 \text{ cm}^{-3}$ is required to thermalize the sublevels of the H20 α RRL that occurs at 764 GHz in the submillimeter wavelength range.

We conclude that the sublevels of hydrogen atoms in H II regions are thermally populated throughout the radio range, even into the submillimeter range. In this regime, collisional redistribution among ℓ -states occurs faster than radiative depopulation, establishing an LTE population of the sublevels even though the principal quantum states n themselves may *not* be in LTE. The situation could be very different for H II region RRLs in the near-IR and optical ranges and for RRLs from low-density environments.

D.2 Calculation of Oscillator Strengths

Various texts on atomic theory give formulas for calculating oscillator strengths for hydrogen and for hydrogenic atoms such as singly ionized helium. In general, all of these derive from Menzel and Pekeris (1935). In

particular, Goldwire (1968) discusses in detail the methodology involved in these calculations and tabulates the level-averaged absorption oscillator strengths $f(n_1, n_2)$ for a wide range of principal quantum numbers. As before, $n_2 > n_1$.

The details are as follows. Using the results of Pengelly and Seaton (1964), Goldwire calculated the level-averaged absorption oscillator strength as

$$f(n_1, n_2) n_1^2 = \sum_{\ell_1=1}^{n_1-1} [(2\ell_1 + 1)f(n_1, \ell_1; n_2, \ell_1 - 1)] \\ + \sum_{\ell_1=0}^{n_1-1} [(2\ell_1 + 1)f(n_1, \ell_1; n_2, \ell_1 + 1)], \quad (\text{D.2})$$

where the oscillator strengths of the sublevels are weighted by $(2\ell_1 + 1)/n_1^2$ to account for a full population, i.e., LTE. The sublevel absorption oscillator strengths result from

$$f(n_1, \ell_1; n_2, \ell_2) = \frac{1}{2\ell_1 + 1} \sum_{m_1=-\ell_1}^{\ell_1} \sum_{m_2=-\ell_2}^{\ell_2} f(n_1, \ell_1, m_1; n_2, \ell_2, m_2) \quad (\text{D.3}) \\ = \frac{1}{3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \frac{\max(\ell_1, \ell_2)}{2\ell_1 + 1} \frac{1}{a_0 t^2} \\ \times [\Re_{ZM}(n_2, \ell_2; n_1, \ell_1)]^2, \quad (\text{D.4})$$

where the first Bohr radius of the hydrogenic system is

$$a_0 t \equiv \left(\frac{R_\infty}{R_M} \right) \frac{a_0}{Z} \quad (\text{D.5})$$

and a_0 , M , and Z are the Bohr radius of hydrogen, the species mass, and the net electronic charge, respectively. Gordon (1929) gave equations to evaluate the radial matrix integrals of $\Re_{ZM}(n_2, \ell_2; n_1, \ell_1)$; these are also given by (4) and (5) of Goldwire (1968) or may be found in any standard text on atomic theory. In particular, Hoang-Binh (1990) has found a simple method of evaluating them given in Appendix D.3.

The formal, direct equation given by (D.2) is necessary for calculations involving alternative weightings of the degenerate sublevels. However, other expressions for oscillator strengths are available for LTE. Menzel and Pekeris (1935) give a parallel formula for the thermalized emission oscillator strength $f(n_2, n_1)$ of hydrogen in their (1.15). Perhaps, the simplest expression to use is a modified form of an expression given by Kardashev (1959), giving the thermalized absorption oscillator strength as

$$f(n_1, n_2) = \frac{2}{3} n_2^2 \frac{(4n_2 n_1)^{2n_1+2} (n_2 - n_1)^{2n_2-2n_1-4}}{(n_2 + n_1)^{2n_2+2n_1+3}} A, \quad (\text{D.6})$$

where the coefficient A is

$$A \equiv \begin{pmatrix} n_2 - 1 \\ n_2 - n_1 - 1 \end{pmatrix}^2 F^2(-n_1, -n_1, n_2 - n_1; x^{-1}) - \begin{pmatrix} n_2 \\ n_2 - n_1 + 1 \end{pmatrix}^2 x^{-2} F^2(-n_1 + 1, -n_1 + 1, n_2 - n_1 + 2; x^{-1}). \quad (\text{D.7})$$

In (D.7), $F(a, b, c; z)$ is a hypergeometric function described in many mathematical reference books such as Abramowitz and Stegun (1964), and

$$x \equiv -\frac{4n_2n_1}{(n_2 - n_1)^2}. \quad (\text{D.8})$$

Goldwire (1968) notes that the excellent agreement between LTE level-averaged absorption oscillator strengths derived from (D.2) and (D.6) confirms the accuracy of the latter expression.

D.3 FORTRAN Code for Evaluating Radial Matrix Integrals

FORTRAN code for calculating radial integrals for hydrogen is listed below (Hoang-Binh, 1990).

D.3.1 Radial Matrix Integrals $\mathfrak{R}_H(n_2, \ell_1 - 1; n_1, \ell_1)$

```

c   * Program AS-RFA
c   * Written by D. Hoang-Binh, 1990 Astron. Astrophys. 238:449
*-----
c   * Hydrogen atom, Z= 1
c   * This program computes the radial integral
c   * R= R(nup, lu; nlo, lo)= ain; R**2= ain2
c   * nup= principal quantum number of upper state
c   * nlo= principal quantum number of lower state
c   * lu= orbital quantum number of upper state
c   * lo= orbital quantum number of lower state
c   * lu= lo-1
c   * SUBROUTINE FA(nlo,nup,lu,ain,ain2)
*-----
c   * INPUT:  nlo, lo, ni, nf, inup (file='AS.r')
c   * ni= initial value of nup
c   * nf= final value of nup
c   * inup= step of increase of nup
c   * OUTPUT: R**2, f, A; for nup= ni (inup) nf (file='AS.dat')
           f= absorption oscillator strength
           A= Einstein coefficient
*-----
           RH= 109677.576
           RKAY= RH

```

```

        iunit=5
        iread=2
        open(file='AS.dat',unit=iunit)
        open(file='AS.r',unit=iread)
*-----
1   format('NLO,LO, NUP,LUP,R**2,f, A' //)
2   format('AS-RFA'/)
5   format(2(i4,3x), 2(1pe11.4,3x))
6   format (6(i4,3x))
7   format('nlo, lo, ni, nf, inup')
9   format('INPUT'/)
11  format('/OUTPUT')
55  format(4(i4,3x), 4(1pe11.4,3x))
*-----
        read (iread,6) nlo, lo, ni, nf, inup
*-----
        write(iunit,2)
        write (iunit,9)
        write (iunit,7)
        write (iunit,6) nlo, lo, ni, nf, inup
        write(iunit,11)
        write(iunit,1)
        write(9,1)
*-----
        lu=lo-1
        do 33 nup = ni, nf, inup
        call FA(nlo,nup,lu,ain,ain2)
*-----
        XUP=NUP
        XLO=NLO
        EUP=RKAY-RKAY/XUP**2
        EL0=RKAY-RKAY/XLO**2
        OS=1./3.*(XUP+XLO)*(XUP-XLO)/(XUP*XLO)**2*MAX(LU,LO)/
           2 (2.*LO+1.)*AIN2
        EIN=0.66704*(2.*LO+1.)/(2.*LU+1.)*(EUP-EL0)**2*OS
*-----
        write(9,55) NLO,LO, NUP,LU, AIN2, OS, EIN
        write(iunit,55) NLO,LO, NUP,LU, AIN2, OS, EIN
33  CONTINUE
*-----
        stop
        end

SUBROUTINE FA(n,np,lp,ain,ain2)
c   * RECURRENCE ON A

        dimension h(1100)

        l=lp+1
        B0= 0.-np+1+0.
        C0=2.*1

        B=B0
        C = C0
        cc4=1.*(n-np)
        X=-4.*N*NP/cc4**2

        h(1)=1.
        h(2)= 1.-B*X/C
        e1=0
        e2=e1
        i1=(n-1+1)

        do 4 i=2,i1
        j=i+1
        A=1.-i+0.
        H1=-A*(1.-X)*h(j-2) /(A-C)

```

```

H2=(A*(1.-X)+(A+B*X-C))*h(j-1)/(A-C)
h(j)= H1+H2
if(abs(h(j)) .gt. 1.e+25) go to 30
go to 4

30  continue
    h(j)=h(j)/1.e+25
    h(j-1)=h(j-1)/1.e+25
    h(j-2)=h(j-2)/1.e+25
    e1=e1+25.
    e2=e1

4    continue
    p1=h(i1-1)
    p2 = h(i1+1)

    cc4=1.*(n-np)
    cc4= abs(cc4)
    cc5=n+np
    FF= P1*(1.-cc4**2/cc5**2*P2/P1*10.**(e2-e1))
    alof=log10(ABS(FF))+e1

    i2=(2*L-1)
    S1=0.

    do 6 I=1,i2
        ai=i
        S1=S1+ alog10(ai)
6    continue

    C1= - (alog10(4.)+S1)
    S=0.

    i3=(N+L)
    si3=0.

    do 7 I=1,i3
        ai=i
        si3=si3+ alog10(ai)
7    continue

    S=S+si3

    i4=(NP+L-1)
    si4=0.

    do8 I=1,i4
        ai=i
        si4=si4+ alog10(ai)
8    continue

    S=S+si4

    i5=n-1-1
    si5=0.
    if(i5 .eq. 0) go to 2

    do 9I=1,i5
        ai=i
        si5=si5+ alog10(ai)
9    continue

    S=S-si5

2    continue

    i6=np-1

```

```

    si6=0.
    if(i6 .eq. 0) go to 3

    do 12 I=1,i6
    ai=i
    si6=si6+ alog10(ai)
12  continue

    S=S-si6

3   continue
    C2=S/2.
    cc3=4.*n*np
    C3= (1+1.)*alog10(cc3)

    cc4=cc4
    C4= (N+NP-2.*L-2.)*alog10(cc4)

    cc5=n*np
    C5=(-N-NP)*alog10(cc5)

    C=C1+C2+C3+C4+C5

    ali =alof+C
    ain = 10.**ali
    ain2=ain**2

    return
    end

*-----
AS.r
0006  0001  0007  0020  0001
read (iread,6) nlo, lo, ni, nf, inup
*-----

*-----
AS.dat
AS-RFA

INPUT

nlo, lo, ni, nf, inup
  6      1      7      20      1
read (iread,6) nlo, lo, ni, nf, inup

OUTPUT
NLO,LO, NUP,LUP,R**2,f, A

  6      1      7      0  1.1811E+02  9.6717E-02  1.2644E+05
  6      1      8      0  1.6873E+01  2.2784E-02  8.1001E+04
  6      1      9      0  5.5390E+00  9.4976E-03  5.4447E+04
  6      1     10      0  2.5575E+00  5.0519E-03  3.8434E+04
  6      1     11      0  1.4192E+00  3.0770E-03  2.8203E+04
  6      1     12      0  8.8253E-01  2.0429E-03  2.1344E+04
  6      1     13      0  5.9271E-01  1.4397E-03  1.6561E+04
  6      1     14      0  4.2070E-01  1.0600E-03  1.3120E+04
  6      1     15      0  3.1129E-01  8.0704E-04  1.0577E+04
  6      1     16      0  2.3789E-01  6.3096E-04  8.6552E+03
  6      1     17      0  1.8656E-01  5.0407E-04  7.1753E+03
  6      1     18      0  1.4942E-01  4.0993E-04  6.0160E+03
  6      1     19      0  1.2180E-01  3.3844E-04  5.0950E+03
  6      1     20      0  1.0078E-01  2.8305E-04  4.3536E+03
*-----

```

D.3.2 Radial Matrix Integrals $\mathfrak{R}_H(n_2, \ell_1 + 1; n_1, \ell_1)$

```

c   * Program BS-RFA
c   * Written by D. Hoang-Binh, 1990 Astron. Astrophys. 238:449
*-----
c   * Hydrogen atom, Z= 1
c   * This program computes the radial integral
c   * R= R(nup, lu; nlo, lo)= ain; R**2= ain2
c   * nup= principal quantum number of upper state
c   * nlo= principal quantum number of lower state
c   * lu= orbital quantum number of upper state
c   * lo= orbital quantum number of lower state
c   * lu=lo+1
c   * SUBROUTINE FR(nup,nlo,lo,ain,ain2)
*-----
c   * INPUT: nlo, lo, ni, nf, inup (file='BS.r')
c   * ni= initial value of nup
c   * nf= final value of nup
c   * inup= step of increase of nup

c   * OUTPUT: R**2, f, A; for nup= ni (inup) nf (file='BS.dat')
f= absorption oscillator strength
A= Einstein coefficient
*-----
      RH= 109677.576
      RKAY= RH
*-----
      iunit=5
      iread=2
      open(file='BS.dat',unit=iunit)
      open(file='BS.r',unit=iread)
*-----
1    format(' NLO,LO,NUP,LU,R2,f, A'/)
2    format('BS-RFA'/)
5    format(2(i4,3x), 2(1pe11.4,3x))
6    format( 6(i4,3x))
7    format('nlo, lo, ni, nf, inup')
9    format('INPUT'/)
11   format('/OUTPUT'/)
55   format(4(i4,3x), 4(1pe11.4,3x))
*-----
      read (iread,6) nlo, lo, ni, nf, inup
*-----
      write(iunit,2)
      write (iunit,9)
      write (iunit,7)
      write (iunit,6) nlo, lo, ni, nf, inup
      write(iunit,11)
      write(9,1)
      write(iunit,1)
*-----
      lu=lo+1
      do 33 nup = ni, nf, inup
      CALL FR(nup,nlo,lo,ain,ain2)
*-----
      XUP=NUP
      XLO=NLO
      EUP=RKAY-RKAY/XUP**2
      ELO=RKAY-RKAY/XLO**2
      OS=1./3.*(XUP+XLO)*(XUP-XLO)/(XUP*XLO)**2*MAX(LU,LO)/
        2 (2.*LO+1.)*AIN2
      EIN=0.66704*(2.*LO+1.)/(2.*LU+1.)*(EUP-ELO)**2*OS
*-----
      write(9,55) NLO,LO, NUP,LU, AIN2, OS, EIN
      write(iunit,55) NLO,LO, NUP,LU, AIN2, OS, EIN
33  CONTINUE

```

```

*-----
      stop
      end

      subroutine FR(n,np,lp,ain,ain2)
c  RECURRENCE ON B
      dimension h(1100)
      l=lp+1
      A01=-n+1+1.
      A02=A01-2.
      B0=-np+1
      C0=2.*1
      c = C0
      X=-4.*N*NP/(N-NP)**2
      B=A01
      e1=0.
      h(1)=1.
      h(2)=1.-B*X/C
      i1=(NP-L)
      if(i1 .eq. 0) go to 40
      if(i1 .eq. 1) go to 41

      do 4 I=2,i1
        j=i+1
        A=-I+1.
        H1=-A*(1.-X)*h(j-2)/(A-C)
        H2=(A*(1.-X)+(A+B*X-C))*h(j-1)/(A-C)
        h(j)= H1+H2
        if(abs(h(j)) .gt. 1.e+25) go to 30
        go to 4

30    continue
      h(j)=h(j)/1.e+25
      h(j-1)=h(j-1)/1.e+25
      e1=e1+25.

4     continue
      P1=h(i1+1)
      go to 50

40    continue
      p1=h(1)
      go to 50

41    continue
      p1=h(2)
      go to 50

50    continue
      B=A02
      E2=0.
      h(1)=1.
      h(2)=1.- B*X/C

      i1=(NP-L)
      if(i1 .eq. 0) go to 42
      if(i1 .eq. 1) go to 43

      do 5 I=2,i1
        j=i+1
        A=-I+1.
        H1=-A*(1.-X)*h(j-2)/(A-C)
        H2=(A*(1.-X)+(A+B*X-C))*h(j-1)/(A-C)
        h(j)= H1+H2
        if(abs(h(j)) .gt. 1.e+25) go to 31
        go to 5

```



```

31  continue
    h(j)=h(j)/1.e+25
    h(j-1)=h(j-1)/1.e+25
    e2=e2+25.

5   continue
    P2=h(i1+1)
    go to 51

42  continue
    p2=h(1)
    go to 51

43  continue
    p2=h(2)
    go to 51

51  continue
    cc4= n-np
    cc5=n+np
    FF= P1*(1.-cc4**2/cc5**2*P2/P1*10.**(e2-e1))
    alof=log10(ABS(FF))+e1

c   REM CAL OF C1, C2, C3, C4, C5
    i2=(2*L-1)
    S1=0.

    do 6 I=1,i2
        ai=i
        S1=S1+ alog10(ai)
6   continue

    C1= - (alog10(4.)+S1)

    S=0.
    i3=(N+L)
    si3=0.

    do 7 I=1,i3
        ai=i
        si3=si3+ alog10(ai)
7   continue

    S=S+si3
    i4=(NP+L-1)
    si4=0.

    do8 I=1,i4
        ai=i
        si4=si4+ alog10(ai)
8   continue

    S=S+si4
    i5=n-l-1
    si5=0.

    if(i5 .eq. 0) go to 2
    do 9I=1,i5
        ai=i
        si5=si5+ alog10(ai)
9   continue

    S=S-si5
2   continue

    i6=i1
    si6=0.

```

```

      if(i6 .eq. 0)go to 3

      do 12I=1,i6
      ai=i
      si6=si6+ alog10(ai)
12  continue

      S=S-si6

3   continue
      C2=S/2.
      cc3=4.*n*np
      C3= (1+1.)*alog10(cc3)
      cc4=n-np
      cc4= abs(cc4)
      C4= (N+NP-2.*L-2.)*alog10(cc4)
      cc5=n+np
      C5=(-N-NP)*alog10(cc5)
      C=C1+C2+C3+C4+C5

      ali =alof+C
      ain =i0.**ali
      ain2=ain**2
      return
      end
*-----

*-----
BS.r
0006   0000   0007   0020   0001
*-----

*-----
BS-RFA

INPUT

nlo, lo, ni, nf, inup
  6      0      7      20      1

OUTPUT
NLO,LO,NUP,LU,R**2,f, A

      6      0      7      1      2.7419E+02      6.7356E-01      9.7844E+04
      6      0      8      1      4.2342E+01      1.7153E-01      6.7756E+04
      6      0      9      1      1.4544E+01      7.4817E-02      4.7656E+04
      6      0     10      1      6.9164E+00      4.0986E-02      3.4646E+04
      6      0     11      1      3.9173E+00      2.5480E-02      2.5949E+04
      6      0     12      1      2.4725E+00      1.7170E-02      1.9932E+04
      6      0     13      1      1.6793E+00      1.2237E-02      1.5641E+04
      6      0     14      1      1.2023E+00      9.0880E-03      1.2499E+04
      6      0     15      1      8.9579E-01      6.9673E-03      1.0146E+04
      6      0     16      1      6.8837E-01      5.4775E-03      8.3485E+03
      6      0     17      1      5.4230E-01      4.3958E-03      6.9525E+03
      6      0     18      1      4.3599E-01      3.5884E-03      5.8513E+03
      6      0     19      1      3.5653E-01      2.9720E-03      4.9712E+03
      6      0     20      1      2.9577E-01      2.4922E-03      4.2591E+03
*-----

```

Appendix E

Departure Coefficients

E.1 FORTRAN Code for Calculating b_n Values

This listing is a program for calculating departure coefficients and values of β for hydrogenic atoms. Brocklehurst and Salem (1977) wrote the original version in Fortran IV for the Cambridge University (England) IBM 370/165. Walmsley (1990) modified the program to calculate departure coefficients down to level $n = 10$ when needed. Gordon converted the program to standard FORTRAN 77 at the same time.

This code does not consider collisions with detailed angular momentum levels. Consequently, departure coefficients calculated by Storey and Hummer (1995) are better for small quantum numbers and low electron densities, as described in Sect. 2.3.10.5. Where the principal quantum levels are completely degenerate, i.e., at quantum numbers appropriate to meter, decimeter, and centimeter wavelengths, and for the electron densities found in most H II regions, agreement between departure coefficients produced by the two codes is excellent.

The program elements are:

| | |
|--------------|---|
| DATAIN.TXT | Input data for above (see Brocklehurst and Salem reference above for details) |
| BNMAIN2.FOR | Source code for main program |
| BNFNCTN.FOR | Source code for internal functions |
| BNSBRTN2.FOR | Source code for internal subroutines |

Typically, modern PCs with GHz-clock rates will run the executable code in a few seconds. Calls to the subroutines will appear on the PC screen to advise of progress.

Multiple cases given in DATAIN.TXT will appear sequentially in the output file DATAOUT.TXT.

The output appears in a ASCII file on the A: drive entitled DATAOUT.TXT. This file must be renamed or deleted for each new execution because the output file specification is "NEW."

Recomputing some of the tables published by Salem and Brocklehurst (1979) generally verified the output. The results should also agree with the coefficients published by Walmsley (1990), who used the VAX Fortran version of this program.

```

      program bsubn
c
c  This program has been modified to execute in FORTRAN 77 on an
c  IBM PC or compatible, using Microsoft Fortran4.0. The original
c  JCL cards used to run it on an IBM370/165 have been commented
c  with small c's, and new code has usually been written in lower case.
c      M.A. Gordon, March 1988.
c
c
c  ACXIGENERAL BN PROGRAM.  RADIO RECOMBINATION LINES FROM H REGIONS AND  ACXI0000
c1  COLD INTERSTELLAR CLOUDS: COMPUTATION OF THE BN FACTORS.                ACXI0000
c2  M. BROCKLEHURST, M. SALEM.                                              ACXI0000
cREF. IN COMP. PHYS. COMMUN. 13 (1977) 39                                ACXI0000
cJOB MS13 2355 BN PROGRAM TEST                                             ACXI0001
cROUTE PRINTER WESTCAM,POST WCAV,NOTIFY                                    ACXI0002
cMSGLEVEL=1 ACXI0003
cLIMSTORE 175K,COMP 75 SECS,PRINTER 5000                                  ACXI0004
c//ONE EXEC FTG1CLG,REGG=175K,LISTC=SOURCE,MAPL=MAP  COMPIL, LOAD, GO  ACXI0005
c//FORT.SYSIN DD *  SOURCE PROGRAMCARDS                                    ACXI0006
c      WIDE TEMPERATURE RANGE BN PROGRAM                                  ACXI0007
c      *****                                                            ACXI0008
c                                                                           ACXI0009
c  PROGRAM FOR CALCULATION OF COEFFICIENTS OF DEPARTURE FROM              ACXI0010
c  THERMODYNAMIC EQUILIBRIUM, BN, FOR HYDROGENIC ATOMS, AT ELECTRON        ACXI0011
c  TEMPERATURES FROM 10K TO 20 000K                                         ACXI0012
c
c  The original card deck, specified below, is now contained in a data
c  file named DATAIN.TXT c                                               ACXI0013
c  INPUT CARDS -                                                            ACXI0014
c  ** ALPHANUMERIC TITLE CARD.                                             ACXI0015
c  ** NORMALLY, THE FOLLOWING 6 CARDS (SEE WRITEUP FOR EXPLANATION) -      ACXI0016
c  75  2  4                                                                ACXI0017
c  30 31 32 33 34 35 37 39 41 43 46 49 52 55 58 61 64 68 72 76 80 84 88  ACXI0018
c  9297102107112217122127132138144150156162168174180187194201208215222  ACXI0019
c  C23023824625426227029288297306315325335345355365375386397408419430441 ACXI0020
c  C452463474485496507                                                    ACXI0021
c  75 72 69 66                                                            ACXI0022
c                                                                           ACXI0023
c  ** ONE CARD WITH RADIATION TEMPERATURE AND EMISSION MEASURE OF         ACXI0024
c  BACKGROUND RADIATION FIELD. FORMAT 2E10.3. IF EMISSION MEASURE READ    ACXI0025
c  IS GE 10**10, IT IS TAKEN TO BE INFINITE. THIS CARD IS READ BY        ACXI0026
c
c  FUNCTION COR(N,ISW). IF THIS SUBPROGRAM IS REPLACED BY THE USER, ANY  ACXI0027
c  CARDS READ BY COR WHEN CALLED WITH ISW=0 SHOULD BE PLACED HERE.        ACXI0028
c  (BLANK CARD = NO FIELD).                                                ACXI0029
c                                                                           ACXI0030
c  ** (ONE CARD FOR EACH CASE TO BE CALCULATED)TEMPERATURE, DENSITY,      ACXI0031

```

```

C CASE (THIN=A - 1, THICK=B -2), PRINT CYCLE (PRINT EVERY K-TH LEVEL), ACXI0032
C NPLO AND NPHI, WHERE OUTPUT IS TO BE PUNCHED FROM N=NPLO TO NPHI ACXI0033
C (NO PUNCHED OUTPUT IF NPHI=0), ALPHANUMERIC LABEL TO BE PUNCHED IN ACXI0034
C COLUMNS 77-79 OF CARD OUTPUT. ACXI0035
C FORMAT 2E10.5, 4I5, 37X, A3 (I.E., LABEL IN COLS. 78-80) ACXI0036
C
c I've modified the code to accept one more parameter, ilim, to limit
c the print out- MAG
C
C ACXI0037
C ** A BLANK CARD (WHICH ENDS EXECUTION) ACXI0038
C
C ACXI0039
C ACXI0040
    character none,nbl,nskip
    character*8 icase
    COMMON /EXPDAT/ CXP(707),MAXN ACXI0041
    COMMON /FITDAT/ AFIT(4,4),IVAL(4),NFIT ACXI0042
    COMMON /INOUT/ IREAD,IWRITE,IPUNCH,icrt ACXI0043
    COMMON /PARMS/ DENS,T,ITM ACXI0044
    COMMON /TDEP/ TE32,TE12,CTE ACXI0045
    DIMENSION CO(75), DVAL(507), IND(75,2), IPIV(75), KBOUT(507), ACXI0046
    1 KCOUT(507), MVAL(75), SK(75,75),TITLE(10), VAL(507) ACXI0047
C
C ACXI0048
C MANY MACHINES DO NOT REQUIRE THE FOLLOWING DOUBLE PRECISION ACXI0049
C STATEMENT IN MAIN OR SUBPROGRAMS ACXI0050
C
C ACXI0051
    DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI0052
C
C ACXI0053
    DOUBLE PRECISION AFIT,ARG,CO,COR,CTE,CX,CXP,D,DENS,DVAL,H,HH,RATIO ACXI0054
    1,SK,T,T1,TE12,TE32,TITLE,VAL,X,XXXI ACXI0055
c
c DATA NONE,NBL/1H0,1H /,LPPG/45/ ACXI0056
    data none,nbl/' ',' ' /,lppg/45/
    data icase/' (Case B) ' /
c
c Open the input and output files. IREAD, IWRITE specified by BLOCK DATA
c If desired, you can write directly to your printer by commenting out
c the OPEN statement for DATAOUT.TXT and resetting the IWRITE = 6 in
c BLOCK DATA section. Then the default of UNIT = 6 will be the printer.
c
    open(unit=IREAD,file='A:DATAIN.TXT',form='FORMATTED',access=
    1'SEQUENTIAL',status='OLD')
    open(unit=IWRITE,file='A:DATAOUT.TXT',form='FORMATTED',access=
    1'SEQUENTIAL',status='UNKNOWN')
c
    write (5,190)
190 format (//, ' Begin Calculation of Departure Coefficients' /
    1' Coprocessor or Emulator version of program BSUBN2.EXE' /
    5' with Malcolm Walmsley's extensions to n < 20' /
    2' [See Salem & Brocklehurst, Ap.J.Supp. 39:633 (1979)' /
    3' Brocklehurst & Salem, Comp. Phys. Comm. 13:39 (1977)' /
    4' Adapted to FORTRAN 77 by M. A. Gordon, March 1988]' /)
c
c Read the data needed to set up the calculations c
    write (5,195)
195 format (' Read Initial Parameters from DATAIN.TXT') c
    READ (IREAD,70) TITLE ACXI0057
    READ (IREAD,80) IC,IR,NFIT ACXI0058
    READ (IREAD,80) (MVAL(I),I=1,IC) ACXI0059

```



```

210  format(1x, 'Call Colion')
      CALL COLION (0,1,T,H)
      write (icrt,220)
220  format (6x, 'Call Radcol')
      CALL RADCOL (T,MVAL,IC,NMIN)
      write(icrt,230)
30   format(11x, 'Call Reduce')
230  CALL REDUCE (MVAL,IC,IR,SK)
      write(icrt,240)
240  format(16x, 'Call RHS')
      CALL RHS (CO,MVAL,IC)
      write(icrt,250)
250  format(21x, 'Call JMD')
      CALL JMD (SK,CO,MVAL,IC)
      write(icrt,260)
260  format(26x, 'Call Matinv')
      CALL MATINV (SK,IC,CO,1,D,ERROR,75,IPIV,IND)
      write (icrt,270)
270  format(31x, 'Call Interp')
      CALL INTERP (MVAL,CO,VAL,DVAL,IC,2)
      J=MVAL(1)
      K=MVAL(IC)
c   limits the printout to ilim lines
      if (ilim.ne.0) k=ilim
      IPUN=0
      DO 60 I=J,K
      RATIO=DVAL(I)/VAL(I)
      H=T*DFLOAT(I)**3*RATIO/3.158D5
      HH=1.D0-H
      XXXI=DEXP(15.778D4/(DFLOAT(I*I)*T))*VAL(I)
      X=-HH*XXXI*100.D0/T
      IF (MOD(I,ICYC).NE.0) GO TO 50
      NSKIP=NBL
c   IF (MOD(NLINE,5).EQ.0) NSKIP=NONE
      if (mod(nline,5).eq.0) write (IWRITE,161)
      IF (MOD(NLINE,LPPG).NE.0) GO TO 40
      write(icrt,280) npage
280  format(1x, 'Writing page ',I2,' to DATAOUT.TXT')
      WRITE (IWRITE,140) T,DENS,NMIN,icase,NPAGE
      WRITE (IWRITE,150)
      NPAGE=NPAGE+1
40   NLINE=NLINE+1
c   WRITE (IWRITE,160) NSKIP,I,VAL(I),HH,DVAL(I),RATIO,H,X
      write (IWRITE,160) I,VAL(I),HH,DVAL(I),RATIO,H,X
50   IF (NPHI.EQ.0) GO TO 60
      IF (I.LT.NPLO) GO TO 60
      IF (I.GT.NPHI) GO TO 60
      IPUN=IPUN+1
      KBOUT(IPUN)=VAL(I)*1.D4
      KCOUT(IPUN)=DLOG10(RATIO)*1.D4
60   CONTINUE
      T1=T
      IF (NPHI.NE.0) CALL BCPCH (KBOUT,KCOUT,T,DENS,NMIN,LABEL,NPLO,NPHIACX10125
1,ND)
      GO TO 10
65  endfile (unit=IREAD)
      close (unit=IREAD)

```

```

        endfile (unit=IWRITE)
        close (unit=IWRITE)
        STOP 'End of BSUBN'
C
C
70  FORMAT (10A8)
80  FORMAT (23I3)
C
C  Output formats modified for 80-character line
C
C 90  FORMAT(20X,10A8/20X,20(4H****)///)
90  FORMAT (' ',10A8/24(3H***)//)
C 100 FORMAT (7H MVAL (,I3,10H VALUES) -(1X,24I5))
100 FORMAT (7H MVAL (,I3,10H VALUES) -(1X,12I5)/(1X,12I5))
C 110 FORMAT (7HOIVAL (,I3,10H VALUES) -(1X,24I5))
110 FORMAT (7H IVAL (,I3,10H VALUES) -(1X,12I5)/(1X,12I5))
120 FORMAT (5H IR =,I3)
130 FORMAT (2G10.3,5I5,36X,A3)
140 FORMAT (1H1,14H TEMPERATURE =,F6.0,14H K, DENSITY =,1PG10.3,
115HCM*-3, NMNIN = ,I3,1X,A8/32X,4HPAGE,I3)
150 FORMAT (1x,3H N,3X,2HBN,10X,6HbsBETA,5X,6HDBN/DN,5X,
112HD(LN(BN))/DN,2X,8H1-bsBETA,6X,4HZETA)
160 FORMAT (1x,I3,1P,6(G12.5))
161 FORMAT (1X)
END

BLOCK DATA
DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP
COMMON /GAUNTS/ A1(50),A2(50),A3(50),A4(50),A5(50),A6(50),A7(50),AACX10198
18(50),A9(50),A10(50),A11(50),A14(50),A17(50),A20(50),A25(50),A30(5ACX10199
20),A40(50),A50(50),A100(50),A150(50),A225(50),A500(50),IXV(12) ACX10200
COMMON /RCMB/ SVOA(33),SV0B(33),SVOC(33),SV1A(33),SV1B(33),SV1C(33ACX10201
1),SV2A(33),SV2B(33),SV2C(33) ACX10202
COMMON /GAUSS/ VALUE(12) ACX10203
COMMON /INOUT/ IREAD,IWRITE,IPUNCH,icrt ACX10204
DOUBLE PRECISION VALUE ACX10205
C
C INPUT/OUTPUT UNITS. IREAD IS CARD READER, IWRITE IS PRINTER, IPUNCH ACX10207
C IS CARD PUNCH. ACX10208
DATA IREAD,IWRITE,IPUNCH,icrt/2,3,7,6/ ACX10209
C
C NOTE - SOME COMPILERS REQUIRE AN IMPLIED DO LOOP WHEN THE VALUES OF ACX10210
C A WHOLE ARRAY ARE SET IN A DATA STATEMENT. THE FOLLOWING STATEMENTS ACX10211
C WILL NEEDCHANGING TO (A1(I),I=1,50), ETC. (THE ANS STANDARD FORTRAN ACX10212
C FORM IS EXCESSIVELY LENGTHY FOR LARGE ARRAYS.) ACX10213
C
C RADIATIVE GAUNT FACTORS ACX10215
C
C
C
DATA A1/.7166,.7652,.7799,.7864,.7898,.7918,.7931,.7940,.7946,.795ACX10218
11,.7954,.7957,.7959,.7961,.7963,.7964,.7965,.7966,.7967,.796ACX10219
28,.7968,.7968,.7969,.7969,.7969,.7970,.7970,.7970,.7970,.797ACX10220
31,.7971,.7971,.7971,.7971,.7971,.7971,.7971,.7971,.7972,.7972,.797ACX10221
42,.7972,.7972,.7972,.7972,.7972,.7972,.7972/.7972/ ACX10222
DATA A2/.7566,.8217,.8441,.8549,.8609,.8647,.8672,.8690,.8702,.871ACX10223
12,.8720,.8726,.8730,.8734,.8737,.8740,.8742,.8744,.8746,.8747,.874ACX10224
29,.8750,.8751,.8751,.8752,.8753,.8753,.8754,.8755,.8755,.8755,.875ACX10225
36,.8756,.8756,.8757,.8757,.8757,.8757,.8758,.8758,.8758,.8758,.875ACX10226

```



```

48, .8759, .8759, .8759, .8759, .8759, .8759, .8759/          ACXIO227
DATA A3/.7674, .8391, .8653, .8784, .8861, .8910, .8944, .8968, .8986, .900ACXIO228
10, .9011, .9019, .9026, .9032, .9037, .9041, .9044, .9047, .9049, .9052, .905ACXIO229
24, .9055, .9057, .9058, .9059, .9060, .9061, .9062, .9063, .9064, .9064, .906ACXIO230
35, .9066, .9066, .9067, .9067, .9067, .9068, .9068, .9068, .9069, .9069, .906ACXIO231
49, .9070, .9070, .9070, .9070, .9070, .9071, .9071/          ACXIO232
DATA A4/.7718, .8467, .8750, .8896, .8984, .9041, .9081, .9110, .9132, .914ACXIO233
19, .9163, .9173, .9182, .9190, .9196, .9201, .9205, .9209, .9213, .9215, .921ACXIO234
28, .9220, .9222, .9224, .9226, .9227, .9228, .9230, .9231, .9232, .9233, .923ACXIO235
33, .9234, .9235, .9235, .9236, .9237, .9237, .9238, .9238, .9238, .9239, .923ACXIO236
49, .9240, .9240, .9240, .9240, .9241, .9241, .9241/          ACXIO237
DATA A5/.7741, .8507, .8804, .8960, .9055, .9118, .9162, .9195, .9220, .924ACXIO238
10, .9255, .9268, .9278, .9287, .9294, .9300, .9306, .9310, .9314, .9318, .932ACXIO239
21, .9324, .9326, .9329, .9331, .9332, .9334, .9335, .9337, .9338, .9339, .934ACXIO240
30, .9341, .9342, .9343, .9344, .9344, .9345, .9345, .9346, .9347, .9347, .934ACXIO241
48, .9348, .9348, .9349, .9349, .9349, .9350, .9350/          ACXIO242
DATA A6/.7753, .8531, .8837, .8999, .9099, .9167, .9215, .9251, .9278, .930ACXIO243
10, .9317, .9331, .9343, .9352, .9361, .9368, .9374, .9379, .9384, .9388, .939ACXIO244
22, .9395, .9398, .9400, .9403, .9405, .9407, .9408, .9410, .9412, .9413, .941ACXIO245
34, .9415, .9416, .9417, .9418, .9419, .9420, .9420, .9421, .9422, .9422, .942ACXIO246
43, .9423, .9424, .9424, .9425, .9425, .9426, .9426/          ACXIO247
DATA A7/.7761, .8547, .8858, .9025, .9130, .9200, .9251, .9289, .9318, .934ACXIO248
12, .9360, .9376, .9389, .9399, .9408, .9416, .9423, .9429, .9434, .9439, .944ACXIO249
23, .9447, .9450, .9453, .9455, .9458, .9460, .9462, .9464, .9466, .9467, .946ACXIO250
38, .9470, .9471, .9472, .9473, .9474, .9475, .9476, .9477, .9477, .9478, .947ACXIO251
49, .9479, .9480, .9480, .9481, .9481, .9482, .9482/          ACXIO252
DATA A8/.7767, .8558, .8873, .9044, .9151, .9224, .9277, .9317, .9348, .937ACXIO253
13, .9393, .9409, .9423, .9434, .9444, .9453, .9460, .9467, .9472, .9477, .948ACXIO254
22, .9486, .9489, .9493, .9496, .9498, .9501, .9503, .9505, .9507, .9509, .951ACXIO255
30, .9512, .9513, .9514, .9515, .9517, .9518, .9519, .9519, .9520, .9521, .952ACXIO256
42, .9522, .9523, .9524, .9524, .9525, .9525, .9526/          ACXIO257
DATA A9/.7771, .8565, .8884, .9058, .9167, .9242, .9297, .9338, .9370, .939ACXIO258
16, .9417, .9434, .9449, .9461, .9472, .9481, .9489, .9496, .9502, .9507, .951ACXIO259
22, .9517, .9520, .9524, .9527, .9530, .9533, .9535, .9537, .9539, .9541, .954ACXIO260
33, .9545, .9546, .9548, .9549, .9550, .9551, .9552, .9553, .9554, .9555, .955ACXIO261
46, .9557, .9557, .9558, .9559, .9559, .9560, .9561/          ACXIO262
DATA A10/.7773, .8571, .8892, .9068, .9179, .9256, .9312, .9354, .9388, .94ACXIO263
114, .9436, .9454, .9470, .9482, .9494, .9503, .9512, .9519, .9526, .9531, .95ACXIO264
237, .9541, .9545, .9549, .9553, .9556, .9559, .9561, .9564, .9566, .9568, .95ACXIO265
370, .9572, .9573, .9575, .9576, .9577, .9579, .9580, .9581, .9582, .9583, .95ACXIO266
484, .9585, .9585, .9586, .9587, .9588, .9588, .9589/          ACXIO267
DATA A11/.7775, .8575, .8898, .9076, .9188, .9267, .9324, .9367, .9402, .94ACXIO268
129, .9452, .9470, .9486, .9500, .9511, .9521, .9530, .9538, .9545, .9551, .95ACXIO269
256, .9561, .9566, .9570, .9573, .9577, .9580, .9583, .9585, .9588, .9590, .95ACXIO270
392, .9594, .9595, .9597, .9599, .9600, .9601, .9603, .9604, .9605, .9606, .96ACXIO271
407, .9608, .9609, .9610, .9610, .9611, .9612, .9612/          ACXIO272
DATA A14/.7779, .8583, .8910, .9091, .9207, .9288, .9347, .9393, .9429, .94ACXIO273
159, .9483, .9503, .9520, .9535, .9548, .9559, .9569, .9578, .9585, .9592, .95ACXIO274
298, .9604, .9609, .9614, .9618, .9622, .9625, .9629, .9632, .9634, .9637, .96ACXIO275
339, .9642, .9644, .9646, .9647, .9649, .9651, .9652, .9654, .9655, .9656, .96ACXIO276
457, .9659, .9660, .9661, .9662, .9662, .9663, .9664/          ACXIO277
DATA A17/.7781, .8587, .8916, .9099, .9217, .9300, .9361, .9408, .9446, .94ACXIO278
176, .9502, .9523, .9541, .9557, .9571, .9582, .9593, .9602, .9611, .9618, .96ACXIO279
225, .9631, .9637, .9642, .9646, .9651, .9655, .9658, .9662, .9665, .9668, .96ACXIO280
370, .9673, .9675, .9677, .9679, .9681, .9683, .9685, .9686, .9688, .9689, .96ACXIO281
491, .9692, .9693, .9694, .9695, .9696, .9697, .9698/          ACXIO282
DATA A20/.7782, .8590, .8920, .9105, .9224, .9307, .9370, .9418, .9456, .94ACXIO283

```

| | |
|---|---|
| | 188,.9514,.9536,.9555,.9571,.9586,.9598,.9609,.9619,.9628,.9636,.96ACXIO284 |
| | 243,.9649,.9655,.9661,.9666,.9670,.9675,.9679,.9682,.9686,.9689,.96ACXIO285 |
| | 392,.9694,.9697,.9699,.9702,.9704,.9706,.9708,.9709,.9711,.9713,.97ACXIO286 |
| | 414,.9716,.9717,.9718,.9719,.9721,.9722,.9723/ACXIO287 |
| | DATA A25/.7784,.8592,.8924,.9110,.9230,.9315,.9378,.9428,.9467,.95ACXIO288 |
| | 100,.9527,.9550,.9569,.9586,.9601,.9614,.9626,.9637,.9646,.9655,.96ACXIO289 |
| | 262,.9669,.9676,.9682,.9687,.9692,.9697,.9701,.9705,.9709,.9712,.97ACXIO290 |
| | 315,.9718,.9721,.9724,.9727,.9729,.9731,.9733,.9735,.9737,.9739,.97ACXIO291 |
| | 441,.9742,.9744,.9745,.9747,.9748,.9749,.9751/ACXIO292 |
| | DATA A30/.7784,.8594,.8926,.9113,.9234,.9319,.9383,.9433,.9474,.95ACXIO293 |
| | 107,.9534,.9558,.9578,.9595,.9611,.9625,.9637,.9648,.9657,.9666,.96ACXIO294 |
| | 274,.9682,.9689,.9695,.9701,.9706,.9711,.9715,.9720,.9724,.9727,.97ACXIO295 |
| | 331,.9734,.9737,.9740,.9743,.9745,.9748,.9750,.9752,.9754,.9756,.97ACXIO296 |
| | 458,.9760,.9762,.9763,.9765,.9766,.9768,.9769/ACXIO297 |
| | DATA A40/.7785,.8595,.8928,.9116,.9237,.9324,.9389,.9440,.9480,.95ACXIO298 |
| | 114,.9542,.9567,.9587,.9606,.9622,.9636,.9649,.9660,.9670,.9680,.96ACXIO299 |
| | 288,.9696,.9703,.9710,.9716,.9722,.9727,.9732,.9737,.9741,.9745,.97ACXIO300 |
| | 349,.9752,.9756,.9759,.9762,.9765,.9768,.9770,.9773,.9775,.9777,.97ACXIO301 |
| | 479,.9781,.9783,.9785,.9787,.9788,.9790,.9791/ACXIO302 |
| | DATA A50/.7785,.8596,.8929,.9117,.9239,.9326,.9391,.9443,.9484,.95ACXIO303 |
| | 118,.9547,.9571,.9592,.9611,.9627,.9642,.9655,.9666,.9677,.9687,.96ACXIO304 |
| | 296,.9704,.9711,.9718,.9724,.9730,.9736,.9741,.9746,.9751,.9755,.97ACXIO305 |
| | 359,.9763,.9766,.9770,.9773,.9776,.9779,.9781,.9784,.9786,.9789,.97ACXIO306 |
| | 491,.9793,.9795,.9797,.9799,.9801,.9803,.9804/ACXIO307 |
| | DATA A100/.7785,.8597,.8931,.9119,.9242,.9329,.9395,.9447,.9489,.9ACXIO308 |
| | 1523,.9553,.9578,.9599,.9619,.9635,.9651,.9664,.9676,.9687,.9698,.9ACXIO309 |
| | 2707,.9716,.9724,.9731,.9738,.9744,.9750,.9756,.9761,.9766,.9771,.9ACXIO310 |
| | 3775,.9779,.9783,.9787,.9791,.9794,.9797,.9801,.9803,.9806,.9809,.9ACXIO311 |
| | 4812,.9814,.9817,.9819,.9821,.9823,.9825,.9827/ACXIO312 |
| | DATA A150/.7786,.8597,.8931,.9119,.9242,.9330,.9396,.9448,.9490,.9ACXIO313 |
| | 1525,.9554,.9579,.9601,.9620,.9637,.9652,.9666,.9678,.9690,.9700,.9ACXIO314 |
| | 2710,.9718,.9726,.9734,.9741,.9747,.9754,.9759,.9765,.9770,.9775,.9ACXIO315 |
| | 3779,.9783,.9787,.9791,.9795,.9799,.9802,.9805,.9808,.9811,.9814,.9ACXIO316 |
| | 4817,.9819,.9822,.9824,.9827,.9829,.9831,.9833/ACXIO317 |
| | DATA A225/.7786,.8597,.8931,.9120,.9243,.9330,.9396,.9448,.9490,.9ACXIO318 |
| | 1525,.9554,.9580,.9602,.9621,.9638,.9653,.9667,.9680,.9691,.9701,.9ACXIO319 |
| | 2711,.9720,.9728,.9735,.9742,.9749,.9755,.9761,.9766,.9772,.9776,.9ACXIO320 |
| | 3781,.9785,.9790,.9793,.9797,.9801,.9804,.9808,.9811,.9814,.9817,.9ACXIO321 |
| | 4819,.9822,.9825,.9827,.9829,.9832,.9834,.9836/ACXIO322 |
| | DATA A500/.7786,.8597,.8931,.9120,.9243,.9330,.9396,.9448,.9490,.9ACXIO323 |
| | 1525,.9555,.9580,.9602,.9621,.9639,.9654,.9668,.9680,.9692,.9702,.9ACXIO324 |
| | 2712,.9720,.9729,.9736,.9743,.9750,.9756,.9762,.9768,.9773,.9778,.9ACXIO325 |
| | 3782,.9787,.9791,.9795,.9799,.9802,.9806,.9809,.9812,.9815,.9818,.9ACXIO326 |
| | 4821,.9824,.9827,.9829,.9831,.9834,.9836,.9838/ACXIO327 |
| | DATA IXV/11,14,17,20,25,30,40,50,100,150,225,507/ACXIO328 |
| C | ACXIO329 |
| | DATA VALUE/.4975936099985107D0,.4873642779856548D0,.46913727600136ACXIO330 |
| | 164D0,.4432077635022005D0,.4100009929869515D0,.3700620957892772D0,.ACXIO331 |
| | 23240468259684878D0,.2727107356944198D0,.2168967538130226D0,.157521ACXIO332 |
| | 33398480817D0,.0955594337368082D0,.0320284464313028D0/ACXIO333 |
| C | ACXIO334 |
| C | DATA FOR CALCULATION OF HYDROGENIC RECOMBINATION COEFFICIENTS ACXIO335 |
| | DATA SVOA/.06845,.07335,.07808,.08268,.08714,.09148,.09570,.09982,ACXIO336 |
| | 1.10385,.10778,.11163,.1209,.1297,.1382,.1462,.1540,.1615,.1687,.17ACXIO337 |
| | 257,.1824,.1889,.1953,.2015,.2133,.2245,.2352,.2454,.2552,.2646,.27ACXIO338 |
| | 336,.2823,.2906,.2987/ACXIO339 |
| | DATA SVOB/.3140,.3284,.3419,.3547,.3668,.3783,.3892,.3996,.4096,.4ACXIO340 |

```

1191,.4413,.4615,.4799,.4968,.5124,.5269,.5404,.5530,.5648,.5759,.5ACXI0341
2864,.5963,.6146,.6311,.6461,.6598,.6724,.6840,.6947,.7047,.7140,.7ACXI0342
3226,.7384/
ACXI0343
DATA SV0C/.7524,.7649,.7761,.7862,.7955,.8039,.8117,.8188,.8254,.8ACXI0344
1399,.8521,.8626,.8716,.8795,.8865,.8927,.8982,.9032,.9077,.9118,.9ACXI0345
2156,.9223,.9279,.9328,.9370,.9408,.9441,.9471,.9498,.9522,.9544,2*ACXI0346
3.9544/
ACXI0347
DATA SV1A/.00417,.00444,.00469,.00493,.00516,.00538,.00558,.00578,ACXI0348
1.00597,.00614,.00631,.0067,.0070,.0073,.0076,.0078,.0080,.0082,.00ACXI0349
283,.0085,.0086,.0087,.0087,.0088,.0088,.0088,.0087,.0086,.0084,.00ACXI0350
382,.0080,.0077,.0074/
ACXI0351
DATA SV1B/.0068,.0060,.0053,.0044,.0035,.0025,.0016,+.0005,-.0005,ACXI0352
1-.0016,-.0044,-.0072,-.0101,-.0130,-.0160,-.0190,-.0220,-.0250,-.0ACXI0353
2279,-.0308,-.0337,-.0366,-.0422,-.0478,-.0532,-.0586,-.0638,-.0689ACXI0354
3,-.0739,-.0788,-.0835,-.0882,-.0972/
ACXI0355
DATA SV1C/-.1058,-.1141,-.1220,-.1295,-.1368,-.1438,-.1506,-.1571,ACXI0356
1-.1634,-.1784,-.1923,-.2053,-.2174,-.2289,-.2397,-.2499,-.2596,-.2ACXI0357
2689,-.2777,-.2862,-.2944,-.3099,-.3242,-.3376,-.3502,-.3622,-.3736ACXI0358
3,-.3844,-.3948,-.4047,-.4147,2*-.4147/
ACXI0359
DATA SV2A/-.00120,-.00131,-.00142,-.00153,-.00164,-.00175,-.00186,ACXI0360
1-.00197,-.00207,-.00218,-.00228,-.0025,-.0028,-.0030,-.0033,-.0035ACXI0361
2,-.0038,-.0040,-.0043,-.0045,-.0047,-.0050,-.0052,-.0056,-.0061,-.ACXI0362
30065,-.0070,-.0074,-.0078,-.0082,-.0086,-.0091,.0095/
ACXI0363
DATA SV2B/-.0103,-.0110,-.0118,-.0126,-.0133,-.0141,-.0148,-.0155,ACXI0364
1-.0162,-.0170,-.0187,-.0204,-.0221,-.0237,-.0253,-.0269,-.0284,-.0ACXI0365
2299,-.0314,-.0329,-.0344,-.0359,-.0388,-.0416,-.0444,-.0471,-.0497ACXI0366
3,-.0523,-.0549,-.0575,-.0600,-.0625,-.0674/
ACXI0367
DATA SV2C/-.0722,-.0768,-.0814,-.0859,-.0904,-.0947,-.0989,-.1031,ACXI0368
1-.1072,-.1174,-.1272,-.1367,-.146,-.155,-.1638,-.1723,-.1807,-.189ACXI0369
2,-.197,-.2049,-.2127,-.228,-.243,-.257,-.272,-.285,-.299,-.312,-.3ACXI0370
325,-.337,3*-.35/
ACXI0371
END

FUNCTION BK (N,NDASH,IS)
ACXI0374
C
ACXI0375
C CALLS APPROPRIATE ROUTINES FOR CALCULATION OF ATOMIC DATA FOR
ACXI0376
C ARRAY SK (IN MAIN)
ACXI0377
C
ACXI0378
C N=INITIAL LEVEL, NDASH=FINAL LEVEL, IS=SUBSCRIPT WHICH IDENTIFIES
ACXI0379
C VALUE OF N IN CONDENSED MATRIX
ACXI0380
C
ACXI0381
COMMON /EXPDAT/ CXP(707),MAXN
ACXI0382
COMMON /PARMS/ DENS,T,ITM
ACXI0383
COMMON /RCRATS/ RADTOT(75),COLTOT(75)
ACXI0384
COMMON /TDEP/ TE32,TE12,CTE
ACXI0385
DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP
ACXI0386
DOUBLE PRECISION A,AN,ANDASH,BK,C,COLTOT,COR,CTE,CX,CXP,DENS,G,RADACXI0387
1TOT,RT,T,TE12,TE32,TEMP
ACXI0388
IF (N-NDASH) 20,10,50
ACXI0389
C
ACXI0390
C NDASH=N
ACXI0391
10 CALL COLION (N,1,T,RT)
ACXI0392
BK=-RADTOT(IS)-(COLTOT(IS)+RT)*DENS
ACXI0393
IF (N.LE.20) RETURN
ACXI0394
BK=BK+COR(N,3)
ACXI0395
RETURN
ACXI0396
C
ACXI0397

```

```

C NDASH GT N
20 CALL RAD (A,N,NDASH,G)
   C=COLRAT(N,NDASH,T,TE12)
   AN=N
   ANDASH=NDASH
   IF (NDASH.GT.707) GO TO 30
   CX=CXP(NDASH)
   IF (CX.LT.1.D-30) GO TO 30
   CXN=CXP(N)
   IF (CXN.LT.1.D-30) GO TO 30
   TEMP=CXN/CX
   GO TO 40
30 TEMP=DEXP(-CTE*(1.DO/AN**2-1.DO/ANDASH**2))
40 TEMP=(ANDASH/AN)**2*TEMP
   BK=(A+DENS*C)*TEMP
   IF (N.LE.20) RETURN
   IF (NDASH.NE.N+1) RETURN
   BK=BK+COR(N,1)
   RETURN
C
C NDASH LT N
50 C=COLRAT(NDASH,N,T,TE12)
   BK=C*DENS
   IF (NDASH.NE.N-1) RETURN
   IF (N.LE.20) RETURN
   BK=BK+COR(N,2)
   RETURN
END

FUNCTION CAPPA (T,TL,T32,F)
C
C COMPUTES FREE-FREE ABSORPTION COEFFICIENT CAPPA, GIVEN ELECTRON
C TEMPERATURE (T), LOG(T) (TL), T**1.5 (T32), AND FREQUENCY IN GHZ (F)
C
   V=0.6529+.6666667*ALOG10(F)-TL
   IF (V.GT.-2.6) GO TO 10
   ALIV=-1.1249*V+0.3788
   GO TO 30
10 IF (V.GT.-0.25) GO TO 20
   ALIV=-1.2326*V+0.0987
   GO TO 30
20 ALIV=-1.0842*V+0.1359
30 CAPPA=4.646*EXPM1(.047993*F/T)*EXP(2.302585*ALIV)/(F**2.33333*T32)
   RETURN
END

FUNCTION COLGL (N,NDASH,T,TE12)
C
C CALCULATES COLLISION RATES FROM LEVEL N TO HIGHER LEVEL NDASH AT
C ELECTRON TEMPERATURE T. TE12 IS SQRT(T). USES GAUSS-LAGUERRE
C INTEGRATION OF CROSS-SECTIONS (FUNCTION CROSS) OVER MAXWELL
C DISTRIBUTION. FUNCTION COLGL IS USED FOR VALUES OUTSIDE THE REGION
C OF VALIDITY OF FUNCTION COLRAT.
C
   DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP
   DIMENSION XGL(10), WGL(10)
   DOUBLE PRECISION T,TE12

```

ACXI0398
ACXI0399
ACXI0400
ACXI0401
ACXI0402
ACXI0403
ACXI0404
ACXI0405
ACXI0406
ACXI0407
ACXI0408
ACXI0409
ACXI0410
ACXI0411
ACXI0412
ACXI0413
ACXI0414
ACXI0415
ACXI0416
ACXI0417
ACXI0418
ACXI0419
ACXI0420
ACXI0421
ACXI0422
ACXI0423
ACXI0424
ACXI0425

ACXI0427
ACXI0428
ACXI0429
ACXI0430
ACXI0431
ACXI0432
ACXI0433
ACXI0434
ACXI0435
ACXI0436
ACXI0437
ACXI0438
ACXI0439
ACXI0440
ACXI0441
ACXI0442

ACXI0444
ACXI0445
ACXI0446
ACXI0447
ACXI0448
ACXI0449
ACXI0450
ACXI0451
ACXI0452
ACXI0453
ACXI0454

| | | |
|----|--|----------|
| | DATA NGL/./10/ | ACXI0455 |
| C | DATA XGL/./1377935,.7294545,1.808343,3.401434,5.552496,8.330153, | ACXI0456 |
| C | A 11.84379,16.27926,21.99659,29.92070/ | ACXI0457 |
| C | DATA WGL/./3084411,.4011199,.2180683,6.208746E-2,9.501517E-3, | ACXI0458 |
| C | A 7.530084E-4,2.825923E-5,4.249314E-7,1.839565E-9,9.911827E-13/ | ACXI0459 |
| | DATA XGL/./4157746,2.294280,6.289945,7*0./ | ACXI0460 |
| | DATA WGL/./7110930,.2785177,1.038926E-2,7*0./ | ACXI0461 |
| | DATA NGL/3/ | ACXI0462 |
| | BETA=1.58D5/T | ACXI0463 |
| | EN2=N*N | ACXI0464 |
| | END2=NDASH*NDASH | ACXI0465 |
| | DE=1./EN2-1./END2 | ACXI0466 |
| | COLGL=0. | ACXI0467 |
| | DO 10 I=1,NGL | ACXI0468 |
| | E=XGL(I)/BETA+DE | ACXI0469 |
| | COLGL=COLGL+WGL(I)*CROSS(N,NDASH,E)*E*BETA | ACXI0470 |
| 10 | CONTINUE | ACXI0471 |
| | COLGL=COLGL*6.21241E5*SNGL(TE12)*(EN2/END2) | ACXI0472 |
| | RETURN | ACXI0473 |
| | END | ACXI0474 |
| | FUNCTION COLRAT (N,NP,T,TE12) | ACXI0510 |
| C | | ACXI0511 |
| C | CALCULATES RATE OF COLLISIONS FROM LEVEL N TO HIGHER LEVEL NP | ACXI0512 |
| C | AT ELECTRON TEMPERATURE T. TE12 IS SQRT(T). SETS RATE=0 FOR NP-N | ACXI0513 |
| C | GT 40, BUT THIS IS EASILY MODIFIED. | ACXI0514 |
| C | | ACXI0515 |
| C | THIS FUNCTION MUST BE INITIALIZED BY BEING CALLED WITH N=0 BEFORE | ACXI0516 |
| C | ANY COLLISION RATES ARE COMPUTED. | ACXI0517 |
| C | THEORY: GEE, PERCIVAL, LODGE AND RICHARDS, MNRAS 175, 209-215 (1976) | ACXI0518 |
| C | | ACXI0519 |
| C | RANGE OF VALIDITY OF GPLR RATES IS 10**6/N**2 LT T LTT 3*10**9 | ACXI0520 |
| C | OUTSIDE THIS RANGE, NUMERICAL INTEGRATION OF THE GPLR CROSS-SECTIONS | ACXI0521 |
| C | IS RESORTED TO.THESE CROSS SECTIONS ARE VALID DOWN TO ENERGIES OF | ACXI0522 |
| C | 4/N**2 RYDBERGS; THE CROSS-SECTION FORMULA CAN BE USED AT LOWER | ACXI0523 |
| C | ENERGIES FOR BN CALCULATIONS, THE INACCURACY IN THE CROSS-SECTIONS | ACXI0524 |
| C | HAVING LITTLE EFFECT ON THE BN'S. | ACXI0525 |
| C | | ACXI0526 |
| | COMMON /COLINF/ AL18S4(506),S23TRM(506) | ACXI0527 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI0528 |
| | DOUBLE PRECISION DRT,T,TE12 | ACXI0529 |
| | REAL L,J1,J2,J3,J4 | ACXI0530 |
| | IF (N.LE.0) GO TO 60 | ACXI0531 |
| | COLRAT=0. | ACXI0532 |
| | IS=NP-N | ACXI0533 |
| | IF (IS.GT.40) RETURN | ACXI0534 |
| | S=IS | ACXI0535 |
| | EN2=N*N | ACXI0536 |
| | IF (SNGL(T).LT.1.E6/EN2) GO TO 50 | ACXI0537 |
| | EN=N | ACXI0538 |
| | ENP=NP | ACXI0539 |
| | IPOW=1+IS+IS | ACXI0540 |
| | POW=IPOW | ACXI0541 |
| | ENNP=N*NP | ACXI0542 |
| | BETA=1.58D5/T | ACXI0543 |
| | BETA1=1.4*SQRT(ENNP) | ACXI0544 |
| | BETRT=BETA1/BETA | ACXI0545 |

```

      BETSUM=BETA1+BETA
      F1=0.2*S/ENNP
      IF (F1.GT.0.02) GO TO 10
      F1=1.-POW*F1
      GO TO 20
10    F1=(1.-F1)**IPOW
20    A=(2.666667/S)*(ENP/(S*EN))**3*S23TRM(IS)*F1
      L=0.85/BETA
      L=ALOG((1.+0.53*L*L*ENNP)/(1.+0.4*L))
      J1=1.333333*A*L*BETRT/BETSUM
      DRT=DSQRT(2.DO-DFLOAT(N*N)/DFLOAT(NP*NP))
      F1=0.3*S/ENNP
      IF (F1.GT.0.02) GO TO 30
      F1=1.-POW*F1
      GO TO 40
30    F1=(1.-F1)**IPOW
40    J2=0.
      ARG=BETA/BETA1
      IF (ARG.LE.150.) J2=1.777778*F1*(ENP*SNGL(DRT+1.DO)/((EN+ENP)*S))*ACXI0546
1*3*EXP(-ARG)/(BETA/(1.-AL18S4(IS)))
      XI=2./(EN2*SNGL(DRT-1.DO))
      Z=0.75*X1*BETSUM
      EXPZ=0.
      IF (Z.LE.150.) EXPZ=EXP(-Z)
      J4=2./(Z*(2.+Z*(1.+EXPZ)))
      J3=0.25*(EN2*X1/ENP)**3*J4*ALOG(1.+0.5*BETA*X1)/BETSUM
      COLRAT=EN2*EN2*(J1+J2+J3)/SNGL(TE12)**3
      RETURN
C
C GPLR COLLISION RATE FORMULAS INVALID AT LOW TEMPERATURES. INTEGRATE ACXI0575
C CROSS SECTIONS.
C
C
50  COLRAT=COLGL(N,NP,T,TE12)
      RETURN
C
C INITIALIZE
C
60  DO 70 I=1,506
      Z=I
      AL18S4(I)=ALOG(18.*Z)/(4.*Z)
      S23TRM(I)=(0.184-0.04*Z**(-.6666667))
70  CONTINUE
      COLRAT=0.
      RETURN
      END
      ACXI0547
      ACXI0548
      ACXI0549
      ACXI0550
      ACXI0551
      ACXI0552
      ACXI0553
      ACXI0554
      ACXI0555
      ACXI0556
      ACXI0557
      ACXI0558
      ACXI0559
      ACXI0560
      ACXI0561
      ACXI0562
      ACXI0563
      ACXI0564
      ACXI0565
      ACXI0566
      ACXI0567
      ACXI0568
      ACXI0569
      ACXI0570
      ACXI0571
      ACXI0572
      ACXI0573
      ACXI0574
      ACXI0575
      ACXI0576
      ACXI0577
      ACXI0578
      ACXI0579
      ACXI0580
      ACXI0581
      ACXI0582
      ACXI0583
      ACXI0584
      ACXI0585
      ACXI0586
      ACXI0587
      ACXI0588
      ACXI0589
      ACXI0590
      ACXI0592
      ACXI0593
      ACXI0594
      ACXI0595
      ACXI0596
      ACXI0597
      ACXI0598
      ACXI0599
      ACXI0600
      ACXI0601
      ACXI0602
      FUNCTION COR (N,ISW)
C
C CORRECTION TO RATE OF POPULATION OF LEVEL N DUE TO RADIATION FIELD.
C SEE WRITEUP. THIS SUBPROGRAM MAY BE REPLACED BY THE USER IF OTHER
C THAN A DILUTE BLACKBODY RADIATION FIELD IS REQUIRED.
C
C THE MAIN PROGRAM CALLS COR ONCE WITH ISW=0 BEFORE STARTING THE
C CALCULATIONS. ANY INITIALIZATION (INCLUDING THE READING OF DATA IF
C REQUIRED) SHOULD BE CARRIED OUT DURING THIS FIRST CALL.
C
C COMPUTES TERMS OF

```

```

C  D(NU)RHO(NU) (N(N+1)B(N+1,N)+N(N-1)B(N-1,N)-N(N) (B(N,N-1)+B(N,N+1)) ACXI0603
C  WITH REMOVED FACTOR ACXI0604
C  (C/4PI)(H*H/2PI M KT)**3/2 N*N NE NI EXP(CHI1/N*N KT) ACXI0605
C ACXI0606
      COMMON /EXPDAT/ CXP(707),MAXN ACXI0607
      COMMON /INOUT/ IREAD,IWRITE,IPUNCH,icrt ACXI0608
      DIMENSION DILT(508), DX(508) ACXI0609
      DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI0610
      DOUBLE PRECISION A,AM,AP,COR,CXP,EX,G ACXI0611
      LOGICAL NOFLD ACXI0612
      DATA DILT/508*0.5/,NOFLD/.FALSE./ ACXI0613
      IF (ISW.GT.0) GO TO 40 ACXI0614
      READ (IREAD,90) TBCK,EBCK ACXI0615
      IF (EBCK.EQ.0. .OR. TBCK.EQ.0.) NOFLD=.TRUE. ACXI0616
      IF (.NOT.NOFLD) GO TO 10 ACXI0617
      WRITE (IWRITE,100) ACXI0618
      write (icrt,111)
111  format (' No background info detected.')
```

```

      GO TO 80 ACXI0619
10  WRITE (IWRITE,110) TBCK,EBCK ACXI0620
      write (icrt,112)
112  format (' Background info detected. This part of the code',
1' has not be checked.')
```

```

      C15=15.778E4/TBCK ACXI0621
      MAXP=MAXN+1 ACXI0622
      DO 20 I=1,MAXP ACXI0623
        AI=I*(I+1) ACXI0624
        ARG=C15*FLOAT(2*I+1)/AI**2 ACXI0625
        DX(I)=EXPM1(ARG) ACXI0626
20  CONTINUE ACXI0627
      IF (EBCK.GE.0.9999E10) GO TO 80 ACXI0628
      TBCK32=TBCK*SQRT(TBCK) ACXI0629
      TL=ALOG10(TBCK) ACXI0630
      DO 30 I=1,MAXP ACXI0631
        FREQG=6.58E6/FLOAT(I)**3 ACXI0632
        TOW=EBCK*CAPPA(TBCK,TL,TBCK32,FREQG) ACXI0633
        IF (TOW.LE.20.) DILT(I)=-0.5*EXPM1(-TOW) ACXI0634
30  CONTINUE ACXI0635
      GO TO 80 ACXI0636
40  IF (NOFLD) GO TO 80 ACXI0637
      IF (N.GT.MAXN) WRITE (IWRITE,120) N ACXI0638
      GO TO (50,60,70), ISW ACXI0639
C  N(N+1)B(N+1,N) ACXI0640
50  CALL RAD (A,N,N+1,G) ACXI0641
      EX=DX(N) ACXI0642
      COR=(DFLOAT((N+1)**2)/DFLOAT(N*N))*DILT(N)*A/(EX*(EX+1.DO)) ACXI0643
      RETURN ACXI0644
C  N(N-1)B(N-1,N) ACXI0645
60  CALL RAD (A,N-1,N,G) ACXI0646
      EX=DX(N-1) ACXI0647
      COR=DILT(N)*A/(EX*(EX+1.DO)) ACXI0648
      RETURN ACXI0649
C  N(N) (B(N,N-1) + B(N,N+1)) ACXI0650
70  CALL RAD (AM,N-1,N,G) ACXI0651
      CALL RAD (AP,N,N+1,EX) ACXI0652
      EX=DX(N-1) ACXI0653
      G=DX(N) ACXI0654
```


| | | |
|----|--|----------|
| | FUNCTION DMJ1 (K,KM) | ACXIO714 |
| | COMMON /FITDAT/ AFIT(4,4),IVAL(4),NFIT | ACXIO715 |
| | COMMON /HIGHER/ STORE1(224,4),STORE2(224),VAL(24),B,STORE3(224,5), | ACXIO716 |
| | 1RTVAL(24),LIMIT | ACXIO717 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXIO718 |
| | DOUBLE PRECISION AFIT,B,DMJ1,RTVAL,STORE1,STORE2,STORE3,VAL | ACXIO719 |
| | DMJ1=0.DO | ACXIO720 |
| | DO 10 J=1,NFIT | ACXIO721 |
| 10 | DMJ1=DMJ1+AFIT(J,KM)*STORE1(K,J) | ACXIO722 |
| | RETURN | ACXIO723 |
| | END | ACXIO724 |
| | FUNCTION DMJ2 (K) | ACXIO726 |
| | COMMON /FITDAT/ AFIT(4,4),IVAL(4),NFIT | ACXIO727 |
| | COMMON /HIGHER/ STORE1(224,4),STORE2(224),VAL(24),B,STORE3(224,5), | ACXIO728 |
| | 1RTVAL(24),LIMIT | ACXIO729 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXIO730 |
| | DOUBLE PRECISION AFIT,B,DMJ2,RTVAL,STORE1,STORE2,STORE3,SUM,TOT,VAACXIO731 | |
| | 1L | ACXIO732 |
| | DMJ2=0.DO | ACXIO733 |
| | TOT=0.DO | ACXIO734 |
| | DO 20 J=1,NFIT | ACXIO735 |
| | SUM=0.DO | ACXIO736 |
| | DO 10 I=1,NFIT | ACXIO737 |
| 10 | SUM=SUM+AFIT(J,I) | ACXIO738 |
| 20 | TOT=TOT+SUM*STORE1(K,J) | ACXIO739 |
| | DMJ2=1.DO-TOT | ACXIO740 |
| | RETURN | ACXIO741 |
| | END | ACXIO742 |
| | FUNCTION DPHI (IQ,LG,ITAU,N) | ACXIO744 |
| C | | ACXIO745 |
| C | LAGRANGIAN INTERPOLATION | ACXIO746 |
| C | | ACXIO747 |
| | DIMENSION IQ(8) | ACXIO748 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXIO749 |
| | DOUBLE PRECISION A,DPHI | ACXIO750 |
| | DPHI=0.DO | ACXIO751 |
| | DO 20 M=1,LG | ACXIO752 |
| | IF (M.EQ.ITAU) GO TO 20 | ACXIO753 |
| | A=1.DO | ACXIO754 |
| | DO 10 L=1,LG | ACXIO755 |
| | IF (L.EQ.ITAU.OR.L.EQ.M) GO TO 10 | ACXIO756 |
| | A=A*DFLOAT(N-IQ(L)) | ACXIO757 |
| 10 | CONTINUE | ACXIO758 |
| | DPHI=DPHI+A | ACXIO759 |
| 20 | CONTINUE | ACXIO760 |
| | RETURN | ACXIO761 |
| | END | ACXIO762 |
| | FUNCTION EXPM1 (X) | ACXIO765 |
| C | | ACXIO766 |
| C | COMPUTES EXP(X)-1 ACCURATELY, EVEN FOR SMALL X | ACXIO767 |
| C | | ACXIO768 |
| | IF (ABS(X).LE.0.1) GO TO 10 | ACXIO769 |
| | EXPM1=EXP(X)-1. | ACXIO770 |
| | RETURN | ACXIO771 |

| | | |
|----|---|----------|
| 10 | EXPM1=X*(.9999997+X*(0.5+X*(.1667708+X*.4166667E-1))) | ACXI0772 |
| | RETURN | ACXI0773 |
| | END | ACXI0774 |
| | FUNCTION PHI (IQ,LG,ITAU,N) | ACXI1007 |
| C | | ACXI1008 |
| C | LAGRANGIAN INTERPOLATION | ACXI1009 |
| C | | ACXI1010 |
| | DIMENSION IQ(8) | ACXI1011 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI1012 |
| | DOUBLE PRECISION PHI | ACXI1013 |
| | PHI=1.DO | ACXI1014 |
| | DO 10 L=1,LG | ACXI1015 |
| | IF (L.EQ.ITAU) GO TO 10 | ACXI1016 |
| | PHI=PHI*DFLOAT(N-IQ(L)) | ACXI1017 |
| 10 | CONTINUE | ACXI1018 |
| | RETURN | ACXI1019 |
| | END | ACXI1020 |
| | FUNCTION POL (K,KM) | ACXI1022 |
| | COMMON /HIGHER/ STORE1(224,4),STORE2(224),VAL(24),B,STORE3(224,5) | ACXI1023 |
| | RTVAL(24),LIMIT | ACXI1024 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI1025 |
| | DOUBLE PRECISION B,POL,RTVAL,STORE1,STORE2,STORE3,VAL | ACXI1026 |
| | IND=K+LIMIT | ACXI1027 |
| | POL=RTVAL(K)*STORE3(IND,KM)*STORE2(IND) | ACXI1028 |
| | RETURN | ACXI1029 |
| | END | ACXI1030 |
| | SUBROUTINE BCPCH (KB,KC,T,DENS,NMIN,LABEL,NPLO,NPHI,NDIM) | ACXI0144 |
| C | | ACXI0145 |
| C | PUNCHES BN AND CN VALUES IN STANDARD FORMAT | ACXI0146 |
| C | | ACXI0147 |
| | COMMON /INOUT/ IREAD,IWRITE,IPUNCH,icrt | ACXI0148 |
| | DIMENSION KB(NDIM), KC(NDIM), K1(19), K2(12) | ACXI0149 |
| | DOUBLE PRECISION DENL,DENS,T,TL | ACXI0150 |
| | EQUIVALENCE (K1(1),K2(1)) | ACXI0151 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI0152 |
| | CHARACTER ALPHA(35) | ACXI0153 |
| C | SOME COMPILERS REQUIRE (ALPHA(I),I=1,35) IN DATA STATEMENT | ACXI0154 |
| | DATA ALPHA /'1','2','3','4','5','6','7','8','9','A','B', | ACXI0155 |
| | 'C','D','E','F','G','H','I','J','K','L','M','N','O','P','Q','R', | ACXI0156 |
| | 'S','T','U','V','W','X','Y','Z'/ | ACXI0157 |
| | N=NPHI-NPLO+1 | ACXI0158 |
| | TL=DLOG10(T) | ACXI0159 |
| | DENL=DLOG10(DENS) | ACXI0160 |
| | WRITE (IPUNCH,50) TL,DENL,NMIN,NPLO,NPHI,LABEL,ALPHA(1) | ACXI0161 |
| | NB=N/19 | ACXI0162 |
| | IF (19*NB.NE.N) NB=NB+1 | ACXI0163 |
| | NC=N/12 | ACXI0164 |
| | IF (12*NC.NE.N) NC=NC+1 | ACXI0165 |
| | IPOINT=0 | ACXI0166 |
| | DO 20 I=1,NB | ACXI0167 |
| | DO 10 J=1,19 | ACXI0168 |
| | IPOINT=IPOINT+1 | ACXI0169 |
| | K=0 | ACXI0170 |
| | IF (IPOINT.LE.N) K=KB(IPOINT) | ACXI0171 |

```

      K1(J)=K                                ACXI0172
10  CONTINUE                                ACXI0173
      WRITE (IPUNCH,60) K1,LABEL,ALPHA(I+1) ACXI0174
20  CONTINUE                                ACXI0175
      IPOINT=0                                ACXI0176
      DO 40 I=1,NC                            ACXI0177
      DO 30 J=1,12                            ACXI0178
      IPOINT=IPOINT+1                        ACXI0179
      K=0                                      ACXI0180
      IF (IPOINT.LE.N) K=KC(IPOINT)          ACXI0181
      K2(J)=K                                ACXI0182
30  CONTINUE                                ACXI0183
      NBI=NB+I+1                             ACXI0184
      WRITE (IPUNCH,70) K2,LABEL,ALPHA(NBI) ACXI0185
40  CONTINUE                                ACXI0186
      WRITE (IWRITE,80) IPUNCH               ACXI0187
      RETURN                                  ACXI0188
C                                            ACXI0189
50  FORMAT (2F10.6,3I5,4I1X,A3,A1)          ACXI0190
60  FORMAT (19I4,A3,A1)                     ACXI0191
70  FORMAT (12I6,4X,A3,A1)                  ACXI0192
80  FORMAT (///27H *** OUTPUT PUNCHED ON UNIT,I3) ACXI0193
      END                                    ACXI0194

      SUBROUTINE COLION (N,IONZ,T,QI)          ACXI0476
C                                            ACXI0477
C  COMPUTES COLLISIONAL IONIZATION RATE, QI, FROM LEVEL N FOR IONS
C  OF EFFECTIVE CHARGE IONZ AT ELECTRON TEMPERATURE T. ACXI0478
C                                            ACXI0479
C                                            ACXI0480
C  WHEN CALLED WITH N=0, COLION COMPUTES AND STORES QUANTITIES WHICH
C  DEPEND ONLY UPON TEMPERATURE AND EFFECTIVE CHARGE. IT IS ASSUMED
C  THAT T AND IONZ REMAIN CONSTANT UNTIL THE NEXT CALL WITH N=0. ACXI0481
C                                            ACXI0482
C                                            ACXI0483
C                                            ACXI0484
      COMMON /TDEP/ TE32,TE12,CTE            ACXI0485
      DIMENSION EXPX(507)                    ACXI0486
      DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI0487
      DOUBLE PRECISION CTE,QI,T,TE12,TE32    ACXI0488
      QI=0.D0                                ACXI0489
      IF (N.NE.0) GO TO 20                   ACXI0490
C  INITIALIZE                                ACXI0491
c  Malcolm Walmsley has modified this program to consider collisional
c  ionization from level 10. The original program considered levels only
c  down to n = 20.
      CONS=DFLOAT(IONZ*IONZ)*CTE             ACXI0492
      nlow=9
      nlow1=nlow+1
c      DO 10 I=21,507                        ACXI0493
      do 10 i=nlow1,507
10  EXPX(I)=EXP(-CONS/FLOAT(I*I))            ACXI0494
      RETURN                                  ACXI0495
c20 IF (N.LE.20) RETURN                      ACXI0496
20  if (n.le.507) return
      X=CONS/DFLOAT(N*N)                     ACXI0497
      DXP=EXPX(N)                            ACXI0498
      IF (N.GT.507) DXP=EXP(-X)              ACXI0499
      IF (X.LE.1.) GO TO 30                  ACXI0500
      E1=((.250621+X*(2.334733+X))/(1.681534+X*(3.330657+X)))*DXP/X ACXI0501

```

```

GO TO 40
ACXI0502
30 E1=-.57721566+X*(.9999193+X*(-.24991055+X*(.5519968E-1+X*(-.976004ACXI0503
1E-2+X*.107857E-2))))-ALOG(X)
ACXI0504
40 EI=DXP*(1.666667-.6666667*X)/X+E1*(.6666667*X-1.)-0.5*E1*EI/DXP
ACXI0505
QI=5.444089*EI/SNGL(TE32)
ACXI0506
RETURN
ACXI0507
END
ACXI0508

SUBROUTINE HELPME (AID,KM)
ACXI0776
COMMON /HIGHER/ STORE1(224,4),STORE2(224),VAL(24),B,STORE3(224,5),ACXI0777
1RTVAL(24),LIMIT
ACXI0778
DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP
ACXI0779
DOUBLE PRECISION AID,B,C,POL,RTVAL,STORE1,STORE2,STORE3,SUM,VAL,Y
ACXI0780
SUM=0.DO
ACXI0781
DO 10 K=1,LIMIT
ACXI0782
C=STORE2(K)*STORE3(K,KM)
ACXI0783
10 SUM=SUM+C
ACXI0784
SUM=SUM-0.5D0*C
ACXI0785
Y=.6170614899993600D-2*(POL(1,KM)+POL(2,KM))
ACXI0786
Y=Y+.1426569431446683D-1*(POL(3,KM)+POL(4,KM))
ACXI0787
Y=Y+.2213871940870990D-1*(POL(5,KM)+POL(6,KM))
ACXI0788
Y=Y+.2964929245771839D-1*(POL(7,KM)+POL(8,KM))
ACXI0789
Y=Y+.3667324070554015D-1*(POL(9,KM)+POL(10,KM))
ACXI0790
Y=Y+.4309508076597664D-1*(POL(11,KM)+POL(12,KM))
ACXI0791
Y=Y+.4880932605205694D-1*(POL(13,KM)+POL(14,KM))
ACXI0792
Y=Y+.5372213505798282D-1*(POL(15,KM)+POL(16,KM))
ACXI0793
Y=Y+.5775283402686280D-1*(POL(17,KM)+POL(18,KM))
ACXI0794
Y=Y+.6083523646390170D-1*(POL(19,KM)+POL(20,KM))
ACXI0795
Y=Y+.6291872817341415D-1*(POL(21,KM)+POL(22,KM))
ACXI0796
Y=Y+.6396909767337608D-1*(POL(23,KM)+POL(24,KM))
ACXI0797
AID=SUM+B*Y
ACXI0798
RETURN
ACXI0799
END
ACXI0800

SUBROUTINE INTERP (M,CO,VAL,DVAL,IC,IR)
ACXI0802
C
ACXI0803
C COMPUTES SOLUTIONS AT ALL N FROM THOSE OBTAINED AT THE CONDENSED
ACXI0804
C POINTS
ACXI0805
C
ACXI0806
DIMENSION CO(75),DVAL(507),IQ(8),M(75),VAL(507)
ACXI0807
DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP
ACXI0808
DOUBLE PRECISION CO,COT,DFL,DPhi,DVAL,FL,PHI,PHITAU,VAL
ACXI0809
C
ACXI0810
LG=2*(IR+1)
ACXI0811
IB=IC-IR
ACXI0812
IBB=IB-1
ACXI0813
ICC=IC-1
ACXI0814
C
ACXI0815
K=M(IC)
ACXI0816
DO 10 I=1,K
ACXI0817
VAL(I)=0.DO
ACXI0818
DVAL(I)=0.DO
ACXI0819
10 CONTINUE
ACXI0820
C
ACXI0821
DO 20 IT=1,IC
ACXI0822
IA=IT
ACXI0823
MIT=M(IT)
ACXI0824

```

| | | |
|----|--|----------|
| | VAL(MIT)=CO(IT) | ACXI0825 |
| | IF ((M(IT+1)-MIT).GT.1) GO TO 30 | ACXI0826 |
| 20 | CONTINUE | ACXI0827 |
| C | | ACXI0828 |
| 30 | IF (IA.EQ.IC) GO TO 90 | ACXI0829 |
| | IF (IA.EQ.IB) GO TO 60 | ACXI0830 |
| C | | ACXI0831 |
| | DO 50 IT=IA,IBB | ACXI0832 |
| | N1=M(IT)+1 | ACXI0833 |
| | N2=M(IT+1) | ACXI0834 |
| | ITR1=IT-IR-1 | ACXI0835 |
| | DO 40 ITAU=1,LG | ACXI0836 |
| | IND=ITR1+ITAU | ACXI0837 |
| 40 | IQ(ITAU)=M(IND) | ACXI0838 |
| | DO 50 ITAU=1,LG | ACXI0839 |
| | PHITAU=1.DO/PHI(IQ,LG,ITAU,IQ(ITAU)) | ACXI0840 |
| | DO 50 N=N1,N2 | ACXI0841 |
| | FL=PHI(IQ,LG,ITAU,N)*PHITAU | ACXI0842 |
| | DFL=DPHI(IQ,LG,ITAU,N)*PHITAU | ACXI0843 |
| | IND=ITR1+ITAU | ACXI0844 |
| | COT=CO(IND) | ACXI0845 |
| | VAL(N)=VAL(N)+FL*COT | ACXI0846 |
| 50 | DVAL(N)=DVAL(N)+DFL*COT | ACXI0847 |
| 60 | IF (IR.EQ.O) GO TO 90 | ACXI0848 |
| C | | ACXI0849 |
| | ICLG=IC-LG | ACXI0850 |
| | DO 70 ITAU=1,LG | ACXI0851 |
| | IND=ICLG+ITAU | ACXI0852 |
| 70 | IQ(ITAU)=M(IND) | ACXI0853 |
| | DO 80 ITAU=1,LG | ACXI0854 |
| | PHITAU=1.DO/PHI(IQ,LG,ITAU,IQ(ITAU)) | ACXI0855 |
| | DO 80 IT=IB,ICC | ACXI0856 |
| | N1=M(IT)+1 | ACXI0857 |
| | N2=M(IT+1) | ACXI0858 |
| | DO 80 N=N1,N2 | ACXI0859 |
| | FL=PHI(IQ,LG,ITAU,N)*PHITAU | ACXI0860 |
| | DFL=DPHI(IQ,LG,ITAU,N)*PHITAU | ACXI0861 |
| | IND=ICLG+ITAU | ACXI0862 |
| | COT=CO(IND) | ACXI0863 |
| | VAL(N)=VAL(N)+FL*COT | ACXI0864 |
| 80 | DVAL(N)=DVAL(N)+DFL*COT | ACXI0865 |
| C | | ACXI0866 |
| 90 | RETURN | ACXI0867 |
| | END | ACXI0868 |
| | | |
| | SUBROUTINE JMD (SK,CO,MVAL,IC) | ACXI0870 |
| | COMMON /FITDAT/ AFIT(4,4),IVAL(4),NFIT | ACXI0871 |
| | COMMON /GAUSS/ VALUE(12) | ACXI0872 |
| | COMMON /HIGHER/ STORE1(224,4),STORE2(224),VAL(24),B,STORE3(224,5), | ACXI0873 |
| | IRTVAL(24),LIMIT | ACXI0874 |
| | COMMON /PARMS/ DENS,T,ITM | ACXI0875 |
| | DIMENSION AZ(4),CO(75),IND(4,2),IPIV(4),MVAL(75),SK(75,75) | ACXI0876 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI0877 |
| | DOUBLE PRECISION A,AC,AFIT,AID,AJ,AK,AKK,AZ,B,BK,CO,D,DENS,DMJ1,DMACXI0878 | |
| | 1J2,RTVAL,SK,SOS,STORE1,STORE2,STORE3,T,VAL,VALUE | ACXI0879 |
| | SOS(I,A)=DSQRT(-A)**(2*I+ITM)/DLOG(-A) | ACXI0880 |
| | LIMIT=200 | ACXI0881 |

| | | |
|-----|--|----------|
| | NG=24 | ACXI0882 |
| | DO 10 J=1,NFIT | ACXI0883 |
| | K=IVAL(J) | ACXI0884 |
| | AJ=-1.DO/DFLOAT(MVAL(K))*2 | ACXI0885 |
| | DO 10 I=1,NFIT | ACXI0886 |
| 10 | AFIT(J,I)=SOS(I,AJ) | ACXI0887 |
| | CALL MATINV (AFIT,NFIT,AZ,0,D,ERROR,4,IPIV,IND) | ACXI0888 |
| | B=1.DO/DFLOAT(MVAL(IC)+LIMIT)**2 | ACXI0889 |
| | A=-0.5DO*B | ACXI0890 |
| | NH=NG/2 | ACXI0891 |
| | DO 20 K=1,NH | ACXI0892 |
| | VAL(2*K-1)=A+VALUE(K)*B | ACXI0893 |
| 20 | VAL(2*K)=A-VALUE(K)*B | ACXI0894 |
| | DO 30 K=1,LIMIT | ACXI0895 |
| | A=MVAL(IC)+K | ACXI0896 |
| | AC=-1.DO/A**2 | ACXI0897 |
| | DO 30 J=1,NFIT | ACXI0898 |
| 30 | STORE1(K,J)=SOS(J,AC) | ACXI0899 |
| | DO 40 K=1,NG | ACXI0900 |
| | DO 40 J=1,NFIT | ACXI0901 |
| | INP=K+LIMIT | ACXI0902 |
| 40 | STORE1(INP,J)=SOS(J,VAL(K)) | ACXI0903 |
| | KK=LIMIT+NG | ACXI0904 |
| | DO 60 K=1,KK | ACXI0905 |
| | DO 50 J=1,NFIT | ACXI0906 |
| 50 | STORE3(K,J)=DMJ1(K,J) | ACXI0907 |
| 60 | STORE3(K,NFIT+1)=DMJ2(K) | ACXI0908 |
| | DO 100 J=1,IC | ACXI0909 |
| | I=MVAL(J) | ACXI0910 |
| | DO 70 K=1,LIMIT | ACXI0911 |
| | KK=MVAL(IC)+K | ACXI0912 |
| | AKK=KK | ACXI0913 |
| 70 | STORE2(K)=BK(I,KK,0) | ACXI0914 |
| | DO 80 K=1,NG | ACXI0915 |
| | AK=DSQRT(-1.DO/VAL(K)) | ACXI0916 |
| | RTVAL(K)=AK**3*0.5DO | ACXI0917 |
| | KK=AK | ACXI0918 |
| | INP=K+LIMIT | ACXI0919 |
| | STORE2(INP)=BK(I,KK,0) | ACXI0920 |
| 80 | CONTINUE | ACXI0921 |
| | CALL HELPME (AID,NFIT+1) | ACXI0922 |
| | CO(J)=CO(J)-AID | ACXI0923 |
| | DO 90 KM=1,NFIT | ACXI0924 |
| | CALL HELPME (AID,KM) | ACXI0925 |
| | L=IVAL(KM) | ACXI0926 |
| 90 | SK(J,L)=SK(J,L)+AID | ACXI0927 |
| 100 | CONTINUE | ACXI0928 |
| | RETURN | ACXI0929 |
| | END | ACXI0930 |
| | SUBROUTINE MATINV (A,N,B,L,D,ERROR,NDA,IPIV,IND) | ACXI0932 |
| | DIMENSION A(NDA,NDA), B(NDA), IND(NDA,2), IPIV(NDA) | ACXI0933 |
| | DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP | ACXI0934 |
| | DOUBLE PRECISION A,AMAX,ATEMP,B,D | ACXI0935 |
| C | | ACXI0936 |
| C | SOLVES SIMULTANEOUS EQUATIONS IF L=1 | ACXI0937 |
| C | | ACXI0938 |


```

      J=N+1-I                                ACXI0994
      IF (IND(J,1)-IND(J,2)) 210,230,210    ACXI0995
210  IROW=IND(J,1)                          ACXI0996
      ICOL=IND(J,2)                          ACXI0997
      DO 220 K=1,N                           ACXI0998
      AMAX=A(K,IROW)                         ACXI0999
      A(K,IROW)=A(K,ICOL)                   ACXI1000
220  A(K,ICOL)=AMAX                         ACXI1001
230  CONTINUE                               ACXI1002
240  IRROR=0                                ACXI1003
      RETURN                                ACXI1004
      END                                    ACXI1005

      SUBROUTINE RAD (A,N,NDASH,G)           ACXI1032
C                                           ACXI1033
C COMPUTES EINSTEIN RADIATIVE RATES A AND GAUNT FACTORS G FOR ACXI1034
C TRANSITIONS FROM NDASH TO N              ACXI1035
C                                           ACXI1036
      COMMON /GAUNTS/ GAUNT(50,22),IXV(12)  ACXI1037
      DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI1038
      DOUBLE PRECISION A,ALF2,EN,EN2,END2,G,TERM1,TERM2,TERM3 ACXI1039
C ARRAY GAUNT IS NOT DOUBLE PRECISION      ACXI1040
      EN=N                                  ACXI1041
      EN2=N*N                              ACXI1042
      END2=NDASH*NDASH                     ACXI1043
      NDMN=NDASH-N                         ACXI1044
      IF (NDMN.GT.50) GO TO 30              ACXI1045
      IF (N.LE.10) GO TO 20                 ACXI1046
      DO 10 J=2,12                          ACXI1047
      IVJ=IXV(J)                           ACXI1048
      IVJ1=IXV(J-1)                        ACXI1049
      IF (N.GT.IVJ) GO TO 10                 ACXI1050
      P=1./FLOAT(IVJ-IVJ1)                  ACXI1051
      Q=FLOAT(IVJ-N)*P                      ACXI1052
      P=FLOAT(N-IVJ1)*P                     ACXI1053
      G=Q*GAUNT(NDMN,J+9)+GAUNT(NDMN,J+10)*P ACXI1054
      GO TO 40                              ACXI1055
10  CONTINUE                               ACXI1056
20  G=GAUNT(NDMN,N)                        ACXI1057
      GO TO 40                              ACXI1058
30  ALF2=(EN2/END2)                         ACXI1059
      TERM1=(1.DO-ALF2)                     ACXI1060
      TERM1=(TERM1*EN)**.6666666666666666D0 ACXI1061
      TERM2=0.1728D0*(1.DO+ALF2)/TERM1      ACXI1062
      TERM3=0.0496D0*(1.DO-1.33333333333333D0*ALF2+ALF2*ALF2)/TERM1**2 ACXI1063
      G=1.DO-TERM2-TERM3                   ACXI1064
40  A=G*15.7457D9/(EN*(END2-EN2)*DFLOAT(NDASH)*END2) ACXI1065
      RETURN                                ACXI1066
      END                                    ACXI1067

      SUBROUTINE RADCOL (T,MVAL,IC,NMIN)     ACXI1069
      COMMON /EXPDAT/ CXP(707),MAXN         ACXI1070
      COMMON /RCRATS/ RADTOT(75),COLTOT(75) ACXI1071
      COMMON /TDEP/ TE32,TE12,CTE          ACXI1072
      DIMENSION MVAL(75)                   ACXI1073
      DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI1074
      DOUBLE PRECISION A,AKK,AN,C,COLTOT,CTE,CX,CXP,G,Q,RADTOT,T,TE12,TEACX ACXI1075

```



```

132,TOT ACXI1076
C ACXI1077
C RADIATIVE CASCADE COEFFICIENTS ACXI1078
C ACXI1079
C SUM FROM N TO ALL LEVELS DOWN TO NMIN (=1 OR 2) ACXI1080
C ACXI1081
DO 20 M=1,IC ACXI1082
J=MVAL(M) ACXI1083
TOT=0.DO ACXI1084
IF (J.LE.NMIN) GO TO 20 ACXI1085
K=J-1 ACXI1086
DO 10 I=NMIN,K ACXI1087
CALL RAD (A,I,J,G) ACXI1088
10 TOT=TOT+A ACXI1089
20 RADTOT(M)=TOT ACXI1090
C ACXI1091
C COLLISIONAL RATE TOTALS ON TO LEVEL N ACXI1092
C ACXI1093
C SUMMED FROM NMIN TO INFINITY ACXI1094
C ACXI1095
c Here also, Malcolm Walmsley has changed the lower principal quantum
c number from 20 to 1 (CASE A) or 2 (CASE B) for the rates.
nlo=nmin
DO 70 M=1,IC ACXI1096
N=MVAL(M) ACXI1097
AN=N ACXI1098
TOT=0.DO ACXI1099
L=N-1 ACXI1100
c DO 30 KK=20,L ACXI1101
do 30 kk=nlo,l
C=COLRAT(KK,N,T,TE12) ACXI1102
30 TOT=TOT+C ACXI1103
L=N+1 ACXI1104
MAX=N+40 ACXI1105
DO 60 KK=L,MAX ACXI1106
AKK=KK ACXI1107
C=COLRAT(N,KK,T,TE12) ACXI1108
CX=CXP(KK) ACXI1109
IF (CX.LT.1.D-30) GO TO 40 ACXI1110
CXN=CXP(N) ACXI1111
IF (CXN.LT.1.D-30) GO TO 40 ACXI1112
Q=CXN/CX ACXI1113
GO TO 50 ACXI1114
40 Q=DEXP(15.778D4*(1.DO/AKK**2-1.DO/AN**2)/T) ACXI1115
50 C=C*(AKK/AN)**2*Q ACXI1116
60 TOT=TOT+C ACXI1117
70 COLTOT(M)=TOT ACXI1118
RETURN ACXI1119
END ACXI1120

SUBROUTINE RECOMB (Z,ETEMP,N1,ALPHA) ACXI1122
C ACXI1123
C COMPUTES RECOMBINATION COEFFICIENT, ALPHA, ONTO LEVEL N1 FOR IONS ACXI1124
C OF EFFECTIVE CHARGE Z AT ELECTRON TEMPERATURE ETEMP ACXI1125
C ACXI1126
COMMON /RCMB/ SV0(99),SV1(99),SV2(99) ACXI1127
C SV0, SV1, SV2 NOT DOUBLE PRECISION ACXI1128

```

```

DIMENSION XV(99) ACXI1129
DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI1130
DOUBLE PRECISION ALPHA,CONST,ETEMP,F,FL,P,Q,SO,S1,S2,TE,U,V,X,XV,XACXI1131
1VI,XVI1,XXX,Y,Z,Z ACXI1132
Z2=Z*Z ACXI1133
TE=ETEMP*1.0D-4 ACXI1134
CONST=15.778D0/TE ACXI1135
FL=1.D0/(CONST*Z2)**.333333333333333D0 ACXI1136
CONST=5.197D-14*Z2*DSQRT(CONST) ACXI1137
XV(1)=.02D0 ACXI1138
DO 10 N=2,11 ACXI1139
10 XV(N)=XV(N-1)+.002D0 ACXI1140
DO 20 N=12,23 ACXI1141
20 XV(N)=XV(N-1)+.005D0 ACXI1142
DO 30 N=24,33 ACXI1143
30 XV(N)=XV(N-1)+.01D0 ACXI1144
DO 40 N=34,65 ACXI1145
40 XV(N)=XV(N-32)*10.D0 ACXI1146
DO 50 N=66,97 ACXI1147
50 XV(N)=XV(N-32)*10.D0 ACXI1148
F=N1 ACXI1149
X=15.778D0*Z2/(F*F*TE) ACXI1150
IF (X.LT.0.02D0) GO TO 80 ACXI1151
IF (X.GT.20.D0) GO TO 70 ACXI1152
DO 60 I=2,99 ACXI1153
XVI=XV(I) ACXI1154
IF (X.GT.XVI) GO TO 60 ACXI1155
IM1=I-1 ACXI1156
XVI1=XV(IM1) ACXI1157
P=1.D0/(XVI-XVI1) ACXI1158
Q=(XVI-X)*P ACXI1159
P=(X-XVI1)*P ACXI1160
S0=P*SVO(I)+Q*SVO(IM1) ACXI1161
S1=P*SV1(I)+Q*SV1(IM1) ACXI1162
S2=P*SV2(I)+Q*SV2(IM1) ACXI1163
GO TO 90 ACXI1164
60 CONTINUE ACXI1165
70 U=1.D0/X ACXI1166
V=U/3.D0 ACXI1167
XXX=X** .3333333333333333D0 ACXI1168
S0=1.D0-U*(1.D0-2.D0*U*(1.D0-3.D0*U*(1.D0-4.D0*U))) ACXI1169
S1=-.1728D0*XXX*(1.D0-V*(8.D0-V*(70.D0-V*(800.D0-V*11440.D0)))) ACXI1170
S2=-.0496D0*XXX**2*(1.D0-V*(3.D0-V*(32.D0-V*448.D0))) ACXI1171
GO TO 90 ACXI1172
80 S0=X*DEXP(X)*(-DLOG(X)-.5772D0+X) ACXI1173
XXX=X** .3333333333333333D0 ACXI1174
S1=.4629D0*X*(1.D0+4.D0*X)-1.0368D0*XXX**4*(1.D0+1.875D0*X) ACXI1175
S2=-.0672D0*X*(1.D0+3.D0*X)+.1488D0*XXX**5*(1.D0+1.8D0*X) ACXI1176
90 Y=(S0+FL*(S1+FL*S2))/F ACXI1177
ALPHA=CONST*Y ACXI1178
RETURN ACXI1179
END ACXI1180

SUBROUTINE REDUCE (M,IC,IR,SK) ACXI1182
C ACXI1183
C GIVEN A SET OF INTEGERS ACXI1184
C M(IT),IT=1,IC,SUCH THAT- ACXI1185

```

```

C      1) M(IT+1)=M(IT)+1 FOR IT.LE. IA ACXI1186
C      WHERE IA.GE.1 AND ACXI1187
C      2) (M(IT+1) - M(IT)).GT.1 FOR IT.GE. IA, ACXI1188
C      AND GIVEN A FUNCTION SUBPROGRAM BK ACXI1189
C      WHICH CALCULATES THE ACXI1190
C      ELEMENTS OF A LARGE ACXI1191
C      M(IC)*M(IC) MATRIX, ACXI1192
C      THIS SUBROUTINE USES LAGRANGE ACXI1193
C      INTERPOLATION OF ORDER ACXI1194
C      2*(IR+1) TO CALCULATE A ACXI1195
C      SMALLER IC*IC MATRIX SK ACXI1196
C      REQUIRES A FUNCTION SUBPROGRAM ACXI1197
C      PHI ACXI1198
C      IR MUST BE .LE. (IA-1) ACXI1199
C ACXI1200
C      DIMENSION IQ(8), M(75), SK(75,75) ACXI1201
C      DIMENSION STORE1(8), STORE2(8) ACXI1202
C      DOUBLE PRECISION DABS,DSQRT,DBLE,DLOG,DLOG10,DFLOAT,DEXP ACXI1203
C      DOUBLE PRECISION BK,DUCKIT,FL,PHI,PHITAU,SK,STORE1,STORE2 ACXI1204
C ACXI1205
C      LG=2*(IR+1) ACXI1206
C      IB=IC-IR ACXI1207
C      IBB=IB-1 ACXI1208
C      ICC=IC-1 ACXI1209
C ACXI1210
C      DO 10 IS=1,IC ACXI1211
C      DO 10 IT=1,IC ACXI1212
10 SK(IS,IT)=BK(M(IS),M(IT),IS) ACXI1213
C ACXI1214
C      DO 20 IT=1,IC ACXI1215
C      IA=IT ACXI1216
C      IF ((M(IT+1)-M(IT)).GT.1) GO TO 30 ACXI1217
20 CONTINUE ACXI1218
C ACXI1219
30 IF (IA.EQ.IC) GO TO 110 ACXI1220
C IF (IA.EQ.IB) GO TO 80 ACXI1221
C ACXI1222
C      DO 70 IT=IA,IBB ACXI1223
C      N1=M(IT)+1 ACXI1224
C      N2=M(IT+1)-1 ACXI1225
C      DO 40 ITAU=1,LG ACXI1226
C      IND=IT-IR-1+ITAU ACXI1227
40 IQ(ITAU)=M(IND) ACXI1228
C      DO 50 ITAU=1,LG ACXI1229
50 STORE1(ITAU)=PHI(IQ,IG,ITAU,IQ(ITAU)) ACXI1230
C      DO 70 N=N1,N2 ACXI1231
C      DO 60 ITAU=1,LG ACXI1232
60 STORE2(ITAU)=PHI(IQ,IG,ITAU,N) ACXI1233
C      DO 70 IS=1,IC ACXI1234
C      DUCKIT=BK(M(IS),N,IS) ACXI1235
C      DO 70 ITAU=1,LG ACXI1236
C      FL=STORE2(ITAU)/STORE1(ITAU) ACXI1237
C      IND=IT-IR-1+ITAU ACXI1238
70 SK(IS,IND)=SK(IS,IND)+DUCKIT*FL ACXI1239
C ACXI1240
80 IF (IR.EQ.0) GO TO 110 ACXI1241
C ACXI1242

```

```

DO 90 ITAU=1, LG
IND=IC-LG+ITAU
90 IQ(ITAU)=M(IND)
DO 100 ITAU=1, LG
PHITAU=1.DO/PHI(IQ, LG, ITAU, IQ(ITAU))
DO 100 IT=IB, ICC
N1=M(IT)+1
N2=M(IT+1)-1
DO 100 N=N1, N2
FL=PHI(IQ, LG, ITAU, N)*PHITAU
DO 100 IS=1, IC
IND=IC-LG+ITAU
100 SK(IS, IND)=SK(IS, IND)+BK(M(IS), N, IS)*FL
C
110 RETURN
END

SUBROUTINE RHS (CO, MVAL, IC)
C
ACXI1261
C COMPUTES THE RIGHT HAND SIDE OF EQUATIONS (2.7) OF BROCKLEHURST,
C MNRAS 148, 417 (1970).
C
COMMON /EXPDAT/ CXP(707), MAXN
COMMON /PARMS/ DENS, T, ITM
COMMON /TDEP/ TE32, TE12, CTE
DIMENSION MVAL(75), CO(75)
DOUBLE PRECISION DABS, DSQRT, DBLE, DLOG, DLOG10, DFLOAT, DEXP
DOUBLE PRECISION ALFA, CO, CTE, CXP, DENS, RT, T, TE12, TE32
DO 10 I=1, IC
J=MVAL(I)
CALL COLION (J, 1, T, RT)
CALL RECOMB (1.DO, T, J, ALFA)
CO(I)=-ALFA*CXP(J)*TE32*0.24146879D16/DFLOAT(J*J)-RT*DENS
10 CONTINUE
RETURN
END

C-----File: DATAIN.TXT-----
C Pivot Points for calculation
75 2 4
5 6 7 8 9 10 11 12 13 14 15 17 19 21 23 26 29 32 35 38 41 44 48
52 56 60 64 68 72 77 82 87 92 97 102 107 112 118 124 130 136 142 148 154 160 167
174 181 188 195 202 210 218 226 234 242 250 259 268 277 286 295 305 315 325 335 345 355 366
377 388 399 410 421 432
75 72 69 66
+0.000E+00+0.000E+00 +4.000E+03+1.000E+060000200001000010000000099
+6.000E+03+1.000E+060000200001000010000000099

C-----File: DATAOUT.TXT-----
C Pivot Points from DATAIN.TXT
*****

MVAL (75 VALUES) -
5 6 7 8 9 10 11 12 13 14 15 17

```

```

19  21  23  26  29  32  35  38  41  44  48  52
56  60  64  68  72  77  82  87  92  97 102 107
112 118 124 130 136 142 148 154 160 167 174 181
188 195 202 210 218 226 234 242 250 259 268 277
286 295 305 315 325 335 345 355 366 377 388 399
410 421 432
IVAL (4 VALUES) -
75  72  69  66
IR = 2

NO BACKGROUND FIELD.

```

```

1 TEMPERATURE = 4000. K,  DENSITY = 1.000E+06CM**-3,  NMIN =  2
(Case B)

```

| PAGE 1 | | | | | | |
|--------|-------------|---------|-------------|--------------|----------|--------------|
| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
| 5 | 6.55035E-02 | 1.0000 | .00000 | .00000 | .00000 | -7.93295E-03 |
| 6 | .11023 | 1.0000 | .00000 | .00000 | .00000 | -8.24342E-03 |
| 7 | .15474 | 1.0000 | .00000 | .00000 | .00000 | -8.65293E-03 |
| 8 | .19628 | 1.0000 | .00000 | .00000 | .00000 | -9.08830E-03 |
| 9 | .23408 | 1.0000 | .00000 | .00000 | .00000 | -9.52348E-03 |
| 10 | .26830 | 1.0000 | .00000 | .00000 | .00000 | -9.95098E-03 |
| 11 | .29930 | 1.0000 | .00000 | .00000 | .00000 | -1.03663E-02 |
| 12 | .32752 | 1.0000 | .00000 | .00000 | .00000 | -1.07681E-02 |
| 13 | .35354 | 1.0000 | .00000 | .00000 | .00000 | -1.11620E-02 |
| 14 | .37873 | 1.0000 | .00000 | .00000 | .00000 | -1.15789E-02 |
| 15 | .40072 | 1.0000 | .00000 | .00000 | .00000 | -1.19375E-02 |
| 16 | .42355 | -1.8973 | 2.36537E-02 | 5.58460E-02 | 2.8973 | 2.34374E-02 |
| 17 | .44784 | -2.4351 | 2.47209E-02 | 5.52007E-02 | 3.4351 | 3.12501E-02 |
| 18 | .47186 | -2.8289 | 2.44580E-02 | 5.18332E-02 | 3.8289 | 3.76914E-02 |
| 19 | .49712 | -3.5877 | 2.62511E-02 | 5.28065E-02 | 4.5877 | 4.97360E-02 |
| 20 | .52490 | -4.6127 | 2.90746E-02 | 5.53902E-02 | 5.6127 | 6.68038E-02 |
| 21 | .55530 | -5.6955 | 3.16957E-02 | 5.70787E-02 | 6.6955 | 8.64650E-02 |
| 22 | .58829 | -6.8003 | 3.40242E-02 | 5.78357E-02 | 7.8003 | .10851 |
| 23 | .62304 | -7.7302 | 3.52945E-02 | 5.66491E-02 | 8.7302 | .12973 |
| 24 | .65839 | -8.4010 | 3.53487E-02 | 5.36899E-02 | 9.4010 | .14808 |
| 25 | .69340 | -8.8514 | 3.45154E-02 | 4.97770E-02 | 9.8514 | .16343 |
| 26 | .72711 | -9.0309 | 3.27622E-02 | 4.50581E-02 | 10.031 | .17403 |
| 27 | .75844 | -8.8901 | 3.00873E-02 | 3.96700E-02 | 9.8901 | .17794 |
| 28 | .78718 | -8.6640 | 2.73595E-02 | 3.47564E-02 | 9.6640 | .17930 |
| 29 | .81311 | -8.3084 | 2.45010E-02 | 3.01323E-02 | 9.3084 | .17700 |
| 30 | .83617 | -7.8567 | 2.16548E-02 | 2.58975E-02 | 8.8567 | .17160 |
| 31 | .85648 | -7.3654 | 1.89875E-02 | 2.21693E-02 | 8.3654 | .16431 |
| 32 | .87423 | -6.8625 | 1.65611E-02 | 1.89436E-02 | 7.8625 | .15588 |
| 33 | .88975 | -6.3968 | 1.44585E-02 | 1.62500E-02 | 7.3968 | .14754 |
| 34 | .90323 | -5.9100 | 1.25369E-02 | 1.38801E-02 | 6.9100 | .13808 |
| 35 | .91491 | -5.4497 | 1.08659E-02 | 1.18764E-02 | 6.4497 | .12873 |
| 36 | .92507 | -5.0404 | 9.45546E-03 | 1.02213E-02 | 6.0404 | .12017 |
| 37 | .93388 | -4.6362 | 8.20399E-03 | 8.78480E-03 | 5.6362 | .11141 |

| | | | | | | |
|--|--------|--------------|-------------|--------------|----------|--------------|
| 38 | .94154 | -4.2611 | 7.12709E-03 | 7.56964E-03 | 5.2611 | .10308 |
| 39 | .94820 | -3.9202 | 6.20930E-03 | 6.54853E-03 | 4.9202 | 9.53702E-02 |
| 40 | .95400 | -3.5991 | 5.41246E-03 | 5.67345E-03 | 4.5991 | 8.79815E-02 |
| 41 | .95906 | -3.3023 | 4.72659E-03 | 4.92836E-03 | 4.3023 | 8.10577E-02 |
| 42 | .96348 | -3.0278 | 4.13539E-03 | 4.29212E-03 | 4.0278 | 7.45800E-02 |
| 43 | .96736 | -2.7755 | 3.62663E-03 | 3.74901E-03 | 3.7755 | 6.85689E-02 |
| 44 | .97076 | -2.5425 | 3.18728E-03 | 3.28329E-03 | 3.5425 | 6.29751E-02 |
| 45 | .97375 | -2.3236 | 2.80393E-03 | 2.87952E-03 | 3.3236 | 5.76772E-02 |
| 46 | .97638 | -2.1253 | 2.47507E-03 | 2.53494E-03 | 3.1253 | 5.28533E-02 |
| 47 | .97871 | -1.9425 | 2.18990E-03 | 2.23753E-03 | 2.9425 | 4.83840E-02 |
| 48 | .98078 | -1.7731 | 1.94159E-03 | 1.97965E-03 | 2.7731 | 4.42252E-02 |
| 49 | .98260 | -1.6110 | 1.72169E-03 | 1.75217E-03 | 2.6110 | 4.02307E-02 |
| 1 TEMPERATURE = 4000. K, DENSITY = 1.000E+06CM**-3, NMIN = 2 | | | | | | |
| (Case B) | | | | | | |
| PAGE 2 | | | | | | |
| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
| 50 | .98423 | -1.4662 | 1.53306E-03 | 1.55763E-03 | 2.4662 | 3.66498E-02 |
| 51 | .98568 | -1.3320 | 1.36807E-03 | 1.38795E-03 | 2.3320 | 3.33252E-02 |
| 52 | .98697 | -1.2070 | 1.22307E-03 | 1.23922E-03 | 2.2070 | 3.02200E-02 |
| 53 | .98813 | -1.0862 | 1.09319E-03 | 1.10632E-03 | 2.0862 | 2.72122E-02 |
| 54 | .98916 | -.97829 | 9.81137E-04 | 9.91886E-04 | 1.9783 | 2.45218E-02 |
| 55 | .99009 | -.87787 | 8.82279E-04 | 8.91106E-04 | 1.8779 | 2.20145E-02 |
| 56 | .99093 | -.78383 | 7.94668E-04 | 8.01940E-04 | 1.7838 | 1.96639E-02 |
| 57 | .99168 | -.69278 | 7.15652E-04 | 7.21652E-04 | 1.6928 | 1.73853E-02 |
| 58 | .99237 | -.61081 | 6.46821E-04 | 6.51797E-04 | 1.6108 | 1.53324E-02 |
| 59 | .99298 | -.53419 | 5.85621E-04 | 5.89761E-04 | 1.5342 | 1.34122E-02 |
| 60 | .99354 | -.46221 | 5.30999E-04 | 5.34452E-04 | 1.4622 | 1.16071E-02 |
| 61 | .99404 | -.39279 | 4.81564E-04 | 4.84449E-04 | 1.3928 | 9.86531E-03 |
| 62 | .99450 | -.32948 | 4.37991E-04 | 4.40412E-04 | 1.3295 | 8.27622E-03 |
| 63 | .99492 | -.27000 | 3.98956E-04 | 4.00992E-04 | 1.2700 | 6.78291E-03 |
| 64 | .99530 | -.21388 | 3.63868E-04 | 3.65585E-04 | 1.2139 | 5.37343E-03 |
| 65 | .99565 | -.15975 | 3.31958E-04 | 3.33409E-04 | 1.1598 | 4.01370E-03 |
| 66 | .99597 | -.10991 | 3.03567E-04 | 3.04796E-04 | 1.1099 | 2.76166E-03 |
| 67 | .99626 | -6.28759E-02 | 2.77960E-04 | 2.79004E-04 | 1.0629 | 1.57984E-03 |
| 68 | .99652 | -1.83102E-02 | 2.54797E-04 | 2.55685E-04 | 1.0183 | 4.60073E-04 |
| 69 | .99677 | 2.47899E-02 | 2.33613E-04 | 2.34371E-04 | .97521 | -6.22883E-04 |
| 70 | .99699 | 6.46838E-02 | 2.14639E-04 | 2.15286E-04 | .93532 | -1.62526E-03 |
| 71 | .99720 | .10245 | 1.97432E-04 | 1.97987E-04 | .89755 | -2.57408E-03 |
| 72 | .99739 | .13830 | 1.81792E-04 | 1.82268E-04 | .86170 | -3.47486E-03 |
| 73 | .99756 | .17318 | 1.67392E-04 | 1.67801E-04 | .82682 | -4.35106E-03 |
| 74 | .99772 | .20552 | 1.54437E-04 | 1.54790E-04 | .79448 | -5.16325E-03 |
| 75 | .99787 | .23618 | 1.42638E-04 | 1.42942E-04 | .76382 | -5.93339E-03 |
| 76 | .99801 | .26533 | 1.31868E-04 | 1.32131E-04 | .73467 | -6.66532E-03 |
| 77 | .99814 | .29311 | 1.22018E-04 | 1.22246E-04 | .70689 | -7.36279E-03 |
| 78 | .99825 | .32023 | 1.12894E-04 | 1.13091E-04 | .67977 | -8.04382E-03 |
| 79 | .99836 | .34539 | 1.04650E-04 | 1.04822E-04 | .65461 | -8.67533E-03 |
| 80 | .99846 | .36934 | 9.70984E-05 | 9.72479E-05 | .63066 | -9.27620E-03 |
| 81 | .99856 | .39217 | 9.01680E-05 | 9.02983E-05 | .60783 | -9.84911E-03 |

| | | | | | | |
|----|--------|--------|-------------|-------------|--------|--------------|
| 82 | .99864 | .41400 | 8.37956E-05 | 8.39095E-05 | .58600 | -1.03967E-02 |
| 83 | .99872 | .43534 | 7.78671E-05 | 7.79666E-05 | .56466 | -1.09319E-02 |
| 84 | .99880 | .45521 | 7.24808E-05 | 7.25680E-05 | .54479 | -1.14303E-02 |
| 85 | .99887 | .47417 | 6.75224E-05 | 6.75989E-05 | .52583 | -1.19057E-02 |
| 86 | .99893 | .49230 | 6.29507E-05 | 6.30179E-05 | .50770 | -1.23601E-02 |
| 87 | .99899 | .50966 | 5.87287E-05 | 5.87878E-05 | .49034 | -1.27953E-02 |
| 88 | .99905 | .52661 | 5.47911E-05 | 5.48431E-05 | .47339 | -1.32200E-02 |
| 89 | .99910 | .54249 | 5.11905E-05 | 5.12364E-05 | .45751 | -1.36179E-02 |
| 90 | .99915 | .55769 | 4.78616E-05 | 4.79021E-05 | .44231 | -1.39984E-02 |
| 91 | .99920 | .57224 | 4.47798E-05 | 4.48157E-05 | .42776 | -1.43628E-02 |
| 92 | .99924 | .58620 | 4.19229E-05 | 4.19546E-05 | .41380 | -1.47123E-02 |
| 93 | .99928 | .59979 | 3.92532E-05 | 3.92813E-05 | .40021 | -1.50526E-02 |
| 94 | .99932 | .61262 | 3.67973E-05 | 3.68223E-05 | .38738 | -1.53735E-02 |

1 TEMPERATURE = 4000. K, DENSITY = 1.000E+06CM**-3, NMIN = 2
(Case B)

| PAGE 3 | | | | | | |
|--------|--------|--------|-------------|--------------|----------|--------------|
| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
| 95 | .99936 | .62490 | 3.45180E-05 | 3.45402E-05 | .37510 | -1.56809E-02 |
| 96 | .99939 | .63670 | 3.24000E-05 | 3.24197E-05 | .36330 | -1.59759E-02 |
| 97 | .99942 | .64803 | 3.04296E-05 | 3.04472E-05 | .35197 | -1.62593E-02 |
| 98 | .99945 | .65905 | 2.85845E-05 | 2.86002E-05 | .34095 | -1.65349E-02 |
| 99 | .99948 | .66949 | 2.68785E-05 | 2.68924E-05 | .33051 | -1.67960E-02 |

1 TEMPERATURE = 6000. K, DENSITY = 1.000E+06CM**-3, NMIN = 2
(Case B)

| PAGE 1 | | | | | | |
|--------|--------|---------|-------------|--------------|----------|--------------|
| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
| 5 | .14001 | 1.0000 | .00000 | .00000 | .00000 | -6.68090E-03 |
| 6 | .19784 | 1.0000 | .00000 | .00000 | .00000 | -6.84550E-03 |
| 7 | .24882 | 1.0000 | .00000 | .00000 | .00000 | -7.09266E-03 |
| 8 | .29287 | 1.0000 | .00000 | .00000 | .00000 | -7.36148E-03 |
| 9 | .33108 | 1.0000 | .00000 | .00000 | .00000 | -7.63444E-03 |
| 10 | .36435 | 1.0000 | .00000 | .00000 | .00000 | -7.89897E-03 |
| 11 | .39387 | 1.0000 | .00000 | .00000 | .00000 | -8.15795E-03 |
| 12 | .41992 | 1.0000 | .00000 | .00000 | .00000 | -8.40096E-03 |
| 13 | .44398 | 1.0000 | .00000 | .00000 | .00000 | -8.64554E-03 |
| 14 | .46613 | 1.0000 | .00000 | .00000 | .00000 | -8.88430E-03 |
| 15 | .48708 | 1.0000 | .00000 | .00000 | .00000 | -9.12443E-03 |
| 16 | .50742 | -2.1042 | 2.02406E-02 | 3.98894E-02 | 3.1042 | 1.97207E-02 |
| 17 | .52777 | -2.6388 | 2.05739E-02 | 3.89827E-02 | 3.6388 | 2.54225E-02 |
| 18 | .54874 | -3.3472 | 2.15287E-02 | 3.92330E-02 | 4.3472 | 3.32005E-02 |
| 19 | .57105 | -4.2895 | 2.31787E-02 | 4.05899E-02 | 5.2895 | 4.39103E-02 |
| 20 | .59529 | -5.4528 | 2.52727E-02 | 4.24543E-02 | 6.4528 | 5.77766E-02 |
| 21 | .62160 | -6.7344 | 2.73239E-02 | 4.39573E-02 | 7.7344 | 7.40556E-02 |
| 22 | .64992 | -8.0594 | 2.91042E-02 | 4.47809E-02 | 9.0594 | 9.21745E-02 |
| 23 | .67955 | -9.2067 | 3.00044E-02 | 4.41533E-02 | 10.207 | .10959 |
| 24 | .70953 | -10.071 | 2.99080E-02 | 4.21520E-02 | 11.071 | .12466 |
| 25 | .73910 | -10.690 | 2.91033E-02 | 3.93768E-02 | 11.690 | .13734 |
| 26 | .76749 | -10.995 | 2.75689E-02 | 3.59208E-02 | 11.995 | .14622 |
| 27 | .79385 | -10.923 | 2.53107E-02 | 3.18833E-02 | 11.923 | .14983 |

| | | | | | | |
|---|--------|---------|-------------|--------------|----------|-------------|
| 28 | .81803 | -10.737 | 2.30205E-02 | 2.81413E-02 | 11.737 | .15138 |
| 29 | .83986 | -10.387 | 2.06393E-02 | 2.45746E-02 | 11.387 | .15002 |
| 30 | .85931 | -9.9155 | 1.82850E-02 | 2.12786E-02 | 10.916 | .14622 |
| 31 | .87648 | -9.3788 | 1.60718E-02 | 1.83368E-02 | 10.379 | .14081 |
| 32 | .89152 | -8.8152 | 1.40553E-02 | 1.57655E-02 | 9.8152 | .13439 |
| 33 | .90471 | -8.2831 | 1.23005E-02 | 1.35961E-02 | 9.2831 | .12795 |
| 34 | .91619 | -7.7176 | 1.06956E-02 | 1.16740E-02 | 8.7176 | .12056 |
| 35 | .92617 | -7.1752 | 9.29497E-03 | 1.00359E-02 | 8.1752 | .11316 |
| 36 | .93487 | -6.6863 | 8.10633E-03 | 8.67108E-03 | 7.6863 | .10632 |
| 37 | .94244 | -6.1994 | 7.05023E-03 | 7.48086E-03 | 7.1994 | 9.92640E-02 |
| 38 | .94902 | -5.7433 | 6.13848E-03 | 6.46823E-03 | 6.7433 | 9.25119E-02 |
| 39 | .95476 | -5.3255 | 5.35869E-03 | 5.61260E-03 | 6.3255 | 8.62214E-02 |
| 40 | .95977 | -4.9293 | 4.68003E-03 | 4.87619E-03 | 5.9293 | 8.01559E-02 |
| 41 | .96415 | -4.5607 | 4.09438E-03 | 4.24661E-03 | 5.5607 | 7.44430E-02 |
| 42 | .96799 | -4.2184 | 3.58857E-03 | 3.70725E-03 | 5.2184 | 6.90783E-02 |
| 43 | .97135 | -3.9018 | 3.15203E-03 | 3.24499E-03 | 4.9018 | 6.40723E-02 |
| 44 | .97431 | -3.6083 | 2.77423E-03 | 2.84737E-03 | 4.6083 | 5.93949E-02 |
| 45 | .97692 | -3.3317 | 2.44422E-03 | 2.50198E-03 | 4.3317 | 5.49559E-02 |
| 46 | .97921 | -3.0799 | 2.16031E-03 | 2.20616E-03 | 4.0799 | 5.08933E-02 |
| 47 | .98125 | -2.8469 | 1.91364E-03 | 1.95021E-03 | 3.8469 | 4.71165E-02 |
| 48 | .98305 | -2.6304 | 1.69851E-03 | 1.72780E-03 | 3.6304 | 4.35918E-02 |
| 49 | .98465 | -2.4231 | 1.50793E-03 | 1.53143E-03 | 3.4231 | 4.02037E-02 |
| 1 TEMPERATURE = 6000. K, DENSITY = 1.000E+06CM**-3, NMN = 2 | | | | | | |
| (Case B) | | | | | | |
| PAGE 2 | | | | | | |
| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
| 50 | .98608 | -2.2369 | 1.34398E-03 | 1.36296E-03 | 3.2369 | 3.71517E-02 |
| 51 | .98735 | -2.0641 | 1.20037E-03 | 1.21576E-03 | 3.0641 | 3.43107E-02 |
| 52 | .98848 | -1.9026 | 1.07401E-03 | 1.08652E-03 | 2.9026 | 3.16512E-02 |
| 53 | .98950 | -1.7465 | 9.60794E-04 | 9.70992E-04 | 2.7465 | 2.90738E-02 |
| 54 | .99041 | -1.6066 | 8.62911E-04 | 8.71268E-04 | 2.6066 | 2.67598E-02 |
| 55 | .99123 | -1.4761 | 7.76463E-04 | 7.83335E-04 | 2.4761 | 2.45994E-02 |
| 56 | .99196 | -1.3538 | 6.99788E-04 | 7.05456E-04 | 2.3538 | 2.25708E-02 |
| 57 | .99263 | -1.2354 | 6.30641E-04 | 6.35324E-04 | 2.2354 | 2.06046E-02 |
| 58 | .99323 | -1.1285 | 5.70299E-04 | 5.74187E-04 | 2.1285 | 1.88278E-02 |
| 59 | .99377 | -1.0285 | 5.16606E-04 | 5.19844E-04 | 2.0285 | 1.71636E-02 |
| 60 | .99426 | -.93439 | 4.68654E-04 | 4.71358E-04 | 1.9344 | 1.55973E-02 |
| 61 | .99471 | -.84365 | 4.25251E-04 | 4.27513E-04 | 1.8436 | 1.40856E-02 |
| 62 | .99512 | -.76074 | 3.86950E-04 | 3.88849E-04 | 1.7607 | 1.27037E-02 |
| 63 | .99548 | -.68280 | 3.52619E-04 | 3.54218E-04 | 1.6828 | 1.14039E-02 |
| 64 | .99582 | -.60920 | 3.21745E-04 | 3.23095E-04 | 1.6092 | 1.01760E-02 |
| 65 | .99613 | -.53821 | 2.93664E-04 | 2.94805E-04 | 1.5382 | 8.99118E-03 |
| 66 | .99641 | -.47277 | 2.68660E-04 | 2.69628E-04 | 1.4728 | 7.89879E-03 |
| 67 | .99667 | -.41098 | 2.46098E-04 | 2.46921E-04 | 1.4110 | 6.86695E-03 |
| 68 | .99690 | -.35241 | 2.25682E-04 | 2.26383E-04 | 1.3524 | 5.88876E-03 |
| 69 | .99712 | -.29578 | 2.07010E-04 | 2.07608E-04 | 1.2958 | 4.94263E-03 |
| 70 | .99732 | -.24331 | 1.90274E-04 | 1.90785E-04 | 1.2433 | 4.06600E-03 |
| 71 | .99750 | -.19362 | 1.75091E-04 | 1.75530E-04 | 1.1936 | 3.23575E-03 |

| | | | | | | |
|----|--------|--------------|-------------|-------------|--------|--------------|
| 72 | .99767 | -.14643 | 1.61286E-04 | 1.61663E-04 | 1.1464 | 2.44713E-03 |
| 73 | .99782 | -.10052 | 1.48574E-04 | 1.48898E-04 | 1.1005 | 1.67992E-03 |
| 74 | .99797 | -5.79291E-02 | 1.37132E-04 | 1.37411E-04 | 1.0579 | 9.68160E-04 |
| 75 | .99810 | -1.75243E-02 | 1.26705E-04 | 1.26947E-04 | 1.0175 | 2.92881E-04 |
| 76 | .99822 | 2.08957E-02 | 1.17186E-04 | 1.17395E-04 | .97910 | -3.49228E-04 |
| 77 | .99833 | 5.75251E-02 | 1.08476E-04 | 1.08657E-04 | .94247 | -9.61407E-04 |
| 78 | .99844 | 9.33029E-02 | 1.00406E-04 | 1.00563E-04 | .90670 | -1.55934E-03 |
| 79 | .99853 | .12651 | 9.31113E-05 | 9.32480E-05 | .87349 | -2.11422E-03 |
| 80 | .99862 | .15812 | 8.64261E-05 | 8.65453E-05 | .84188 | -2.64246E-03 |
| 81 | .99871 | .18827 | 8.02885E-05 | 8.03926E-05 | .81173 | -3.14641E-03 |
| 82 | .99878 | .21711 | 7.46431E-05 | 7.47340E-05 | .78289 | -3.62831E-03 |
| 83 | .99886 | .24532 | 6.93896E-05 | 6.94691E-05 | .75468 | -4.09954E-03 |
| 84 | .99892 | .27160 | 6.46141E-05 | 6.46838E-05 | .72840 | -4.53860E-03 |
| 85 | .99898 | .29668 | 6.02163E-05 | 6.02775E-05 | .70332 | -4.95771E-03 |
| 86 | .99904 | .32068 | 5.61599E-05 | 5.62137E-05 | .67932 | -5.35851E-03 |
| 87 | .99910 | .34367 | 5.24125E-05 | 5.24598E-05 | .65633 | -5.74254E-03 |
| 88 | .99915 | .36611 | 4.89166E-05 | 4.89583E-05 | .63389 | -6.11738E-03 |
| 89 | .99920 | .38716 | 4.57183E-05 | 4.57551E-05 | .61284 | -6.46887E-03 |
| 90 | .99924 | .40730 | 4.27603E-05 | 4.27928E-05 | .59270 | -6.80516E-03 |
| 91 | .99928 | .42659 | 4.00209E-05 | 4.00497E-05 | .57341 | -7.12738E-03 |
| 92 | .99932 | .44511 | 3.74805E-05 | 3.75060E-05 | .55489 | -7.43660E-03 |
| 93 | .99936 | .46315 | 3.51061E-05 | 3.51287E-05 | .53685 | -7.73774E-03 |
| 94 | .99939 | .48017 | 3.29208E-05 | 3.29409E-05 | .51983 | -8.02185E-03 |

1 TEMPERATURE = 6000. K, DENSITY = 1.000E+06CM**-3, NMIN = 2
(Case B)

PAGE 3

| N | BN | bsBETA | DBN/DN | D(LN(BN))/DN | 1-bsBETA | ZETA |
|----|--------|--------|-------------|--------------|----------|--------------|
| 95 | .99942 | .49649 | 3.08919E-05 | 3.09098E-05 | .50351 | -8.29422E-03 |
| 96 | .99945 | .51216 | 2.90060E-05 | 2.90219E-05 | .48784 | -8.55566E-03 |
| 97 | .99948 | .52722 | 2.72510E-05 | 2.72652E-05 | .47278 | -8.80694E-03 |
| 98 | .99951 | .54186 | 2.56072E-05 | 2.56199E-05 | .45814 | -9.05135E-03 |
| 99 | .99953 | .55575 | 2.40867E-05 | 2.40980E-05 | .44425 | -9.28306E-03 |

Appendix F

Observational Units

Radio telescopes use “antenna temperature,” or T_A , as units of intensity. Gordon et al. (1992) describe the derivation of these units, relate them to the even more peculiar units of T_A^* and T_R^* used in millimeter wave astronomy, and relate all of these to the units of physics suitable for physical analyses. Below, we quote sections from that paper.

F.1 What Radio Telescopes Measure

The definition of “spectral flux density” from a source of specific intensity I_ν is

$$S_\nu \equiv \int_{\text{source}} I_\nu d\Omega, \quad (\text{F.1})$$

and, in the radio range when $h\nu \ll kT$, we use the Rayleigh–Jeans approximation¹ for the specific intensity to obtain

$$S_\nu = \frac{2k}{\lambda^2} \int_{\text{source}} T(\theta, \phi, \nu) d\Omega, \quad (\text{F.2})$$

where $T(\theta, \phi, \nu)$ is the equivalent temperature of a black body that radiates I_ν at the frequency ν in the direction (θ, ϕ) . It parameterizes the specific intensity. For extragalactic molecular lines, astronomers report a somewhat different quantity, F , to characterize the flux density received in the line:

¹ Actually, $h\nu \approx kT$ for many observations in the millimeter wave range. In these cases, consider T to be an *effective radiation temperature*, i.e., a surrogate for a more complex expression (see (2) of Ulich and Haas (1976)). Because of the calibration techniques used with millimeter wave telescopes, expressions involving T still prove to be useful although the parameter may no longer be the radiation temperature.

$$F \equiv \int_{\text{line}} S_\nu d\nu \quad (\text{F.3})$$

$$= \frac{2k}{\bar{\lambda}^2} \int_{\text{source}} \int_{\text{line}} T(\theta, \phi, \nu) d\nu d\Omega \quad (\text{F.4})$$

a quantity that is the integral of spectral flux density over the width of the line. The parameter $\bar{\lambda}$ is the observed wavelength at the center of the spectral line.

In general, $T(\theta, \phi, \nu)$ is not a measured quantity. It varies within the telescope beam and cannot be observed directly. Furthermore, the units used by observers to report F for spectral lines are often in a telescope-dependent form that cannot be easily compared with observations made with other telescopes.

Kutner and Ulich (1981) have considered this problem in some detail. They concentrated upon correction of spectral observations for wide-angle scattering, stray radiation, and atmospheric extinction. In this chapter, we extend their work by considering the coupling of the antenna to sources of angular extent less than the beam so as to derive equations for reporting telescope-independent quantities.

F.2 How Radio Telescopes Measure

We consider below two circumstances. The first case deals with sources smaller than the main beam, i.e., where the source size can range from as small as a point to as large as the distance between the first “nulls” of the main beam. Most observations of spectral lines from external galaxies fall into this category. Therefore, this category is the principal subject of this chapter. The second case deals with objects of an angular size larger than the main beam, such as observations of galactic molecular clouds with millimeter wave telescopes of intermediate to large diameters (≥ 10 m).

F.2.1 Sources Smaller Than the Beam Size

In the commonly used “on-off” observing technique, we measure the direct product² of the telescope response and the source distribution over each point, (θ, ϕ) , within the solid angle of the source, Ω_S . In this case, we require Ω_S to be smaller than the solid angle of the main beam, Ω_B . For simplification, we omit the “subscript” ν in T and in S although most quantities are functions of ν . Here, the measured antenna temperature T_A of a source with a

² Observations made with a moving beam involve a convolution of beam and source rather than a direct product.

brightness temperature distribution $T_R(\theta, \phi)$ observed with an antenna with a normalized beam $f(\theta, \phi)$ is (see, e.g., Baars (1973))

$$T_A = \frac{\eta_R}{\Omega_A} \int_{\text{source}} T_R(\theta, \phi) f(\theta, \phi) d\Omega, \quad (\text{F.5})$$

where η_R is the radiation efficiency of the antenna accounting for ohmic losses, and $\Omega_A = \int_{4\pi} f(\theta, \phi) d\Omega$ is the solid angle of the antenna pattern. Normally, η_R is close to 1 for a well-designed telescope surface. Using the relationship

$$\frac{\eta_R}{\Omega_A} = \frac{G}{4\pi} = \frac{A}{\lambda^2}, \quad (\text{F.6})$$

where G is the antenna gain, λ is the wavelength of the observations, and $A \equiv \eta_A \pi (D/2)^2$ is the effective area of the antenna of diameter D with an *aperture efficiency*, η_A , we obtain

$$T_A = \frac{A}{\lambda^2} T_R \int_{\text{source}} \psi(\theta, \phi) f(\theta, \phi) d\Omega, \quad (\text{F.7})$$

where we have introduced the normalized source brightness distribution function, $\psi(\theta, \phi)$. The parameter T_R is the source brightness temperature at the position $(\theta, \phi) = (0, 0)$.

The substitution of (F.2) into (F.7) yields (Baars, 1973)

$$T_A = \frac{SA}{2k} \frac{1}{\Omega_S} \int_{\text{source}} \psi(\theta, \phi) f(\theta, \phi) d\Omega \quad (\text{F.8})$$

$$= \frac{SA}{2k} \frac{\Omega_S}{\Omega_S}, \quad (\text{F.9})$$

where we have defined the source solid angle

$$\Omega_S \equiv \int_{\text{source}} \psi(\theta, \phi) d\Omega \quad (\text{F.10})$$

and the beam-weighted source solid angle

$$\Omega_\Sigma \equiv \int_{\text{source}} \psi(\theta, \phi) f(\theta, \phi) d\Omega. \quad (\text{F.11})$$

The factor $K \equiv \Omega_S/\Omega_\Sigma$ corrects the measured antenna temperature for the weighting of the source distribution by the large antenna beam. Therefore, (F.9) gives the spectral flux density of a source smaller than the beam as

$$S = \frac{2k}{A} K T_A, \quad (\text{F.12})$$

and the flux density received in a spectral line, given by (F.3), becomes

$$F = \frac{8k}{\pi D^2} \frac{K}{\eta_A} \int_{\text{line}} T_A d\nu, \quad \Omega_B > \Omega_S, \quad (\text{F.13})$$

in terms of observational units.

For sources with Gaussian or disk distributions, the correction factor K can be written explicitly as

$$K = \begin{cases} 1 + x^2 & \text{Gaussian source} \\ \frac{x^2}{1 - \exp(-x^2)} & \text{disk source} \end{cases} \quad x \leq 1, \quad (\text{F.14})$$

where the quantity x is defined by

$$x = \begin{cases} \theta_S/\theta_B & \text{Gaussian source} \\ \sqrt{\ln 2} \theta_D/\theta_B & \text{disk source} \end{cases} \quad (\text{F.15})$$

where θ_S and θ_B are the widths of the source and beam at half-intensity, respectively, and θ_D is the angular diameter of the disk source. Table F.1 tabulates K as a function of source size for both a Gaussian and disk source.

Note that the basic characteristic of the antenna required to evaluate (F.13) is the aperture efficiency η_A that can normally be accurately determined³ from the observation of a point source ($K = 1$) or a small source of known size and brightness distribution such as a planet where K may be determined from Table F.1. As long as the source is smaller than the beam, there is no need to invoke the *beam efficiency*, defined as

$$\eta_B \equiv \frac{1}{\Omega_A} \int_{\text{mainbeam}} f(\theta, \phi) d\Omega = \Omega_B/\Omega_A, \quad (\text{F.16})$$

which is more difficult to determine since the entire main beam shape must be measured.

Table F.1 Correction factor K

| x | K | |
|-----|----------|-------|
| | Gaussian | Disk |
| 0.0 | 1.000 | 1.000 |
| 0.1 | 1.010 | 1.005 |
| 0.2 | 1.040 | 1.020 |
| 0.3 | 1.090 | 1.046 |
| 0.4 | 1.160 | 1.082 |
| 0.5 | 1.250 | 1.130 |
| 0.6 | 1.360 | 1.191 |
| 0.7 | 1.490 | 1.265 |
| 0.8 | 1.640 | 1.354 |
| 0.9 | 1.810 | 1.459 |
| 1.0 | 2.000 | 1.582 |

³ The effect of an error pattern from an imperfect reflector is, in this case, only to decrease η_A – precisely the quantity that we measure by observations of a point source of known flux density.

From (F.6) and (F.16), we obtain

$$\eta_B = \frac{\pi \eta_A D^2 \Omega_B}{4 \eta_R \lambda^2}. \quad (\text{F.17})$$

Combining (F.12) and (F.17) and putting $\eta_R = 1$, we find

$$S = \frac{2k}{\lambda^2} \frac{1}{\eta_B} T_A K \Omega_B, \quad (\text{F.18})$$

which for a uniformly bright source that just fills the main beam ($\Omega_S = \Omega_B$) is reduced to the well-known relationship

$$S = \frac{1}{\eta_B} \frac{2k}{\lambda^2} T_A \Omega_S. \quad (\text{F.19})$$

If the beam shape is known, one can convert the antenna efficiency η_A into beam efficiency η_B using (F.17). For example, the solid angle of a symmetrical Gaussian beam is

$$\Omega_B = 1.133 \theta_B^2, \quad (\text{F.20})$$

where the full width at half-flux density is given by

$$\theta_B = F_t \frac{\lambda}{D}. \quad (\text{F.21})$$

For a quadratic illumination function, Table F.2 gives the taper factor F_t as a function of the edge taper.

Assuming a feed with a 12-db taper – a common illumination for parabolic reflectors used in radio astronomy – and substituting (F.20) and (F.21) into (F.17), we find

$$\eta_B = 1.2 \frac{\eta_A}{\eta_R}, \quad (\text{F.22})$$

where, in many cases, the radiation efficiency $\eta_R \approx 1$.

Table F.2 Taper factor F_t

| Taper (db) | F_t |
|------------|-------|
| 0 | 1.020 |
| −8 | 1.115 |
| −10 | 1.135 |
| −12 | 1.155 |
| −14 | 1.170 |
| −16 | 1.186 |
| −18 | 1.198 |
| −20 | 1.208 |
| −22 | 1.218 |
| −24 | 1.227 |
| −∞ | 1.267 |

We now arrive at a telescope-independent expression for the line flux density F of a spectral line observed from a source of angular size less than the beam. Evaluating the factors in (F.13), we find

$$\frac{F}{[\text{W m}^{-2}]} = 3.515 \times 10^{-23} \left(\frac{D}{[\text{m}]} \right)^{-2} \frac{K}{\eta_A} \times \int_{\text{line}} \frac{T_A}{[\text{K}]} \frac{d\nu}{[\text{Hz}]}, \quad \Omega_B > \Omega_S. \quad (\text{F.23})$$

To convert F to ($\text{erg s}^{-1} \text{cm}^{-2}$) or to (Jy Hz), multiply by 10^3 or 10^{26} , respectively.

Note that F is independent of the telescope size. (F.9) shows the measured T_A to be proportional to $\eta_A D^2/K$, thus precisely canceling out the factor $K/(\eta_A D^2)$ in (F.23).

Sanders et al. (1991) have also considered this problem, but their (A6) and (A8) do not quite follow standard antenna theory. Furthermore, the denominator of (A11) is missing a factor of $2 \ln 2 \approx 1.4$. Therefore, the numerical results in their resulting equations (A12)–(A15) that relate the observations to astrophysical quantities need to be multiplied by this factor.

F.2.2 Sources Larger Than the Beam Size

This case is more difficult. The random imperfections of most telescopes give rise to an error beam that is many times wider than the main diffraction beam of the telescope. Even weak radiation entering the error beam can contribute significantly to the resulting spectrum because of the large solid angle of the error beam. The coupling of the overall beam to the source region is often too complex to be corrected by a simple mathematical procedure. This situation is encountered when observing giant molecular clouds in our galaxy with a large millimeter wave antenna.

To obtain accurate measurements of the line flux density F from such extended sources, one needs a detailed knowledge of the antenna pattern out to an angle at least as large as that of the source *and* a detailed knowledge of the source brightness distribution so as to calculate the coupling of the antenna and source. The large-scale antenna pattern can be difficult to measure and the source distribution is, of course, usually unknown.

As a practical approach, we suggest the use of a quantity that we call the *effective beam efficiency*, η'_B ,

$$\eta'_B(\Theta) \equiv \int_{\Theta} f(\theta, \phi) d\Omega / \Omega_A, \quad (\text{F.24})$$

in which the integration is extended over a solid angle Θ equal to that of the source. So, if the source size is known, η'_B will be the best representation of the coupling of the beam to the source. If we also assume that the source is uniformly bright over its angle Θ , then the measured antenna temperature relates to the effective brightness temperature of the source by

$$T_A = \eta'_B T_R. \quad (\text{F.25})$$

Alternatively, (F.19) is valid for this case if η'_B replaces η_B .

Most observers follow procedures described by Kutner and Ulich (1981), which describe observations in terms of the parameter T_R^* that corrects observations for all telescope-dependent parameters *except* the coupling of the antenna to the source brightness distribution. In terms used by Kutner and Ulich (1981), and under the assumption of a uniformly bright source,

$$\eta'_B = \eta_c \eta_s, \quad (\text{F.26})$$

where η_c and η_s are their “coupling” and “extended source” efficiencies, respectively. Unfortunately, η_c generally cannot be measured and can be calculated only with simplifying assumptions.

In our approach, it is possible to estimate η'_B by observing a series of sources of different sizes using the planets (a few arcseconds to an arcminute) and the moon ($\approx 30'$). Interpolation between $1'$ and $30'$ could result in large errors. Extrapolation of η'_B beyond $30'$ could be determined using the complete forward beam efficiency (over 2π sr), which may be obtained from the standard “sky tips” used to measure the atmospheric extinction. Note that the contribution from the main beam, sidelobes, and error pattern are all present in η'_B . Thus, a direct measurement of η'_B is more accurate than any theoretical calculation. We repeat that such measurements require that the source size is known and that the brightness distribution is constant over the source.

If a reasonable estimate of $\eta'_B(\Theta)$ over a source size Θ is available, one could correct the measured antenna temperature at each point into a “main beam” value by multiplying by η_B/η'_B . The mapped source could then be processed as if it had been observed with a “clean beam” of efficiency η_B .

For extragalactic sources that are only a few times larger than the main beam of a well-behaved antenna, we recommend that observers map the source at $\theta_B/2$ (Nyquist sampling) intervals with respect to θ_B given by (F.21). Each measurement of line flux density can be corrected to telescope-independent quantities using expressions given in this chapter. The total line flux density for the source would then be the sum of these measurements. Although significant errors due to beam imperfections would still exist in the sum owing to radiation entering the sidelobes and error beam, the restriction to sources only a few times larger than the beam would minimize these con-

tributions. Although the resulting total F would still overestimate the actual line flux density, we do not know of a better, alternative procedure.

F.2.3 Antenna Temperature Scale

Filled-aperture, centimeter wave telescopes calibrated by hot and cold loads placed in front of the receiver produce spectra in units of T_A described in this chapter. We assume that such observations have been corrected for atmospheric extinction, if present.

Unlike centimeter wave telescopes, millimeter wave telescopes use the atmosphere in calibration procedures involving choppers or vanes and produce spectra in intensity units of T_A^* , T_R^* , or derivatives thereof (see especially Kutner and Ulich (1981), Guilleoteau (1988), and Downes (1989)). In effect, these units presume angular sizes for the emitting region. Using the definitions given by Kutner and Ulich (1981),⁴ we find

$$T_A = T_A^* \eta_l \quad (\text{F.27})$$

and

$$T_A = T_R^* \eta_s, \quad (\text{F.28})$$

to relate millimeter wave intensity units to our unit T_A . Here, η_l is the “forward beam efficiency” and η_s is the “extended source efficiency” defined to be $\eta_l \eta_{fss}$, where η_{fss} is called the “forward spillover and scattering efficiency.” The efficiency η_l results from a sky tip by extrapolation of the measured antenna temperature as a function of air mass to the point where the air mass is zero. The determination of η_s is less straightforward, because it involves a choice for the size of the “diffraction” beam as described by Kutner and Ulich (1981). Usually, η_s is measured by observations of the moon.

The NRAO 12-m telescope produces spectra in intensity units of T_R^* . The temperature scale of its spectra can be converted into our units by using an efficiency η_s of ≈ 0.64 for observations from 70 to 310 GHz and ≈ 0.59 for observations from 330 to 360 GHz (Jewell, 1990).

The IRAM 30-m telescope produces spectral intensities in a variety of units depending upon what the observer enters in the command SET EFFICIENCY of the observing program OBS. Entering the “forward efficiency” (η_l) produces spectra in units of T_A^* ; entering the “extended efficiency” (η_s), T_R^* ; and entering the “main beam efficiency,” T_{mb} .

Table F.3 lists efficiencies that obtain for the IRAM 30-m telescope at this writing that have been taken from Thum (1986), Mauersberger et al. (1989), Baars et al. (1989), and Greve (1992). Depending upon which efficiency was

⁴ Some of these definitions are clarifications of ones originally given by Ulich and Haas (1976).

Table F.3 Efficiencies for the IRAM 30-m telescope

| Frequency (GHz) | η_l | η_s | η_{mb} |
|-----------------|----------|----------|-------------|
| 80–115 | 0.90 | 0.69 | 0.59 |
| 140–160 | 0.86 | 0.68 | 0.55 |
| 210–260 | 0.90 | 0.75 | 0.46 |
| 330–360 | 0.84 | 0.60 | 0.20 |

entered into OBS, either (F.27) or (F.28) may be used to convert spectral intensities taken with the IRAM 30-m telescope into the general units of T_A used in this chapter. In addition, if spectra are reported in units of main beam brightness temperature, one should multiply these intensities by η_{mb} to convert them into our traditional units of T_A .

Similar procedures should apply to the temperature scales used at other millimeter wave telescopes.

References

- Abramowitz, M. and Stegun, I. A.: 1964, *Handbook of Mathematical Functions*, United States Government Printing Office, Washington, DC.
- Adler, D. S., Wood, D. O. S. and Goss, W. M.: 1996, *Astrophys. J.* **471**, 871.
- Afflerbach, A., Churchwell, E. B., Accord, J. M., Hofner, P., Kurtz, S. and DePree, C. G.: 1996, *Astrophys. J., Suppl. Ser.* **106**, 423.
- Altenhoff, W. J., Mezger, P. G., Wendker, H. and Westerhout, G.: 1960, Meßprogramme bei der Wellenlänge 11 cm am 25m-Radioteleskop Stockert. IV. Die Durchmusterung der Milchstraße und die Quellen-Durchmusterung bei 2.7 GHz, *Technical Report 59*, Veröff. Univ. Sternwarte Bonn.
- Altenhoff, W. J., Strittmatter, P. A. and Wendker, H. J.: 1981, *Astron. Astrophys.* **93**, 48.
- Anantharamaiah, K. R.: 1986, *J. Astrophys. Astron.* **7**, 131.
- Anantharamaiah, K. R., Erickson, W. C., Payne, H. E. and Kantharia, N. G.: 1994, *Astrophys. J.* **430**, 682.
- Anantharamaiah, K. R., Erickson, W. C. and Radhakrishnan, V.: 1985, *Nature* **315**, 647.
- Anantharamaiah, K. R. and Goss, W. M.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 267.
- Anantharamaiah, K. R., Goss, W. M. and Dewdney, P. E.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 123.
- Anantharamaiah, K. R. and Kantharia, N. G.: 1999, in A. R. Taylor, T. L. Landecker and G. Jonas (eds), *New Perspective on the Interstellar Medium*, Vol. CS-167, ASP Conf. Ser., Astron. Soc. Pacific, Provo, Utah, p. 123.
- Anantharamaiah, K. R., Payne, H. E. and Erickson, W. C.: 1988, *Mon. Not. R. Astron. Soc.* **235**, 151.
- Anantharamaiah, K. R., Viallefond, F., Mohan, N. R., Goss, W. M. and Zhao, J.-H.: 2000, *Astrophys. J.* **537**, 613.
- Anantharamaiah, K. R., Zhao, J.-H., Goss, W. M. and Viallefond, F.: 1993, *Astrophys. J.* **419**, 585.
- Ariskin, V. I., Alekseev, Y. I., Gladyshev, A. S., Lekht, E. E. and Rudnitskij, G. M.: 1979, *Astron. Tsirk.* **1049**, 1.
- Ariskin, V. I., Kolotovkina, S. A., Lekht, E. E., Rudnitskij, G. M. and Sorochenko, R. L.: 1982, *Astron. Zh.* **59**, 38. English translation: 1982 Sov. Astron. – AJ, 26:23.
- Audi, G. and Wapstra, A. H.: 1995, *Nucl. Phys. A* **4**, 409.
- Baars, J. W. M.: 1973, *IEEE Trans. Ant. Prop.* **AP-21**, 461.
- Baars, J. W. M., Güsten, R. and Schultz, A.: 1989, The 345 GHz cooled Schottky receiver at the 30-m telescope, *Technical report 71*, MPIfR-Division of mm-Technology.
- Baker, J. G. and Menzel, D. H.: 1938, *Astrophys. J.* **88**, 52.

- Balick, B., Gammon, R. H. and Hjellming, R. M.: 1974, *Proc. Astron. Soc. Pacific* **86**, 616.
- Ball, J. A., Cesarsky, D., Goldberg, L. and Lilley, A. E.: 1970, *Astrophys. J.* **162**, L25.
- Balser, D. S., Bania, T. M., Rood, R. T. and Wilson, T. L.: 1999, *Astrophys. J.* **510**, 759.
- Bania, T. M.: 2001, Private communication to M. A. Gordon. Figure only. The data appear in Bania et al. 2000.
- Bania, T. M., Rood, R. T. and Balser, D. S.: 2000, in L. da Silva, M. Spite and J. R. de Medeiros (eds), *The Light Elements and Their Evolution*, Vol. Proceedings of IAU Symposium 198, Astron. Soc. Pacific, Provo, Utah, p. 214.
- Bania, T. M., Rood, R. T. and Balser, D. S.: 2002, *Nature* **415**, 54.
- Becker, W. and Fenkart, R.: 1963, *Z. Ap* **56**, 257.
- Beigman, I. I.: 1977, *Zh. Ehksp. Teor. Fiz.* **73**, 1729. English translation: 1977 Sov. Phys. – JETP, 46:908.
- Bell, M. B.: 1997, *Publ. Astron. Soc. Pac.* **109**, 609.
- Bell, M. B., Avery, L. W., Seaquist, E. R. and Vallée, J. P.: 2000, *Publ. Astron. Soc. Pac.* **112**, 1236.
- Bell, M. B. and Seaquist, E. R.: 1977, *Astron. Astrophys.* **56**, 461.
- Bell, M. B. and Seaquist, E. R.: 1978, *Astrophys. J.* **233**, 378.
- Bell, M. B., Seaquist, E. R., Mebold, U., Reif, K. and Shaver, P. A.: 1984, *Astron. Astrophys.* **130**, 1.
- Berger, P. S. and Simon, M.: 1972, *Astrophys. J.* **171**, 191.
- Berulis, I. I., Smirnov, G. T. and Sorochenko, R. L.: 1975, *Pis'ma Astron. Zh.* **1**, 28. English translation: 1975 Sov. Astron. Lett. 1:187.
- Berulis, I. I. and Sorochenko, R. L.: 1973, *Astron. Zh.* **50**, 270. English translation: 1977 Sov. Astron. – AJ, 17:179.
- Berulis, I. I. and Sorochenko, R. L.: 1983, in N. V. Andreenko (ed.), *Proceedings of the XV All-Union Conference of Galactic and Extragalactic Radioastronomy*, Inst. Rad. Elect., Kharkov, p. 190.
- Bethe, H. A. and Salpeter, E. E.: 1957, *Quantum Mechanics of One- and Two-Electron Systems*, Academic Press, Inc., New York.
- Biegging, J. H., Goss, W. M. and Wilcots, E. M.: 1991, *Astrophys. J., Suppl. Ser.* **75**, 999.
- Black, J. H. and Dalgarno, A.: 1973, *Astrophys. Lett.* **15**, 79.
- Blake, D. H., Crutcher, R. M. and Watson, W. D.: 1980, *Nature* **287**, 707.
- Blitz, L., Fich, M. and Stark, A. A.: 1982, *Astrophys. J., Suppl. Ser.* **49**, 183.
- Bochkarev, N. G.: 1988, *Astron. Nachr.* **310**, 399.
- Bohr, N.: 1913, *Phil. Mag.* **26**, 1.
- Bohr, N.: 1914, *Fysisk Tidsskrift* **12**, 97.
- Bohr, N.: 1923, *Proc. Phys. Soc.* **35**, 275.
- Boreiko, R. T. and Clark, T. A.: 1986, *Astron. Astrophys.* **157**, 353.
- Bracewell, R. M.: 1965, *The Fourier Transform and Its Applications*, first edn, McGraw-Hill, New York.
- Brackett, F. S.: 1922, *Astrophys. J.* **56**, 154.
- Brault, J. W. and Noyes, R. W.: 1983, *Astrophys. J.* **269**, L61.
- Brocklehurst, M. and Leeman, S.: 1971, *Ap. Letters* **9**, 36.
- Brocklehurst, M. and Salem, M.: 1977, *Computer Physics Communications* **13**, 39.
- Brocklehurst, M. and Seaton, M. J.: 1972, *Mon. Not. R. Astron. Soc.* **157**, 179.
- Brown, R. L. and Gómez-González, J.: 1975, *Astrophys. J.* **200**, 598.
- Brown, R. L. and Lockman, F. J.: 1975, *Astrophys. J.* **200**, L155.
- Burles, S., Nollett, K. M. and Turner, M. S.: 2001, *Astrophys. J.* **552**, L1.
- Burton, W. B.: 1974, in G. L. Vershuur and K. I. Kellermann (eds), *Galactic and Extragalactic Radio Astronomy*, first edn, Springer-Verlag, New York, p. 85.
- Burton, W. B. and Gordon, M. A.: 1978, *Astron. Astrophys.* **63**, 7.
- Cameron, A. G. W.: 1973, *Space Sci. Rev.* **15**, 121.
- Cane, H. V.: 1978, *Aust. J. Phys.* **31**, 561.
- Casse, J. L. and Shaver, P. A.: 1977, *Astron. Astrophys.* **61**, 805.
- Caswell, J. L. and Haynes, R. F.: 1987, *Astron. Astrophys.* **171**, 261.

- Cesarsky, C. J. and Cesarsky, D. A.: 1971, *Astrophys. J.* **169**, 293.
- Cesarsky, D. A. and Cesarsky, C. J.: 1973, *Astrophys. J.* **184**, 83.
- Chaisson, E. J.: 1975, *Astrophys. J.* **197**, L65.
- Chaisson, E. J. and Rodríguez, L. F.: 1977, *Astrophys. J.* **214**, L111.
- Chandrasekhar, S.: 1950, *Radiative Transfer*, Clarendon Press, Oxford.
- Chang, E. S.: 1984, *J. Phys. B* **17**, L11.
- Chang, E. S. and Noyes, R. W.: 1983, *Astrophys. J.* **275**, L125.
- Chapman, S. and Cowling, T. G.: 1960, *The Mathematical Theory of Non-Uniform Gases*, Cambridge University Press, Cambridge.
- Churchwell, E.: 1971, *Astron. Astrophys.* **15**, 90.
- Churchwell, E.: 1975, in T. L. Wilson and D. Downes (eds), *HII Regions and Related Topics*, Springer-Verlag, Berlin, p. 245.
- Churchwell, E. B.: 1980, in P. A. Shaver (ed.), *Radio Recombination Lines*, Reidel, Dordrecht, p. 225.
- Churchwell, E. B., Smith, L. F., Mathis, J., Mezger, P. G. and Huchtmeier, W.: 1978, *Astron. Astrophys.* **70**, 719.
- Churchwell, E. B. and Walmsley, C. M.: 1975, *Astron. Astrophys.* **38**, 451.
- Churchwell, E., Mezger, P. G. and Huchtmeier, W.: 1974, *Astron. Astrophys.* **32**, 283.
- Churchwell, E., Terzian, Y. and Walmsley, C. M.: 1976, *Astron. Astrophys.* **48**, 331.
- Clark, B. G.: 1965, *Astrophys. J.* **142**, 1398.
- Clark, T. A., Naylor, D. A. and Davis, G. R.: 2000a, *Astron. Astrophys.* **357**, 757.
- Clark, T. A., Naylor, D. A. and Davis, G. R.: 2000b, *Astron. Astrophys.* **361**, L60.
- Cohen, M., Bieging, J. H., Dreher, J. W. and Welch, W. J.: 1985, *Astrophys. J.* **292**, 249.
- Colley, D.: 1980, *Mon. Not. R. Astron. Soc.* **193**, 495.
- Cox, P., Martín-Pintado, J., Bachiller, R., Bronfman, L., Cernicharo, J., Lyman, L. A. and Roelfsema, P. R.: 1995, *Astron. Astrophys.* **295**, L39.
- Davies, R. D.: 1971, *Astrophys. J.* **163**, 479.
- Davis, J. T. and Vaughan, J. M.: 1963, *Astrophys. J.* **137**, 1302.
- Dickel, H. R., Goss, W. M. and Condon, G. R.: 1996, *Astrophys. J.* **373**, 158.
- Dieter, N. H.: 1967, *Astrophys. J.* **150**, 435.
- Downes, D.: 1989, in D. M. Alloin and J.-M. Mariotti (eds), *Diffraction-Limited Imaging with Very Large Telescopes*, Kluwer, Dordrecht, p. 53.
- Downes, D., Wilson, T. L., Bieging, J. and Wink, J.: 1980, *Astron. Astrophys. Suppl. Ser.* **40**, 379.
- Dravskikh, A. F. and Dravskikh, Z. V.: 1969, *Astron. Zh.* **46**, 455. English translation: 1969 Sov. Astron.-A.J. 13:360.
- Dravskikh, A. F., Dravskikh, Z. V., Kolbasov, V. A., Mizezhnikov, G. S., Nikulin, D. E. and Shteinsleiger, V. B.: 1965, *Dok. Akad. Nauk SSSR* **163**, 332. English translation: 1966, Sov. Phys. – Dokl. 10:627.
- Dravskikh, Z. V.: 1994, Personal letter to M. A. Gordon.
- Dravskikh, Z. V.: 1996, Email message to M. A. Gordon.
- Dravskikh, Z. V. and Dravskikh, A. F.: 1964, *Astron. Tsirk.* **282**, 2.
- Dravskikh, Z. V. and Dravskikh, A. F.: 1967, *Astron. Zh.* **44**, 35.
- Dravskikh, Z. V., Dravskikh, A. F. and Kolbasov, V. A.: 1966, *Trans. IAU XIIB*, 360. Paper presented by Y. Parijski at the XII General Assembly of the IAU in Hamburg, Germany, 1964.
- Dupree, A. K.: 1968, *Astrophys. J.* **152**, L125.
- Dupree, A. K.: 1974, *Astrophys. J.* **187**, 25.
- Egorova, T. M. and Ryzkov, N. F.: 1960, *Izv. Glavn. Astrofiz. Obs.* **21**, 140.
- Elmegreen, B. G. and Lada, C. J.: 1977, *Astrophys. J.* **214**, 725.
- Elmegreen, B. G. and Morris, M.: 1979, *Astrophys. J.* **229**, 593.
- Emerson, D. T.: 2002, Private communication to M. A. Gordon.
- Erickson, W. C., McConnell, D. and Anantharamaiah, K. R.: 1995, *Astrophys. J.* **454**, 125.
- Ershov, A. A. and Berulis, I. I.: 1989, *Pis'ma Astron. Zh.* **15**, 413. English translation: 1989 Sov. Astron. Lett. 15:413.

- Ershov, A. A., Iljsov, Y. P., Lekht, E. E., Smirnov, G. T., Solodkov, V. T. and Sorochenko, R. L.: 1984, *Pis'ma Astron. Zh.* **10**, 833. English translation: 1984 Sov. Astron. Lett. 10:348.
- Ershov, A. A., Lekht, E. E., Rudnitskij, G. M. and Sorochenko, R. L.: 1982, *Pis'ma Astron. Zh.* **8**, 694. English translation: 1982 Sov. Astron. Lett. 8:374.
- Ershov, A. A., Lekht, E. E., Smirnov, G. T. and Sorochenko, R. L.: 1987, *Pis'ma Astron. Zh.* **13**, 19. English translation: 1987 Sov. Astron. Lett. 13:8.
- Evans, N. J., Munday, L. G., Davies, J. H. and Vanden Bout, P. A.: 1987, *Astrophys. J.* **312**, 344.
- Ewen, H. J. and Purcell, E. M.: 1951, *Nature* **168**, 356.
- Federman, S. R., Glassgold, A. E. and Kwan, J.: 1979, *Astrophys. J.* **227**, 466.
- Ferguson, E. and Shlüter, H.: 1963, *Ann. Phys. (N.Y.)* **22**, 351.
- Field, G. B.: 1974, *Astrophys. J.* **187**, 453.
- Field, G. B., Goldsmith, D. W. and Habing, H. J.: 1969, *Astrophys. J.* **155**, L149.
- Finn, G. D. and Mugglestone, D. E.: 1965, *Mon. Not. R. Astron. Soc.* **129**, 221.
- Fowler, W. A., Reeves, H. and Silk, J.: 1970, *Astrophys. J.* **162**, 49.
- Frey, A., Lemke, D., Fahrback, U. and Thum, C.: 1979, *Astron. Astrophys.* **74**, 133.
- Garay, G., Gathier, R. and Rodríguez, L. F.: 1989, *Astron. Astrophys.* **266**, 101.
- Garay, G., Gómez, Y., Lisano, S. and Brown, R. L.: 1998, *Astrophys. J.* **501**, 699.
- Garay, G., Lizano, S., Gómez, Y. and Brown, R. L.: 1998, *Astrophys. J.* **501**, 710.
- Garay, G. and Rodríguez, L. F.: 1983, *Astrophys. J.* **266**, 263.
- Gardner, F. F. and McGee, R. X.: 1967, *Nature* **213**, 480.
- Gaume, R. A., Wilson, T. L. and Johnston, K. J.: 1994, *Astrophys. J.* **425**, 127.
- Gee, C. S., Percival, I. C., Lodge, I. C. and Richards, D.: 1976, *Mon. Not. R. Astron. Soc.* **175**, 209.
- Georgelin, Y. M. and Georgelin, Y. P.: 1976, *Astron. Astrophys.* **49**, 57.
- Giannani, T., Nisino, B., Lorenzetti, D., DiGiorgio, A. M., Spinoglio, L., Benedettini, M., Saraceno, P., Smith, H. A. and White, G. J.: 2000, *Astron. Astrophys.* **358**, 310.
- Goldberg, L.: 1966, *Astrophys. J.* **144**, 1225.
- Goldberg, L.: 1968, in Y. Terzian (ed.), *Interstellar Ionized Hydrogen*, W. A. Benjamin, Inc., New York, p. 373.
- Goldberg, L. and Dupree, A. K.: 1967, *Nature* **215**, 41.
- Goldreich, P. and Kwan, J.: 1974, *Astrophys. J.* **189**, 441.
- Goldwire, Jr., H. C.: 1968, *Astrophys. J., Suppl. Ser.* **17**(152), 445.
- Goldwire, Jr., H. C. and Goss, W. M.: 1967, *Astrophys. J.* **149**, 15.
- Gómez, Y., Garay, G. and Lizano, S.: 1995, *Astrophys. J.* **453**, 727.
- Gómez, Y., Lebron, M., Rodríguez, L. F., Garay, G., Lizano, S., Escalante, V. and Canto, J.: 1998, *Astrophys. J.* **503**, 297.
- Gómez, Y., Moran, J. M., Rodríguez, L. F. and Garay, G.: 1989, *Astrophys. J.* **345**, 862.
- Gordon, M. A.: 1969, *Astrophys. J.* **158**, 479.
- Gordon, M. A.: 1988, in G. L. Verschuur and K. I. Kellermann (eds), *Galactic and Extra-Galactic Radio Astronomy*, second edn, Springer-Verlag, New York, p. 37.
- Gordon, M. A.: 1989, *Astrophys. J.* **337**, 782.
- Gordon, M. A.: 1992, *Astrophys. J.* **387**, 701.
- Gordon, M. A.: 1994, *Astrophys. J.* **421**, 314.
- Gordon, M. A.: 2003, *Astrophys. J.* **589**, 953.
- Gordon, M. A., Baars, J. W. M. and Cocke, W. J.: 1992, *Astron. Astrophys.* **264**, 337.
- Gordon, M. A., Brown, R. L. and Gottesman, S. T.: 1972, *Astrophys. J.* **178**, 119.
- Gordon, M. A. and Burton, W. B.: 1976, *Astrophys. J.* **208**, 346.
- Gordon, M. A. and Cato, T.: 1972, *Astrophys. J.* **176**, 587.
- Gordon, M. A. and Churchwell, E.: 1970, *Astron. Astrophys.* **9**, 307.
- Gordon, M. A. and Gottesman, S. T.: 1971, *Astrophys. J.* **168**, 361.
- Gordon, M. A., Holder, B. P., Jisonna, L. J., Jorgenson, R. A. and Strelnitski, V. S.: 2001, *Astrophys. J.* **559**, 402.
- Gordon, M. A. and Meeks, M. L.: 1967, *Astrophys. J.* **149**, L21.

- Gordon, M. A. and Walmsley, C. M.: 1990, *Astrophys. J.* **365**, 606.
- Gordon, W.: 1929, *Ann. der Phys.* **2**, 1031.
- Goss, W. M., Kalberla, P. M. W. and Dickel, H. R.: 1984, *Astron. Astrophys.* **139**, 317.
- Gottesman, S. T. and Gordon, M. A.: 1970, *Astrophys. J.* **162**, L93.
- Graf, U. U., Eckart, A., Genzel, R., Harris, A. I., Poglitsch, A., Russell, A. P. G. and Stutzki, J.: 1993, *Astrophys. J.* **405**, 249.
- Greisen, E. W.: 1973, *Astrophys. J.* **184**, 363.
- Greve, A.: 1975, *Sol. Phys.* **40**, 329.
- Greve, A.: 1977, *Sol. Phys.* **52**, 417.
- Greve, A.: 1992, Private communication to J. W. M. Baars.
- Griem, H. R.: 1960, *Astrophys. J.* **149**, 883.
- Griem, H. R.: 1967, *Astrophys. J.* **148**, 547.
- Griem, H. R.: 1974, *Spectral line broadening by plasmas*, Vol. 39 of *Pure and applied physics*, Academic Press, New York.
- Griem, H. R.: 2005, *Astrophys. J.* **609**, L133.
- Griem, H. R., Kolb, A. C. and Shen, K. Y.: 1959, *Phys. Rev.* **116**, 4.
- Gudnov, V. M. and Sorochenko, R. L.: 1967, *Astron. Zh.* **44**, 1001. English translation: 1968 *Sov. Astron. – AJ*, 11:805.
- Guilloteau, S.: 1988, Spectral Line Calibration on the 30-m and 15-m Antennas, *Technical report*, IRAM.
- Gulyaev, S. A. and Nefedov, S. A.: 1989, *Astron. Nachr.* **310**(5), 403.
- Gulyaev, S. A. and Sorochenko, R. L.: 1974, *Astron. Zh.* **51**, 1237. English translation: 1974 *Sov. Astron. – AJ*, 18:737.
- Gulyaev, S. A. and Sorochenko, R. L.: 1983, The Catalogue of Radio Recombination Lines, *Technical Report 145, 146, and 168*, Lebedev. Phys. Inst.
- Gulyaev, S. A., Sorochenko, R. L. and Tsivilev, A. P.: 1997, *Pis'ma Astron. Zh.* **23**, 3. English translation: 1997 *Astron. Lett.* 23:165.
- Hamann, F. and Simon, M.: 1986, *Astrophys. J.* **311**, 909.
- Hamann, F. and Simon, M.: 1988, *Astrophys. J.* **327**, 876.
- Hart, L. and Pedlar, A.: 1976, *Mon. Not. R. Astron. Soc.* **176**, 547.
- Hart, L. and Pedlar, A.: 1980, *Mon. Not. R. Astron. Soc.* **193**, 781.
- Harvey, P. M., Thronson, H. A. and Gatley, I.: 1979, *Astrophys. J.* **231**, 115.
- Heiles, C.: 1994, *Astrophys. J.* **436**, 720.
- Heiles, C., Reach, W. T. and Koo, B.-C.: 1996, *Astrophys. J.* **466**, 191.
- Herrmann, F., Madden, S. C., Nikola, T., Poglitsch, A., Timmermann, R., Geis, N., Townes, C. H. and Stacey, G. J.: 1997, *Astrophys. J.* **481**, 343.
- Hewish, A., Bell, S. J., Pilkington, J. D. H., Scott, P. F. and Collins, R. A.: 1968, *Nature* **217**, 709.
- Hill, J. K.: 1977, *Astrophys. J.* **212**, 692.
- Hjellming, R. M.: 1999, in A. N. Cox (ed.), *Allen's Astrophysical Quantities*, fourth edn, Springer Verlag, New York.
- Hjellming, R. M., Gordon, C. P. and Gordon, K. J.: 1969, *Astron. Astrophys.* **2**, 202.
- Hjellming, R. M., Wade, C. M., Vandenberg, N. R. and Newell, R. T.: 1979, *Astron. J.* **84**, 1619.
- Hjerting, F.: 1938, *Astrophys. J.* **88**, 508.
- Hoang-Binh, D.: 1968, *Astrophys. Lett.* **2**, 231.
- Hoang-Binh, D.: 1972, *Mém. Roy. Soc. Liège* **3**, 367.
- Hoang-Binh, D.: 1982, *Astron. Astrophys.* **112**, L3.
- Hoang-Binh, D.: 1990, *Astron. Astrophys.* **238**, 449.
- Hoang-Binh, D. and Walmsley, C. M.: 1974, *Astron. Astrophys.* **35**, 49.
- Hodge, P. W. and Kennicutt, R. C.: 1983, *Astrophys. J.* **267**, 563.
- Hogerheijde, M. R., Jansen, D. J. and van Dishoeck, E. F.: 1995, *Astron. Astrophys.* **294**, 792.
- Höglund, B. and Mezger, P. G.: 1965, *Science* **150**, 339.
- Hollenbach, D.: 2002, Private communication to M. A. Gordon.

- Hollenbach, D., Johnstone, D., Lizano, S. and Shu, F.: 1994, *Astrophys. J.* **428**, 654.
- Howe, J. E.: 1999, Private communication to R. L. Sorochenko.
- Howe, J. E., Jaffe, D. T., Genzel, R. and Stacey, G. J.: 1991, *Astrophys. J.* **373**, 158.
- Hughes, M. P., Thompson, A. R. and Colvin, R. S.: 1971, *Astrophys. J., Suppl. Ser.* **23**, 323.
- Humason, M. L. and Wahlquist, H. D.: 1955, *Astron. J.* **60**, 254.
- Humphreys, C. J.: 1953, *J. Res. Nat. Bur. Standards* **50**.
- Hunten, D. M., Roach, F. E. and Chamberlain, J. W.: 1956, *J. Atmos. Terr. Phys.* **8**, 345.
- Inglis, D. R. and Teller, E.: 1939, *Astrophys. J.* **90**, 439.
- Innanen, K. A.: 1973, *Astrophys. Space. Sci.* **22**, 343.
- Jackson, P. D. and Kerr, F. J.: 1971, *Astrophys. J.* **168**, 723.
- Jackson, P. D. and Kerr, F. J.: 1975, *Astrophys. J.* **196**, 723.
- Jaffe, D. T. and Pankonin, V.: 1978, *Astrophys. J.* **226**, 869.
- Jaffe, D. T., Zhou, S., Howe, I. E. and Stacey, G. J.: 1994, *Astrophys. J.* **436**, 203.
- Jefferies, J. T.: 1968, *Spectral Line Formation*, Blaisdell, Waltham.
- Jewell, P. R.: 1990, NRAO 12m User's Manual, *Internal report*, National Radio Astronomy Observatory.
- Kantharia, N. G. and Anantharamaiah, K. R.: 2000, in A. P. Rao and G. Swarup (eds), *The Universe at Low Radio Frequencies*, Vol. In Press, IAU Symposium 199, Astron. Soc. Pacific, Provo, Utah, p. 337.
- Kantharia, N. G., Anantharamaiah, K. R. and Payne, H. E.: 1998, *Astrophys. J.* **506**, 758.
- Kardashev, N. S.: 1959, *Astron. Zh.* **36**, 838. English translation: 1960 Sov. Astron. – AJ **3**:813.
- Kardashev, N. S.: 2002, Email to M. A. Gordon.
- Kawamura, J. and Masson, C.: 1996, *Astrophys. J.* **461**, 282.
- Khersonskii, V. K. and Varshalovich, D. A.: 1980, *Astron. Zh.* **57**, 621. English translation: 1980 Sov. Astron. – AJ. **24**:359.
- Kinman, T. D.: 1959, *Mon. Not. R. Astron. Soc.* **119**, 559.
- Kitaev, V. V., Smirnov, G. T., Sorochenko, R. L. and Lekht, E. E.: 1994, *Turkish J. Physics* **18**, 908.
- Knapp, G. R., Brown, R. L., Kuiper, T. B. N. and Kakar, R. K.: 1976, *Astrophys. J.* **204**, 781.
- Kogan, V. I., Lisitsa, V. S. and Sholin, G. V.: 1987, in B. B. Kadomtcev (ed.), *Review of Plasma Physics*, Vol. 13, Consultant Bureau, New York, London, p. 261.
- Konovalenko, A. A.: 1984, *Pis'ma Astron. Zh.* **10**, 846. English translation: 1984 Sov. Astron. Lett. **10**:353.
- Konovalenko, A. A.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 175.
- Konovalenko, A. A.: 1995, Private communication to R. L. Sorochenko.
- Konovalenko, A. A. and Sodin, L. G.: 1979, *Pis'ma Astron. Zh.* **5**, 663. English translation: 1979 Sov. Astron. Lett., **5**:355.
- Konovalenko, A. A. and Sodin, L. G.: 1980, *Nature* **283**, 360.
- Konovalenko, A. A. and Sodin, L. G.: 1981, *Nature* **294**, 135.
- Koo, B.-C., Heiles, C. and Reach, W. T.: 1992, *Astrophys. J.* **390**, 108.
- Krügel, E. and Tenorio-Tagle, G.: 1978, *Astron. Astrophys.* **70**, 51.
- Krügel, E., Thum, C., Martín-Pintado, J. and Pankonin, V.: 1982, *Astron. Astrophys. Suppl. Ser.* **48**, 345.
- Kurucz, R. L.: 1979, *Astrophys. J., Suppl. Ser.* **40**, 1.
- Kutner, M. L. and Ulich, B. L.: 1981, *Astrophys. J.* **250**, 341.
- Kwok, S.: 1999, *The Origin and Evolution of Planetary Nebulae*, number 31 in *Cambridge Astrophysics Series*, Cambridge University Press, Cambridge.
- Lada, C. J.: 1987, in M. Peimbert and J. Jugaku (eds), *Star Formation, Proc. IAU Symp. 115*, IAU, Reidel, Dordrecht, p. 1.
- Lamers, H. J. G. and Cassinelli, J. P.: 1999, *Introduction to Stellar Winds*, Cambridge University Press, Cambridge.

- Lang, K. R. and Lord, S. D.: 1976, *Mon. Not. R. Astron. Soc.* **175**, 217.
- Launay, I. M. and Roueff, E.: 1977a, *J. Phys. B* **10**, 879.
- Launay, I. M. and Roueff, E.: 1977b, *Astron. Astrophys.* **56**, 289.
- Lekht, E. E., Smirnov, G. T. and Sorochenko, R. L.: 1989, *Pis'ma Astron. Zh.* **15**, 396.
English translation: 1989 *Sov. Astron. Lett.* 15:171.
- Lichten, S. M., Rodríguez, L. F. and Chaisson, E. J.: 1979, *Astron. Astrophys.* **229**, 524.
- Lilley, A. E., Menzel, D. H., Penfield, H. and Zuckerman, B.: 1966, *Nature* **209**, 468.
- Lilley, A. E. and Palmer, P.: 1968, *Astrophys. J., Suppl. Ser.* **16**(144), 143.
- Lilley, A. E., Palmer, P., Penfield, H. and Zuckerman, B.: 1966, *Nature* **211**, 174.
- Lin, C. C. and Shu, F. H.: 1964, *Astrophys. J.* **140**, 646.
- Linblad, P. and Langebartel, R. G.: 1953, *Stokholm Obs. Ann.* **17**, 6.
- Lis, D. C., Keene, J. and Schilke, P.: 1997, in W. B. Latter, S. J. E. Radford, P. R. Jewell, J. G. Mangum and J. Bally (eds), *CO: 25 Years of Millimeter Wave Spectroscopy*, Kluwer, Dordrecht, p. 128.
- Lockman, F. J.: 1976, *Astrophys. J.* **209**, 429.
- Lockman, F. J.: 1980, in P. A. Shaver (ed.), *Radio Recombination Lines*, Riedel, Dordrecht, p. 185.
- Lockman, F. J.: 1989, *Astrophys. J., Suppl. Ser.* **71**, 469.
- Lockman, F. J.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 225.
- Lockman, F. J. and Brown, R. L.: 1975a, *Astrophys. J.* **201**, 134.
- Lockman, F. J. and Brown, R. L.: 1975b, *Astrophys. J.* **201**, 134.
- Lockman, F. J., Pisano, D. J. and Howard, G. J.: 1996, *Astrophys. J.* **472**, 173.
- Loren, R. B., Plambeck, R. L., Davis, J. H. and Snell, R. L.: 1981, *Astrophys. J.* **245**, 495.
- Lorentz, H. A.: 1906, *Proc. Acad. Sci. Amsterdam* **8**, 591.
- Luhman, M. I., Jaffe, D. T., Sternberg, A., Herrmann, F. and Poglitsch, A.: 1997, *Astrophys. J.* **482**, 298.
- Makuiti, S., Shibai, H., Okuda, H., Nakagawa, T., Matsuhara, H., Hiromoto, N. and Okumura, K.: 1996, *Publ. Astron. Soc. Jpn.* **48**, L71.
- Martín-Pintado, J., Bachiller, R., Thum, C. and Walmsley, C. M.: 1989, *Astron. Astrophys.* **215**, L13.
- Martín-Pintado, J., Bujarrabal, V., Bachiller, R., Gómez-González, J. and Planesas, P.: 1988, *Astron. Astrophys.* **197**, L15.
- Masson, C. R.: 1986, *Astrophys. J.* **302**, L30.
- Mathis, J. S.: 1971, *Astrophys. J.* **167**, 261.
- Mathis, J. S.: 1980, in P. A. Shaver (ed.), *Radio Recombination Lines*, Riedel, Dordrecht, p. 269.
- Matthews, H. E., Pedlar, A. and Davies, R. D.: 1973, *Mon. Not. R. Astron. Soc.* **165**, 149.
- Mauersberger, R., Guélin, M., Martín-Pintado, J., Thum, C., Hein, H. and Navarro, S.: 1989, *Astron. Astrophys. Suppl. Ser.* **79**, 217.
- Mazing, M. A. and Wrubleskaja, N. A.: 1966, *Sov. Phys. - JETP* **50**, 343.
- McGee, R. X. and Gardner, F. F.: 1967, *Nature* **213**, 579.
- McGee, R. X. and Newton, L. M.: 1981, *Mon. Not. R. Astron. Soc.* **196**, 889.
- McKee, C. F. and Ostriker, J. P.: 1977, *Astrophys. J.* **218**, 148.
- Meaburn, J. and Walsh, J. R.: 1980, *Mon. Not. R. Astron. Soc.* **191**, 5.
- Meeks, M. L. (ed.): 1976, *Astrophysics, Part C: Radio Observations*, Vol. 12 of *Methods of Experimental Physics*, Academic Press, New York, p. 308.
- Menzel, D. H.: 1968, *Nature* **218**, 756.
- Menzel, D. H.: 1969, *Astrophys. J., Suppl. Ser.* **18**(161), 221.
- Menzel, D. H. and Pekeris, C. L.: 1935, *Mon. Not. R. Astron. Soc.* **96**, 77.
- Merrill, P. W. and Burwell, C. G.: 1933, *Astrophys. J.* **78**, 87.
- Mezger, P. G.: 1960, Meßprogramme bei der Wellenlänge 11 cm am 25m-Radioteleskop Stockert. I. Radioteleskop und Meßtechnik bei 2.7 GHz, *Technical Report 59*, Veröff. Univ. Sternwarte Bonn.

- Mezger, P. G.: 1965, Letter to R. L. Sorochenko.
- Mezger, P. G.: 1970, in W. Becker and G. Contopoulos (eds), *The Spiral Structure of Our Galaxy*, IAU Symp. 38, Reidel, Dordrecht, p. 107.
- Mezger, P. G.: 1978, *Astron. Astrophys.* **70**, 565.
- Mezger, P. G.: 1980, in P. A. Shaver (ed.), *Radio Recombination Lines*, Reidel, Dordrecht, p. 81.
- Mezger, P. G.: 1992, *Blick in das kalte Weltall: Protosterne, Staubscheiben und Löcher*, Deutsche Verlag-Anstalt, Stuttgart.
- Mezger, P. G.: 1994, Personal letter to M. A. Gordon.
- Mezger, P. G., Chini, R., Kreysa, E., Wink, J. E. and Salter, C. J.: 1988, *Astron. Astrophys.* **191**, 44.
- Mezger, P. G. and Ellis, S. A.: 1968, *Astrophys. Lett.* **1**, 159.
- Mezger, P. G. and Höglund, B.: 1967, *Astrophys. J.* **147**, 579.
- Mezger, P. G., Pankonin, V., Schmidt-Burgk, J., Thum, C. and Wink, J.: 1979, *Astron. Astrophys.* **80**, L3.
- Mezger, P. G. and Smith, L. F.: 1975, in E. K. Karadze (ed.), *Proceedings of the 3rd European Astronomy Meeting*, Acad. Sci. Georgian SSR, Tbilisi, p. 369.
- Mezger, P. G., Smith, L. F. and Churchwell, E.: 1974, *Astron. Astrophys.* **32**, 269.
- Mihalas, D.: 1978, *Stellar Atmospheres*, second edn, W. H. Freeman, San Francisco, California.
- Minaeva, L. A., Sobelman, I. I. and Sorochenko, R. L.: 1967, *Astron. Zh.* **44**, 995.
- Miranda, L. F., Torrelles, J. M. and Eiroa, C.: 1995, *Astrophys. J.* **446**, L39.
- Mochizuki, K. and Nakagawa, T.: 2000, *Astrophys. J.* **535**, 118.
- Mohr, P. J. and Taylor, B. N.: 1999, Codata recommended values of the fundamental physical constants: 1998, *J. Phys. Chem. Ref. Data* **28**(6).
- Moran, J. M.: 1994, in R. Genzel and A. I. Harris (eds), *The Nuclei of Normal Galaxies*, Kluwer, Dordrecht, p. 475.
- Moran, J. M.: 2002, Email to M. A. Gordon.
- Muller, C. A. and Oort, J. H.: 1951, *Nature* **168**, 357.
- Münch, G.: 1957, *Astrophys. J.* **125**, 42.
- Murcray, F. J., Goldman, A., Murcray, F. H., Bradford, C. M., Murcray, D. G., Coffey, M. T. and Mankin, W. G.: 1981, *Astrophys. J.* **247**, L97.
- Nakagawa, T., Yui, Y. Y., Doi, Y., Okuda, H., Shibai, H., Mochizuki, K., Nishimura, T. and Low, F. J.: 1998, *Astrophys. J., Suppl. Ser.* **115**, 259.
- Natta, A., Walmsley, C. M. and Tielens, A. G. G. M.: 1994, *Astrophys. J.* **428**, 209.
- Odegard, N.: 1985, *Astrophys. J., Suppl. Ser.* **57**, 571.
- O'Dell, C. R.: 1966, *Astrophys. J.* **143**, 168.
- Oks, E.: 2004, *Astrophys. J.* **609**, L25.
- Olmon, F. M.: 1975, *Astron. Astrophys.* **39**, 217.
- Onello, J. S. and Phillips, J. A.: 1995, *Astrophys. J.* **448**, 727.
- Onello, J. S., Phillips, J. A., Benaglia, P., Goss, W. M. and Terzian, Y.: 1994, *Astrophys. J.* **426**, 249.
- Onello, J. S., Phillips, J. A. and Terzian, Y.: 1991, *Astrophys. J.* **383**, 693.
- Oster, L.: 1961, *Rev. Mod. Phys.* **33**, 525.
- Oster, L.: 1970, *Astron. Astrophys.* **9**, 318.
- Osterbrock, D. E.: 1989, *Astrophysics of Gaseous Nebulae and Active Galaxies*, University Science Books, Mill Valley, California.
- Osterbrock, D. E. and Flather, E.: 1959, *Astrophys. J.* **129**, 26.
- Palmer, P.: 1968, *Radio Frequency Recombination Spectra of Diffuse Nebulae*, Ph.d dissertation, Harvard University.
- Palmer, P.: 2001, Email message to M. A. Gordon.
- Palmer, P. and Zuckerman, B.: 1966, *Nature* **209**, 1118.
- Palmer, P., Zuckerman, B., Penfield, H., Lilley, A. E. and Mezger, P. G.: 1967, *Nature* **215**, 40.
- Panagia, N.: 1979, *Mem. Soc. Astron. Ital.* **50**, 79.

- Pankonin, V.: 1980, in P. A. Shaver (ed.), *Radio Recombination Lines*, Reidel, Dordrecht, p. 111.
- Pankonin, V., Walmsley, C. M. and Thum, C.: 1980, *Astron. Astrophys.* **89**, 173.
- Pankonin, V., Walmsley, C. M., Wilson, T. L. and Thomasson, P.: 1977, *Astron. Astrophys.* **57**, 341.
- Parijskij, Y. N.: 2002, Email to M. A. Gordon.
- Payne, H. E., Anantharamaiah, K. R. and Erickson, W. C.: 1989, *Astrophys. J.* **341**, 890.
- Payne, H. E., Anantharamaiah, K. R. and Erickson, W. C.: 1994, *Astrophys. J.* **430**, 690.
- Pedlar, A. and Davies, R. D.: 1972, *Mon. Not. R. Astron. Soc.* **159**, 129.
- Pedlar, A., Davies, R. D., Hart, L. and Shaver, P. A.: 1978, *Mon. Not. R. Astron. Soc.* **182**, 473.
- Peimbert, M.: 1967, *Astrophys. J.* **150**, 825.
- Peimbert, M.: 1979, in W. B. Burton (ed.), *The Large-Scale Characteristics of the Galaxy. IAU Symp. No. 84*, Reidel, Dordrecht, p. 307.
- Peimbert, M. and Goldsmith, D. W.: 1972, *Astron. Astrophys.* **19**, 398.
- Peimbert, M. and Spinrad, H.: 1970, *Astrophys. J.* **159**, 809.
- Peimbert, M. and Torres-Peimbert, S.: 1977, *Mon. Not. R. Astron. Soc.* **179**, 217.
- Penfield, H., Palmer, P. and Zuckerman, B.: 1967, *Astrophys. J.* **148**, 25.
- Pengelly, R. M. and Seaton, M. J.: 1964, *Mon. Not. R. Astron. Soc.* **127**, 165.
- Petuchowski, S. J. and Bennet, C. L.: 1993, *Astrophys. J.* **405**, 591.
- Pfund, A. H.: 1924, *J. Opt. Soc. Am. A* **9**, 193.
- Phookun, B., Anantharamaiah, K. R. and Goss, W. M.: 1998, *Mon. Not. R. Astron. Soc.* **295**, 156.
- Pikelner, S. B.: 1967, *Astron. Zh.* **44**, 915. English translation: 1968 Sov. Astron. Lett. 11:737.
- Planesas, P., Martín-Pintado, J. and Serabyn, E.: 1992, *Astrophys. J.* **386**, L23.
- Ponomarev, V. O.: 1994, *Pis'ma Astron. Zh.* **20**, 184. English translation: 1994 Astron. Lett. 12:155.
- Ponomarev, V. O., Smith, H. A. and Strel'nitski, V. S.: 1994, *Astrophys. J.* **424**, 976.
- Ponomarev, V. O. and Sorochenko, R. L.: 1992, *Pis'ma Astron. Zh.* **18**, 541. English translation: 1992 Sov. Astron. Lett. 18:215.
- Pottasch, S. R.: 1984, *Planetary Nebulae*, Vol. 107 of *Astrophysics and Space Science Library*, Reidel, Dordrecht.
- Price, R. M.: 2002, Private communication to M. A. Gordon.
- Puxley, P. J., Brand, P. W. J. L., Moore, T. J., Mountain, C. M., Nakai, N. and Yamashita, T.: 1989, *Astrophys. J.* **345**, 163.
- Puxley, P. J., Brand, P. W. J. L., Moore, T. J. T., Mountain, C. M. and Nakai, N.: 1991, *Mon. Not. R. Astron. Soc.* **248**, 585.
- Reber, G.: 1944, *Astrophys. J.* **100**, 279.
- Reber, G. and Greenstein, J. L.: 1947, *Observatory* **67**, 15.
- Recillas-Cruz, E. and Peimbert, M.: 1970, *Bol. Obs. Tonantz. Tacub.* **35**, 247.
- Reifenstein, E. C., Wilson, T. L., Burke, B. F. and Altenhoff, W. J.: 1970, *Astron. Astrophys.* **4**, 357.
- Reynolds, R. J.: 1984, *Astrophys. J.* **282**, 191.
- Reynolds, R. J.: 1990, in S. Bowyer and C. Leinert (eds), *Galactic and Extragalactic Background Radiation, Proc. IAU Symp. No. 139*, Kluwer, Dordrecht, p. 157.
- Reynolds, R. J.: 1991, *Astrophys. J.* **372**, L17.
- Reynolds, R. J.: 1993, in J. P. Cassinelli and E. B. Churchwell (eds), *Massive Stars: Their Lives in the Interstellar Medium*, Vol. CS-35 of *Conference Proceedings*, Astron. Soc. Pacific, San Francisco, p. 338.
- Reynolds, S. P.: 1988, in G. L. Verschuur and K. I. Kellermann (eds), *Galactic and Extragalactic Radio Astronomy*, Springer-Verlag, New York, p. 439.
- Roberts, W. W.: 1972, *Astrophys. J.* **173**, 259.
- Roberts, W. W.: 1975, *Vistas Astron.* **19**, 91.

- Roberts, W. W.: 2001.
- Robinson, B.: 2001.
- Roelfsema, P. R., Goss, W. M. and Geballe, T. R.: 1988, *Astron. Astrophys.* **207**, 132.
- Roelfsema, P. R., Goss, W. M. and Geballe, T. R.: 1989, *Astron. Astrophys.* **222**, 347.
- Roelfsema, P. R., Goss, W. M. and Mallik, D. C. V.: 1992, *Astrophys. J.* **394**, 188.
- Roelfsema, P. R., Goss, W. M., Whiteoak, J. B., Gardner, F. F. and Pankonin, V.: 1987, *Astron. Astrophys.* **175**, 219.
- Russell, R. W., Melnick, G., Gull, G. E. and Harwit, M.: 1980, *Astrophys. J.* **240**, L99.
- Rydberg, J. R.: 1890, *Zs. Phys. Chem. (Leipzig)* **5**(227).
- Salem, M. and Brocklehurst, M.: 1979, *Astrophys. J., Suppl. Ser.* **39**, 633.
- Sanders, D. B., Scoville, N. Z. and Soifer, B. T.: 1991, *Astrophys. J.* **370**, 158.
- Schmidt, M.: 1965, Rotation parameters and distribution of mass in the galaxy, in A. Blaauw and M. Schmidt (eds), *Galactic Structure*, Vol. V of *Stars and Stellar Systems*, University of Chicago Press, Chicago, chapter 65, p. 513.
- Seauquist, E. R. and Bell, M. B.: 1977, *Astron. Astrophys.* **60**, L1.
- Seauquist, E. R., Carlstrom, J. E., Bryant, P. M. and Bell, M. B.: 1996, *Astrophys. J.* **465**, 691.
- Seauquist, E. R., Kerton, C. R. and Bell, M. B.: 1994, *Astrophys. J.* **429**, 612.
- Seaton, M. J.: 1959, *Mon. Not. R. Astron. Soc.* **119**, 90.
- Seaton, M. J.: 1964, *Mon. Not. R. Astron. Soc.* **127**, 177.
- Sejnowski, T. J. and Hjellming, R. M.: 1969, *Astrophys. J.* **156**, 915.
- Shaver, P. A.: 1975, *Pramana* **5**, 1.
- Shaver, P. A.: 1976a, *Astron. Astrophys.* **49**, 149.
- Shaver, P. A.: 1976b, *Astron. Astrophys.* **49**, 1.
- Shaver, P. A.: 1978, *Astron. Astrophys.* **68**, 97.
- Shaver, P. A.: 1980, *Astron. Astrophys.* **91**, 279.
- Shaver, P. A., Churchwell, E. and Rots, A. H.: 1977, *Astron. Astrophys.* **55**, 435.
- Shaver, P. A., Churchwell, E. and Walmsley, C. M.: 1978, *Astron. Astrophys.* **64**, 1.
- Shaver, P. A., McGee, R. X., Newton, L. M., Danks, A. C. and Pottasch, S. R.: 1983, *Mon. Not. R. Astron. Soc.* **204**, 53.
- Shaver, P. A., Pedlar, A. and Davies, R. D.: 1976, *Mon. Not. R. Astron. Soc.* **177**, 45.
- Shibai, H., Okuda, H., Nakagawa, T., Matsuhara, H., Maihara, T., Mizutani, K., Kobayashi, Y., Hiromoto, N., Nishimura, T. and Low, F. J.: 1991, *Astrophys. J.* **374**, 522.
- Shimabukuro, F. J. and Wilson, W. J.: 1973, *Astrophys. J.* **183**, 1025.
- Shklovsky, I. S.: 1956a, *Astron. Zh.* **33**, 315.
- Shklovsky, I. S.: 1956b, *Kosmicheskoe Radioisluichenie*, Gostechizdat, Moscow. see Shklovsky (1960) for the English translation.
- Shklovsky, I. S.: 1960, *Cosmic Radio Waves*, Harvard Univ. Press, Cambridge. Translated by R. B. Rodman and C. M. Varsavsky.
- Silverglate, P. R.: 1984, *Astrophys. J.* **278**, 604.
- Silverglate, P. R. and Terzian, Y.: 1978, *Astrophys. J.* **224**, 437.
- Simonson, S. C. and Mader, G. L.: 1973, *Astron. Astrophys.* **27**, 337.
- Simpson, J. P.: 1973a, *Astrophys. Space Sci.* **20**, 187.
- Simpson, J. P.: 1973b, *Proc. Ast. Soc. Pac.* **85**, 479.
- Smirnov, G. T.: 1985, PhD thesis, Lebedev Physical Institute, Moscow, USSR.
- Smirnov, G. T., Kitaev, V. V., Sorochenko, R. L. and Schegolev, A. F.: 1996, *Ann. Sess. of Sci. Council Astrocosmic. Ctr.*, P. N. Lebedev Phys. Inst., Moscow, p. 3.
- Smirnov, G. T., Sorochenko, R. L. and Pankonin, V.: 1984, *Astron. Astrophys.* **135**, 116.
- Smirnov, G. T., Sorochenko, R. L. and Walmsley, C. M.: 1995, *Astron. Astrophys.* **300**, 923.
- Smith, H. A., Strelitski, V. S., Miles, J. W., Kelley, D. M. and Lacy, J. H.: 1997, *Astron. J.* **114**, 2658.
- Smith, M. G. and Weedman, D. W.: 1970, *Astrophys. J.* **160**, 65.
- Sobelman, I. I.: 1992, *Atomic Spectra and Radiative Transitions*, Vol. 12 of *Springer Series on Atoms and Plasmas*, second edn, Springer, New York.

- Sobelman, I. I., Vainshtein, L. A. and Yukov, E. A.: 1995, *Excitation of Atoms and Broadening of Spectral Lines*, Vol. 15 of *Springer Series on Atomic, Optical and Plasma Physics*, second edn, Springer, New York.
- Sorochenko, R. L.: 1965, *Trudy Fiz. Inst., P. N. Lebedeva Akad. Nauk USSR* **28**, 90. English translation: 1966, *Proc. P. N. Lebedev Phys. Inst.* 28:65.
- Sorochenko, R. L.: 1996, *Astron. Astrophys. Trans.* **11**, 199.
- Sorochenko, R. L. and Berulis, I. I.: 1969, *Ap. Letters* **4**, 173.
- Sorochenko, R. L. and Borodzich, E. V.: 1965, *Dokl. Akad. Nauk SSSR* **163**, 603. English translation: 1966, *Sov. Phys. – Dokl.* 10, 588.
- Sorochenko, R. L. and Borodzich, E. V.: 1966, *Trans. IAU XIIB*, 360. Paper presented by Vitkevich at the XII General Assembly of the IAU in Hamburg, Germany, 1964.
- Sorochenko, R. L., Puzanov, V. A., Salomonovich, A. E. and Steinschleiger, V. B.: 1969, *Astrophys. Lett.* **3**, 7.
- Sorochenko, R. L., Rydbeck, G. and Smirnov, G. T.: 1988, *Astron. Astrophys.* **198**, 233.
- Sorochenko, R. L. and Smirnov, G. T.: 1987, *Pis'ma Astron. Zh.* **13**, 191. English translation: 1987 *Sov. Astron. Lett.* 13:77.
- Sorochenko, R. L. and Smirnov, G. T.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 189.
- Sorochenko, R. L., Smirnov, G. T. and Lekht, E. E.: 1991, in Y. N. Parijskij (ed.), *Proceedings of the XXXIII All-Union Conference of Galactic and Extragalactic Radioastronomy*, Physical-Technical Inst., Ilim, Ashkhabad, p. 111.
- Sorochenko, R. L. and Tsivilev, A. P.: 2000, *Astron. Zh.* **77**, 488. English translation: 2000 *Astron. Reports* 44:426.
- Sorochenko, R. L. and Walmsley, C. M.: 1991, *Astron. Astrophys. Trans.* **1**, 31.
- Spitzer, L.: 1978, *Physical Processes in the Interstellar Medium*, Wiley, New York.
- Spitzer, L. and Tomasko, M. G.: 1968, *Astrophys. J.* **152**, 971.
- Stacey, G. J., Viscuso, P. J., Fuller, C. E. and Kurtz, N. T.: 1985, *Astrophys. J.* **289**, 803.
- Stark, J.: 1913, *Sitzungber. Kungl. Akad. Wiss.* p. 932.
- Stepkin, S. V., Konovalenko, A. A., Kanthia, N. G. and Shankar, N. U.: 2007, *Mon. Not. R. Astron. Soc.* **374**, 852.
- Storey, P. J. and Hummer, D. G.: 1995, *Mon. Not. R. Astron. Soc.* **272**, 41.
- Strelnitski, V. S., Haas, M. R., Smith, H. A., Erickson, E. F., Colgan, S. W. J. and Holtenbach, D. J.: 1996, *Science* **272**, 1459.
- Strelnitski, V. S., Ponomarev, V. O. and Smith, H. A.: 1996, *Astrophys. J.* **470**, 1118.
- Strelnitski, V. S., Smith, H. A. and Ponomarev, V. O.: 1996, *Astrophys. J.* **470**, 1134.
- Strömgren, B.: 1939, *Astrophys. J.* **89**, 526.
- Stutzki, J., Stacey, G. J., Harris, A. I., Jaffe, D. T. and Lugten, J. B.: 1988, *Astrophys. J.* **332**, 379.
- Subramanyan, R.: 1992a, *Mon. Not. R. Astron. Soc.* **254**, 292.
- Subramanyan, R.: 1992b, *Mon. Not. R. Astron. Soc.* **254**, 719.
- Subramanyan, R. and Goss, W. M.: 1996, *Mon. Not. R. Astron. Soc.* **281**, 239.
- Sullivan, W. T.: 1982, *Classics in Radio Astronomy*, D. Reidel, Dordrecht.
- Sunyaev, R. A.: 1966, *Astron. Zh.* **43**, 1237.
- Taylor, J. H. and Cordes, J. M.: 1993, *Astrophys. J.* **411**, 674.
- Taylor, J. H. and Manchester, R. N.: 1977, *Astrophys. J.* **215**, 885.
- Terzian, Y.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 141.
- Terzian, Y.: 2002, Personal Communication to M. A. Gordon.
- Terzian, Y., Higgs, L. A., MacLeod, J. M. and Doherty, L. H.: 1974, *Astron. J.* **79**, 1018.
- Terzian, Y. and Parrish, A.: 1970, *Astrophys. Letters* **5**, 261.
- Thum, C.: 1986, Temperature calibration of line observations at the 30-m telescope, *Internal report*, IRAM.

- Thum, C., Martín-Pintado, J. and Bachiller, R.: 1992, *Astron. Astrophys.* **256**, 507.
- Thum, C., Matthews, H. E., Harris, A. I., Taconi, L. J., Shuster, K. F. and Martín-Pintado, J.: 1994, *Astron. Astrophys.* **288**, L25.
- Thum, C. and Morris, D.: 1999, *Astron. Astrophys.* **344**, 923.
- Thum, C., Strelitski, V. S., Martín-Pintado, J., Matthews, H. E. and Smith, H. A.: 1995, *Astron. Astrophys.* **300**, 843.
- Tielens, A. G. G. M. and Hollenbach, D.: 1985, *Astrophys. J.* **291**, 722.
- Towle, J. P., Feldman, P. A. and Watson, J. K. G.: 1996, *Astrophys. J., Suppl. Ser.* **107**, 747.
- Troland, T. H., Crutcher, R. M. and Heiles, C.: 1985, *Astrophys. J.* **298**, 808.
- Tsvilev, A. P., Ershov, A. A., Smirnov, G. T. and Sorochenko, R. L.: 1986, *Pis'ma Astron. Zh.* **12**, 848. English translation: 1986 Sov. Astron. Lett. 12:355.
- Turner, B. E.: 1970, *Astrophys. Lett.* **6**, 99.
- Ulich, B. L. and Haas, R. W.: 1976, *Astrophys. J., Suppl. Ser.* **30**, 247.
- Vallée, J. P.: 1987a, *Astron. Astrophys.* **178**, 237.
- Vallée, J. P.: 1987b, *Astrophys. J.* **317**, 693.
- Vallée, J. P.: 1987c, *Astrophys. Space. Sci.* **133**, 275.
- Vallée, J. P.: 1987d, *Astron. J.* **92**, 204.
- Vallée, J. P.: 1989, *Astrophys. J.* **341**, 238.
- van de Hulst, H. C.: 1945, *Nederladsch Tijdschrift voor Naturkunde* **11**, 230.
- van der Werf, P. P., Goss, W. M. and Vanden Bout, P. A.: 1988, *Astron. Astrophys.* **201**, 311.
- van der Werf, P. P., Stutzki, J., Sternberg, A. and Krabbe, A.: 1996, *Astron. Astrophys.* **313**, 633.
- van Dishoeck, E. F. and Black, J. H.: 1986, *Astrophys. J., Suppl. Ser.* **62**, 109.
- Vázquez, R., Torrelles, J. M., Rodríguez, L. F., Gómez, Y., López, J. A. and Miranda, L. F.: 1999, *Astrophys. J.* **515**, 633.
- Vidal, C. R.: 1964, *Naturforsch.* **A19**, 1947.
- Vidal, C. R.: 1965, *Proc. Seventh. Intern. Conf. on Phenomena in Ionized Gases*, p. 168.
- Viner, M. R., Clarke, J. N. and Hughes, J. N.: 1976, *Astron. J.* **81**, 512.
- Voigt, W.: 1913, *Phys. Zeits.* **14**, 377.
- von Sommerfeld, A.: 1916a, *Ann. Phys. (Leipzig)* **50**, 385.
- von Sommerfeld, A.: 1916b, *Ann. Phys. (Leipzig)* **51**, 1.
- Walmsley, C. M.: 1990, *Astron. Astrophys. Suppl. Ser.* **82**, 201.
- Walmsley, C. M., Churchwell, E. and Terzian, Y.: 1981, *Astron. Astrophys.* **96**, 278.
- Walmsley, C. M. and Watson, W. D.: 1982, *Astrophys. J.* **260**, 317.
- Waltman, W. B., Waltman, E. B., Schwartz, P. R., Johnson, K. J. and Wilson, W. J.: 1973, *Astrophys. J., Lett.* **185**, L135.
- Wannier, P. G., Lichten, S. M. and Morris, M.: 1983, *Astrophys. J.* **255**, 738.
- Watson, J. K. G.: 2002, Email to M. A. Gordon.
- Watson, J. K. G.: 2007, *J. Phys. B* **39**, 1889.
- Watson, W. D., Western, L. R. and Christensen, R. B.: 1980, *Astrophys. J.* **240**, 956.
- Weedman, D. W.: 1966, *Astrophys. J.* **145**, 965.
- Weinreb, S., Barrett, A. H., Meeks, M. L. and Henry, J. C.: 1963, *Nature* **200**, 829.
- Weisskopf, V.: 1932, *Z. Phys.* **75**, 287.
- Weisskopf, V. and Wigner, E.: 1930, *Z. Phys.* **63**, 54.
- White, R. L. and Becker, R. H.: 1985, *Astrophys. J.* **297**, 677.
- Wild, J. P.: 1952, *Astrophys. J.* **115**, 206.
- Wilson, O. C., Münch, G., Flather, E. M. and Coffeen, M. F.: 1959, *Astrophys. J., Suppl. Ser.* **4**, 199.
- Wilson, T. L., Bieging, J. H. and Wilson, W. E.: 1979, *Astron. Astrophys.* **71**, 205.
- Wilson, T. L. and Filges, L.: 1990, in M. A. Gordon and R. L. Sorochenko (eds), *Radio Recombination Lines: 25 Years of Investigation*, IAU Colloquium 125, Kluwer Academic Publishers, Dordrecht, p. 105.

- Wilson, T. L., Hoang-Binh, D., Stark, A. A. and Filges, L.: 1990, *Astron. Astrophys.* **238**, 331.
- Wilson, T. L., Mauersberger, R., Muders, D., Przewodnik, A. and Olano, C. A.: 1993, *Astron. Astrophys.* **280**, 221.
- Wilson, T. L., Mezger, P. G., Gardner, F. F. and Milne, D. K.: 1970, *Astron. Astrophys.* **6**, 364.
- Wilson, T. L. and Pauls, T.: 1984, *Astron. Astrophys.* **138**, 225.
- Wink, J., Wilson, T. L. and Bieging, J. H.: 1983, *Astron. Astrophys.* **127**, 211.
- Wyrowski, F., Schilke, P., Hofner, P. and Walmsley, C. M.: 1997, *Astrophys. J.* **487**, L171.
- Zelik, M. and Lada, C. J.: 1978, *Astrophys. J.* **222**, 896.
- Zhao, J.-H., Anantharamaiah, K. R., Goss, W. M. and Viallefond, F.: 1996, *Astrophys. J.* **472**, 54.
- Zhao, J.-H., Anantharamaiah, K. R., Goss, W. M. and Viallefond, F.: 1997, *Astrophys. J.* **482**, 186.
- Zuckerman, B. and Ball, J. A.: 1974, *Astrophys. J.* **190**, 35.
- Zuckerman, B. and Palmer, P.: 1968, *Astrophys. J.* **153**, L145.

Author Index

- Accord, J. M. 129
Afflerbach, A. 129
Altenhoff, W. J. 124, 134, 137
Anantharamaiah, K. R. 100, 105, 147, 149,
152, 181, 188–190, 192, 193, 201, 202,
206, 207, 233, 234, 236, 237
Ariskin, V. I. 99
Audi, G. 284
- Baars, J. W. M. 331
Bachiller, R. 94, 96, 226, 230
Baker, J. G. 80
Balick, B. 167
Ball, J. A. 167
Balser, D. S. 164
Bania, T. M. 163, 164
Barrett, A. H. 14
Becker, R. H. 227
Becker, W. 158
Beigman, I. I. 46
Bell, M. B. 55, 231, 234, 236, 237
Bell, S. J. 144
Benaglia, P. 175
Benedettini, M. 179
Bennet, C. L. 152
Berulis, I. I. 31, 51, 134
Biegging, J. H. 134, 199, 221
Black, J. H. 212
Bochkarev, N. G. 208
Bohr, N. 115
Boreiko, R. T. 220
Borodzich, E. V. 16, 23, 24, 72
Bracewell, R. M. 25
Bradford, C. M. 220
Brand, P. W. J. L. 232
Brault, J. W. 220
- Brocklehurst, M. 82, 88, 133
Bronfman, L. 230
Brown, R. L. 122, 145, 152, 159, 175, 180
Bryant, P. M. 236, 237
Burke, B. F. 124, 134, 137
Burles, S. 162
Burton, W. B. 141
Burwell, C. G. 221
- Cameron, A. G. W. 173
Canto, J. 174
Carlstrom, J. E. 236, 237
Casse, J. L. 99, 209
Cassinelli, J. P. 229
Caswell, J. L. 139
Cernicharo, J. 230
Chaisson, E. J. 168
Chamberlain, J. W. 152
Chandrasekhar, S. 58
Chang, E. S. 220
Churchwell, E. 72, 149, 156–158
Churchwell, E. B. 129
Clark, B. G. 209
Clark, T. A. 220
Coffeen, M. F. 134
Coffey, M. T. 220
Cohen, M. 221
Collins, R. A. 144
Colvin, R. S. 144
Cox, P. 230
Crutcher, R. M. 196
- Dalgarno, A. 212
Danks, A. C. 156, 161
Davies, R. D. 49, 99, 145
Davis, G. R. 220

- DePree, C. G. 129
 Dewdney, P. E. 181
 Dickel, H. R. 194, 196, 200
 Dieter, N. H. 24
 DiGiorgio, A. M. 179
 Doi, Y. 207
 Dravskikh, A. F. 14, 16, 24
 Dravskikh, Z. V. 14, 16, 24
 Dreher, J. W. 221
 Dupree, A. K. 21, 167
- Elmegreen, B. G. 170
 Emerson, D. T. 117
 Erickson, W. C. 100, 105, 189, 190, 192, 193, 201, 202, 207
 Ershov, A. A. 99–102, 105, 189, 190, 195, 210
 Escalante, V. 174
 Ewen, H. J. 118
- Federman, S. R. 196
 Fenkart, R. 158
 Ferguson, E. 41
 Field, G. B. 143, 150, 173
 Flather, E. 49, 55, 67
 Flather, E. M. 134
 Fowler, W. A. 208
 Fuller, C. E. 205
- Gammon, R. H. 167
 Garay, G. 174, 175, 180, 215
 Gardner, F. F. 24, 124, 137, 158
 Gathier, R. 215
 Gatley, I. 221
 Gaume, R. A. 196
 Geballe, T. R. 158, 174
 Gee, C. S. 81
 Geis, N. 185
 Genzel, R. 180
 Giannani, T. 179
 Glassgold, A. E. 196
 Goldberg, L. 21
 Goldman, A. 220
 Goldreich, P. 194
 Goldsmith, D. W. 143, 150, 156
 Gómez-González, J. 159
 Gómez, Y. 174, 175, 180, 215
 Gordon, C. P. 143, 144, 150
 Gordon, K. J. 143, 144, 150
 Gordon, M. A. 72, 96, 122, 141, 143, 145, 148, 150, 152, 157, 224, 226, 227
 Goss, W. M. 158, 159, 174, 175, 181, 194, 196, 199, 200, 208, 233, 234, 236, 237
 Gottesman, S. T. 143, 145, 148, 150, 152
- Greisen, E. W. 199
 Greve, A. 219
 Griem, H. R. 9, 40–42, 44, 49
 Gudnov, V. M. 24
 Gull, G. E. 181
 Gulyaev, S. A. 82
- Habing, H. J. 143, 150
 Harris, A. I. 180
 Hart, L. 99, 152
 Harvey, P. M. 221
 Harwit, M. 181
 Haynes, R. F. 139
 Heiles, C. 153, 154, 196
 Henry, J. C. 14
 Herrmann, F. 185
 Hewish, A. 144
 Hill, J. K. 175
 Hiromoto, N. 203, 205
 Hjellming, R. M. 67, 82, 143, 144, 150, 167
 Hoang-Binh, D. 288
 Hofner, P. 129, 179, 180, 182
 Hogerheijde, M. R. 179
 Höglund, B. 17, 23, 24, 72, 134
 Hollenbach, D. 174, 182, 183, 187, 207
 Howard, G. J. 141, 149
 Howe, I. E. 185
 Howe, J. E. 180, 185
 Huchtmeier, W. 156–158
 Hughes, M. P. 144
 Humason, M. L. 136
 Hummer, D. G. 82, 98
 Hunten, D. M. 152
- Iljsov, Y. P. 99, 101, 102, 189, 190, 195
 Inglis, D. R. 8
 Innanen, K. A. 142
- Jaffe, D. T. 180, 185
 Jansen, D. J. 179
 Jefferies, J. T. 58
 Jewell, P. R. 336
 Johnston, K. J. 196
- Kalberla, P. M. W. 194, 196, 200
 Kantharia, N. G. 188, 189, 192, 193, 202, 206
 Kardashev, N. S. 9, 68
 Keene, J. 180
 Kerton, C. R. 234
 Kinman, T. D. 136
 Kitaev, V. V. 189, 192, 201, 211
 Kobayashi, Y. 203, 205
 Kogan, V. I. 42

- Kolb, A. C. 40
 Kolbasov, V. A. 14, 16, 24
 Kolotovkina, S. A. 99
 Konovalenko, A. A. 99–101, 113, 187, 189, 190
 Koo, B.-C. 153, 154
 Krabbe, A. 180
 Krügel, E. 157, 175, 176
 Kurtz, N. T. 205
 Kurtz, S. 129
 Kurucz, R. L. 156, 157
 Kwan, J. 194, 196

 Lada, C. J. 170
 Lamers, H. J. G. 229
 Launay, I. M. 183
 Lebron, M. 174
 Lekht, E. E. 99–102, 105, 112, 188–190, 192, 194, 195, 210, 211
 Lichten, S. M. 208
 Lilley, A. E. 17, 19, 23, 24, 72, 155, 165, 166, 181, 245
 Lis, D. C. 180
 Lisano, S. 180
 Lisitsa, V. S. 42
 Lizano, S. 174, 175, 180
 Lockman, F. J. 122, 141, 147, 149, 159
 Lodge, I. C. 81
 Lorentz, H. A. 25
 Lorenzetti, D. 179
 Low, F. J. 203, 205, 207
 Lugten, J. B. 180
 Luhman, M. I. 185
 Lyman, L. A. 230

 Madden, S. C. 185
 Maihara, T. 203, 205
 Mallik, D. C. V. 158, 159
 Mankin, W. G. 220
 Martín-Pintado, J. 94, 96, 157, 176, 224, 226, 230
 Masson, C. R. 217
 Mathis, J. S. 157, 161
 Matsuhara, H. 203, 205
 Matthews, H. E. 145, 224
 Mauersberger, R. 194, 196, 200
 Mazing, M. A. 107
 McConnel, D. 201, 202, 207
 McGee, R. X. 24, 134, 156, 157, 161
 Meaburn, J. 217
 Mebold, U. 237
 Meeks, M. L. 14
 Melnick, G. 181
 Menzel, D. H. 17, 19, 23, 24, 72, 80

 Merrill, P. W. 221
 Mezger, P. G. 17, 23, 24, 72, 124, 134, 137, 140, 152, 156–158, 165, 166, 181
 Milne, D. K. 124, 137
 Minaeva, L. A. 41
 Misezchnikov, G. S. 14, 16, 24
 Mizutani, K. 203, 205
 Mochizuki, K. 207
 Mohan, N. R. 237
 Moore, T. J. T. 232
 Moran, J. M. 215
 Morris, D. 229
 Morris, M. 208
 Mountain, C. M. 232
 Muders, D. 194, 196, 200
 Muller, C. A. 118
 Münch, G. 134, 199
 Murcray, D. G. 220
 Murcray, F. H. 220
 Murcray, F. J. 220

 Nakagawa, T. 203, 205, 207
 Nakai, N. 232
 Natta, A. 182
 Naylor, D. A. 220
 Nefedov, S. A. 82
 Newton, L. M. 134, 156, 157, 161
 Nikola, T. 185
 Nikulin, D. E. 14, 16, 24
 Nishimura, T. 203, 205, 207
 Nisino, B. 179
 Nollett, K. M. 162
 Noyes, R. W. 220

 O'Dell, C. R. 74
 Oks, E. 55
 Okuda, H. 203, 205, 207
 Olano, C. A. 194, 196, 200
 Olnon, F. M. 221
 Onello, J. S. 174, 175
 Oort, J. H. 118
 Osterbrock, D. E. 49, 55, 67

 Palmer, P. 19, 21, 24, 72, 155, 165, 166, 181, 245
 Pankonin, V. 52, 54, 157, 158, 167, 176
 Parijskij, Y. N. 15
 Parrish, A. 66
 Payne, H. E. 105, 188–190, 192, 193, 201
 Pedlar, A. 49, 99, 145, 152
 Peimbert, M. 55, 74, 155, 156
 Penfield, H. 17, 19, 21, 23, 24, 72, 155, 165, 166, 181
 Percival, I. C. 81

- Petuchowski, S. J. 152
 Phillips, J. A. 174, 175
 Phookun, B. 233, 234, 237
 Pikelner, S. B. 143, 150
 Pilkington, J. D. H. 144
 Pisano, D. J. 141, 149
 Poglitsch, A. 185
 Ponomarev, V. O. 82, 85, 91, 96–98, 104, 182, 192, 210, 229
 Pottasch, S. R. 156, 161
 Price, R. M. 14
 Przewodnik, A. 194, 196, 200
 Purcell, E. M. 118
 Puxley, P. J. 232
 Puzanov, V. A. 94

 Radhakrishnan, V. 100, 189, 190
 Reach, W. T. 153, 154
 Reber, G. 8
 Reeves, H. 208
 Reif, K. 237
 Reifenstein, E. C. 124, 134, 137
 Reynolds, R. J. 153
 Richards, D. 81
 Roach, F. E. 152
 Roberts, W. W. 200
 Robinson, B. 17
 Rodríguez, L. F. 174, 215
 Roelfsema, P. R. 158, 159, 174, 230
 Rood, R. T. 164
 Roueff, E. 183
 Rudnitskij, G. M. 99, 105, 190
 Russell, R. W. 181

 Salem, M. 82, 88, 133
 Salomonovich, A. E. 94
 Saraceno, P. 179
 Schegolev, A. F. 201
 Schilke, P. 179, 180, 182
 Scott, P. F. 144
 Seaquist, E. R. 231, 234, 236, 237
 Seaton, M. J. 74, 80, 82
 Sejnowski, T. J. 82
 Shaver, P. A. 99, 112, 122, 145, 156, 161, 209, 234, 237
 Shen, K. Y. 40
 Shibai, H. 203, 205, 207
 Shklovsky, I. S. 213
 Shlüter, H. 41
 Sholin, G. V. 42
 Shteinshleiger, V. B. 14, 16, 24
 Silk, J. 208
 Simpson, J. P. 122

 Smirnov, G. T. 52, 54, 99–102, 104, 112, 134, 145, 182, 188–190, 192, 194, 195, 201, 210, 211
 Smith, H. A. 85, 91, 96–98, 179, 224, 229
 Smith, L. F. 152
 Smith, M. G. 31
 Sobelman, I. I. 41
 Sodin, L. G. 99
 Solodkov, V. T. 99, 101, 102, 189, 190, 195
 Sorochenko, R. L. 9, 12, 16, 23, 24, 31, 40, 41, 51, 52, 54, 72, 82, 94, 99–102, 104, 105, 112, 134, 145, 182, 183, 186–190, 192, 194–196, 201, 210, 211
 Spinoglio, L. 179
 Spinrad, H. 155
 Spitzer, L. 144, 150, 175, 206, 208
 Stacey, G. J. 180, 185, 205
 Steinschleiger, V. B. 94
 Sternberg, A. 180, 185
 Storey, P. J. 82, 98
 Strelnitski, V. S. 85, 91, 96–98, 224, 229
 Strömgren, B. 144
 Stutzki, J. 180
 Sullivan, W. T. 9

 Teller, E. 8
 Tenorio-Tagle, G. 175
 Terzian, Y. 66, 174, 175, 215
 Thomasson, P. 167
 Thompson, A. R. 144
 Thronson, H. A. 221
 Thum, C. 94, 96, 157, 176, 224, 226, 229
 Tielsens, A. G. G. M. 174, 182, 183, 187, 207
 Timmermann, R. 185
 Tomasko, M. G. 144, 150, 208
 Torres-Peimbert, S. 55
 Townes, C. H. 185
 Troland, T. H. 196
 Tsivilev, A. P. 183, 186
 Turner, M. S. 162

 Vallée, J. P. 174
 van de Hulst, H. C. 8
 van der Werf, P. P. 180, 208
 van Dishoeck, E. F. 179
 Vanden Bout, P. A. 208
 Viallefond, F. 233, 234, 236, 237
 Viscuso, P. J. 205
 von Sommerfeld, A. 283

 Wahlquist, H. D. 136
 Walmsley, C. M. 81, 82, 94, 96, 104, 122, 167, 179, 180, 182, 187, 189, 190, 192, 196

- Walsh, J. R. 217
- Wannier, P. G. 208
- Wapstra, A. H. 284
- Watson, J. K. G. 55
- Watson, W. D. 104, 190
- Weedman, D. W. 31
- Weinreb, S. 14
- Weisskopf, V. 33
- Welch, W. J. 221
- White, G. J. 179
- White, R. L. 227
- Whiteoak, J. B. 158
- Wilcots, E. M. 199
- Wilson, O. C. 134
- Wilson, T. L. 124, 134, 137, 164, 167, 194, 196, 200
- Wink, J. 134
- Wrubleskaja, N. A. 107
- Wyrowski, F. 179, 180, 182
- Yui, Y. Y. 207
- Zhao, J.-H. 233, 234, 236, 237
- Zhou, S. 185
- Zuckerman, B. 17, 19, 21, 23, 24, 72, 155, 165–167, 181

Subject Index

- Antenna temperature, 65, 329–337
 definition of, 65
 peculiar units at short wavelengths, 65, 329, 336–337
- Astration
 definition of, 118
- Atoms
 hydrogenic
 definition of, 5
 reduced mass of, 241–243
 Rydberg constant for, *see* Rydberg constant
 size limit to, 107–113
- Black-body radiation, 60
- Bohr atom, 3–7, 112, 283–284
- Bohr radius, 37, 287
 definition of, 4
- Boltzmann equation, 67, 72, 74, 75, 79, 80, 86
- Bremsstrahlung, *see* Free-free radiation
- C II regions, *see* RRLs, carbon
- Carbon monoxide (CO), 118, 169, 175–181, 183, 194–196
 Galactic rotation curve from, 137
 Galactocentric distribution of, 141, 142
- Clumping, *see* H II regions, density inhomogeneities
 definition of, 164
- Collisions
 effect on level population, 11, 42, 79–86, 88, 297
- Cosmic rays
 intensity of, 144
 ionization by, 143, 144, 150, 208–212
- Cosmological abundance, 156–160
 definition of, 155
- Damping constant, 33
 classical, 25
 quantum mechanical, 27
- Departure coefficients, 78–86, 125, 297–327
 “blurring” of sublevels, 85–86, 93
 the “Cases” of the calculations, 80
- Dielectronic “capture”, 104, 167
- Dielectronic recombination, 104–107, 166,
 see RRLs, Solar
- Dispersion measure, 144–145, *see* Pulsars
 definition of, 144
- Doppler shift, *see* Line shift
- Dust
 selective absorption by, 161
- Einstein emission and absorption
 coefficients, 67–71
 energy density form of, 67
 specific intensity form of, 67
- Emission measure (EM), 123–126, 131, 133
 definition of, 10
- Euler’s constant, 63
- Excitation parameter (U), 123
- Extreme Low-Density component (ELD), 147, 148, 152
- Extreme Low-Density Warm Ionized Medium (ELDWIM), 149
- Flux density, 329
 definition of, 59, 329
 free-free radiation, 65–66
 spectral
 definition of, 65

- Forbidden transitions, 133
 - auroral lines, 128
 - definition of, 128
 - nebular lines, 128
- Formaldehyde (H_2CO), 167
 - resolving distances with, 139, 167, 194
- Formyl cation (HCO^+), 236
- Fourier transform, 25, 32
- Free-free radiation, 61–67
 - absorption coefficient
 - Altenhoff approximation to, 63
 - Hjellming approximation to, 64
 - Rayleigh–Jeans form of, 62–63
 - at low frequencies, 64
 - at low temperatures, 64
 - emission coefficient, 64
 - observations compared with theory, 65–66
 - spectrum of, 65–67
 - theory of, 62–64
 - transfer equation, 64–65
 - turnover frequency, 66
- Galactic rotation law, 136–137
- Galactic standard of rest, 136
- Galaxies, external, *see* Sources, special
- Galaxy, The, *see* H II regions, surveys of
 - “chimney”-like structures, 153
 - background radiation, 108
 - density waves, 197–200
 - distance ambiguity, 136–137
 - dust gradient, 161
 - evolution of, 156
 - gradient of $^3\text{He}^+/\text{H}^+$, 163–165
 - He II gradient, 160–162
 - large-scale characteristics of, 1–154
 - metallicity gradient, 162
 - rotation law, 136
 - subcentral point, 136
 - temperature gradient, 161
 - thickness of, 152–154
- Gaunt factor, 62–63
- Giant Molecular Cloud (GMC), 151
- giant molecular cloud (GMC), 149
- H II regions
 - density inhomogeneities, 50, 66
 - surveys of, 137–140
 - temperature gradients, 51, 66
 - ultracompact, 130
- Hamburg IAU
 - detection announcement, 14
- He II regions, *see* RRLs, helium
- Heisenberg Principle, 26
- Helium, *see* RRLs, helium
 - hyperfine transition, 163, *see* Spectral lines, ^3He hyperfine line
 - neutral component, 157
- Hydrogen
 - atomic neutral layers, 174–181
 - molecules, 140, 141, 169, 175–181
- Impact parameter, 33
- Inglis–Teller effect, 9
- Intensity
 - continuum
 - see* Free-free radiation, 163
 - line, 29
 - thermal, 29
- Interstellar Medium (ISM), *see* RRLs, carbon, 165–187
- ISAS
 - BICE, 203
- James Clerk Maxwell, 221
- Kirchhoff’s laws, 60, 64, 88
- Ladenburg f, *see* Oscillator strength
- Line broadening
 - early estimate, 8
 - natural, 25–28
 - pressure, *see* Line broadening, Stark
 - quasistatic, 9
 - radiation, 107–111
 - resonance, 34
- Stark, 9, 11, 19, 30–55, 94, 99–100, 102, 131–132, 281–283
 - apparent absence of, 23, 40
 - Bohr theory, 31
 - C_2 coefficient, 37–38
 - choice between approximations, 36–39
 - collision duration, 35
 - distinction between quasistatic and impact, 35–39
 - early estimate, 8
 - elastic “ion line width”, 43
 - impact, 33, 40–46
 - impact, ions, 43
 - in astronomical plasmas, 32
 - inelastic “electron line width”, 44
 - inelastic “ion line width”, 45
 - inelastic collisions, 42
 - inelastic cross sections, 44
 - level compensation, 40, 44
 - line splitting, 37
 - linear, 31, 34

- nonadiabatic, *see* Line broadening, Stark, inelastic collisions
- onset of, 24
- phase shift, 32–34
- problems with revised theory, 49
- quadratic, 31, 34
- quantum theory, 44
- revised theory for RRLs, 49
- test of revised theory, 49, 51, 53
- theory, 24, 30
- thermal, *see* Line profile, thermal, 28–30
- turbulence, 30, 134–135
- van der Waals, 34
- Line profile
 - Doppler, *see* Line profile, thermal
 - Gaussian, *see* Line profile, thermal
 - Lorentz, 25–28, 32–33, 48, 107–110
 - damping constant, 25
 - errors, 46
 - thermal, 9, 28, 29, 48, 52, 69, 94, 132–134, 173, 184, 188, 201, 211, 222, 225, 274, 281
 - derivation of, 28
 - turbulence, 28, 30–31, 122, 163
 - Voigt, 46–48, 52–54, 132, 133, 188, 189, 215
 - approximate line width, 48
 - characteristics of, 48
 - derivation of, 46
 - Voigt function, 48
- Line shift
 - Euclidean, 28
 - general relativity, 28
- Local Standard of Rest, 1
- local standard of rest, 136
- LTE, *see* Thermodynamic equilibrium
- Maxwell–Boltzmann velocities, *see* Velocity distribution, Maxwell–Boltzmann
- Mean intensity
 - definition of, 79, 107
- Metallicity
 - definition of, 129
- Observatories
 - Bell Labs
 - Holmdel, 94
 - Bonn University
 - Stockert, 17
 - CSIRO
 - Parkes, 17, 21, 201
 - DKR-1000
 - Pushchino, 99, 112, 201
 - ESA
 - ISO, 179
 - Harvard University
 - Agassiz Station, 18, 19, 155
 - IIS
 - Gauribidanur, 188, 201
 - ISAS
 - BICE, 203
 - BIRT, 203–205
 - James Clerk Maxwell
 - Mauna Kea, 95
 - Lebedev
 - Pushchino, 12, 14, 52, 94, 184, 188
 - MIT
 - Haystack, 21
 - MPfIR
 - Effelsberg, 52, 53, 167, 222, 231, 237
 - NAIC
 - Arecibo, 173
 - NASA
 - Kuiper Airborne, 95, 183
 - NEROC, 19
 - NRAO
 - Green Bank, 17–19, 105, 134, 146, 164, 166, 201
 - Kitt Peak, 94, 219
 - Socorro, 127, 158, 159, 175, 180, 199, 215, 217, 227, 232, 233
 - RIU
 - Kharkov, 99–101, 112, 189
 - SAO
 - Pulkovo, 12, 14–16
 - Space Research Center
 - Crimea, 12
 - UC Berkeley
 - Hat Creek, 21
- Opacity, *see* Optical depth
- Optical depth, *see* RRLs, line absorption
 - coefficient or Free-free radiation, absorption coefficient
 - definition of, 57
- Oscillator
 - damping constant
 - classical, 25
 - electric dipole, 25
 - spectrum, 25
- Oscillator strength, 68, 285–288
 - Menzel algorithm for, 68, 109
 - origin of the term, 68
- Parsec
 - definition of, 10
- Photodissociation region (PDR), 176, 179–181, 183, 185–187, 205, 207, 208

- Planck function
 definition of, 59–60
 frequency form, 60
 non-LTE correction to, 76–77, 88–91
 Rayleigh–Jeans approximation to, 60, 108
 wavelength form, 60
 Wien approximation to, 60
- Planetary nebulae, 133, 213–217
 distance to, 213, 215–217
 expansion rates, 217
 Galactic distribution of, 214
 morphology of, 213
 physical characteristics of, 215
 RRLs detected from, 215
 Stark broadening of RRLs, 215–217
- Protostars, 178, 179
- Pulsars, 144–145, 152–153
- Radiation transfer, 56–78
 absorption coefficient, 57
 emission coefficient, 57
 equation of, 57
 integrating factor, 57
 masers, 91, 96–99
 partial maser effect, 74–78, 90, 122
- “Radio”
 etymology of, 117
- Rayleigh unit
 definition of, 152
- RRLs
 carbon, 99–113, 165–174
 width of, 167
 definition, 2
 detectability, 7, 11, 52
 detection of, 12–21
 carbon, 21
 detection of
 helium, 21
 diffuse emission, 119–120, 141–154
 location of, 145
 temperature of, 142–145, 147–149
 Doppler shift, 14, *see* Line shift, Doppler
 helium, 9, 154–165
 in planetary nebulae, 156
 intensity, 9, 11, 52, 67–78, 86–94
 beamwidth effects, 51
 non-LTE, 74–78, 86–94
 inversion regime, 93
 line absorption coefficient, 67–69, 75–77
 line emission coefficient, 70, 76
 line frequencies, 3–6, 155, 241–243, 245–280
 hyperfine structure, 258–280
 line-to-continuum ratio, 7, 9, 77–78, 88–90
 LTE transfer equation, 71–72
 masers, *see* Radiation transfer, masers
 naming convention, 17
 net absorption coefficient, 98
 non-LTE transfer equation, 76–78
 overcooled regime, 93
 overheated regime, 93
 Solar, 218–220
 dielectronic recombination, 218–219
 Stark broadening of, 220
 Zeeman broadening of, 219–220
 sulfur, 172–174
 detection of, 168
 use in cosmological studies, 162–165
- Rydberg constant, 3–5, 241
- Saha–Boltzmann ionization equation, 68, 79, 103, 185
- Source function
 definition of, 59
 relationship to Planck function, 59, 74–78
- Sources, special
 Cassiopeia A, 99–106, 112, 113, 187, 188, 190–201, 206, 209–211
 galaxies, 231–237
 list of detections, 233
 models of, 234–237
 quasars, 232, 237
 radio type, 232
 starburst types, 237
 MWC349, *see* Stars, MWC349
 Sagittarius A, 99, 201
 Sun, *see* RRLs, Solar
- Specific intensity, 329
 definition of, 58–59
 free-free radiation, 64
 properties of, 58
 units of, 58, 59
- Spectral lines
 Balmer series, 3, 41, 115
 Brackett series, 6
 Cl III, 55
 CO, *see* Carbon monoxide
 Fraunhofer, 2
 H α , 41
 H₂, *see* Hydrogen, molecular
 H₂CO, *see* Formaldehyde
 HCO⁺, *see* Formyl cation
³He hyperfine line, 162–165
 He I, 161
 He III, 156

- Humphreys series, 7
- Lyman α , 41
- Lyman series, 3
- N II, 129
- O II, 55, 129, 185
- O III, 55, 129
- Paschen α , 41
- Paschen series, 3, 41
- Pfund series, 6
- RRL, *see* RRLs
- series limit, 3
- Stark splitting, *see* Line broadening, Stark
- Stars
 - formation of, 130, 141, 162, 170, 179, 197, 237
 - η Carina, 230
 - hydrogen burning, 162
 - MWC349, *see* Radiation transfer, masers, 94–96, 221–230
 - Sun, *see* RRLs, Solar
- Statistical equilibrium, 78–80
- Stellar atmospheres
 - blanketed, 156
 - Stellar winds, 118, 149, 159, 175, 221, 226, 228, 229
- Strömgren sphere, 147, 149, 156–158
 - definition of, 144
 - helium, 155–160
- S II regions, *see* RRLs, sulfur
- Supernova, *see* Cassiopeia A, 150, 154
 - 3C391, 143
 - classes of, 208
 - explosion of, 118, 144, 154
- TE, *see* Thermodynamic equilibrium
- Thermodynamic equilibrium, 60, 70
 - definition of, 74, 78
 - stationary models, 79
- Transition probability
 - spontaneous, 27
 - sum of, 27
- Velocity distribution
 - Maxwell–Boltzmann, 28, 38–39, 44, 62
- Weisskopf radius, 33–39
 - definition, 33